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BIANCHI TYPE-VI DARK ENERGY'S COSMOLOGICAL MODEL WITH A PERFECT FLUID IN $f(T)$ THEORY OF GRAVITY

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Abstract: This paper explores the analysis of dark energy in the context of $f(T)$ theory of gravity with a Perfect Fluid of Cosmological Model Bianchi Type-VI. The Dark energy, a hidden component driving the accelerated expansion of the universe, continues to challenge our understanding of fundamental physics. The $f(T)$ theory, an extension of teleparallel gravity, provides an alternative perspective to General Relativity, where T is the torsion scalar. The study investigates the dynamics of the accelerating universe across the established $f(T)$ model, $f(T) = \lambda T$, where λ is free Parameter. The behaviour of different cosmological parameters like pressure, energy density will be analysed. Furthermore, a detailed examination of the physical and geometrical characteristics of the model has been studied with the help of its graphs.

Keywords: Bianchi Type-VI cosmological model, Dark energy, $f(T)$ gravity.

1. Introduction

Cosmology is the field where the whole universe is studied. Our universe is actually the collection of galaxies. Cosmology studies the gigantic structure of universe. The theory of expansion and acceleration of the universe and its contents are explicitly explained by the Cosmological Models. It has been discovered that Dark Energy plays an important role in the expansion of universe. However, the driving force behind this accelerating expansion of the universe is still a subject of debate [1]. According to observations of modern astrophysics in recent years, the universe is not only accelerating but also expanding continuously after 'Big Bang'. It has been increasing from an initial state with high density and high temperature. This is proved by cosmological experiments, such as 'The measurement of type-Ia supernovae (SNeIa)', 'The cosmic microwave background (CMB)', 'large scale structure (LSS)' etc [33, 20, 26, 24, 16, 34]. The matter in our universe is field by the Dark Energy (68%) and Dark Matter (26.8%) and the remaining

(4.5%) is occupied by the other ordinary matter [19, 2, 36]. The concept of dark energy and dark matter is one of the indeterminate problems for modern cosmology. The dark energy is described by the equation of state (EoS) parameter $\omega = p/\rho$, where p and ρ represents pressure and density of dark energy. To explain the acceleration of universe, the simplest parameter for dark energy is the cosmological constant, which represents the energy density associated with vacuum ($\omega = -1$). So many researchers have investigated dark energy problem [2, 1, 36, 37].

To explain the problems like initial singularity, horizon and flatness, General theory of relativity failed. Then it was modified by introducing the term $f(R)$ in Einstein- Hilbert action. It is known as $f(R)$ theory of gravitation [28]. In order to explain how dark energy and dark matter as well as late-time acceleration exist in the universe, there exist several modified theories of gravity with different cosmological implications, such as $f(R)$, $f(T)$, $f(R,T)$, $f(G)$, $f(R,G)$, $f(Q)$, $f(Q,T)$ etc. These theories can take any form of modification to general relativity. The main goal of these theories is to provide a theory that performs in line with general relativity over small distances for astronomical masses and that neatly describes large scale observations such as the universal acceleration. Capozziello et al. (2003), Nojiri and Odintsov (2003b) and Carroll et al. (2004) provide a concise overview of $f(R)$-gravity [16]. So $f(R)$ theory of gravitation could be the key to understanding the late-time cosmic acceleration (Carroll et al. 2004) [10]. This modified gravity has recently been demonstrated to explain the Universe' late-time accelerated expansion. After $f(R)$-gravity, the more general model of modified theory of gravity $f(R,T)$, where R is Ricci Scalar and T is the trace of energy momentum tensor have proposed by Harko et al. (2011) [21]. While Modifying the General Relativity on a large scale, the scalar-tensor theories, $f(R)$ theory, $f(T)$ theory etc. are an alternate way to accommodate the current accelerating expansion of the universe. Out of these theories, the generalized teleparallel theory of gravity has presently attracted a lot of attention as a potential explanation for Dark Energy. $f(T)$ theory of gravity is based on Weitznbock geometry. Gravitation is attributed in this theory to the torsion of a space-time with zero curvature, which acts as a force [12]. In the Lagrangian of teleparallel gravity, the torsion scalar T, is substituted by its generic function $f(T)$ in this generalization [8, 7]. $f(T)$ gravity does not serve as a replacement for General Relativity, its unique formulation enables the assertion that gravity arises not from curvature but from torsion. In $f(T)$ gravity, the dynamics entity is not the metric, but rather a collection for the torsion less Levi-Civita connection in standard general relativity, which ends up replacing curvature with torsion. The advantage of this history is that its dynamics are governed by second order field equations.

Further, Wely put forward an extension of Riemannian geometry, in which he established the first unified theory of gravity and electromagnetism, where the non- metricity of spacetime generated the electromagnetic field. The result is, the symmetric teleparallel representation is the third generalization of General Relativity. The development of dark energy parameter for spatially homogeneous and anisotropic Bianchi type-I universes within the context of $f(T)$ theory of gravity investigated by Chirde and Shekh [13]. $f(T)$

cosmology at the levels of background and disturbance studied by Dent et al. [18]. Pawar et al. have studied the dynamics of the Bianchi type-I model in the context of $f(T)$ gravity, that expands upon teleparallel gravity by incorporating a general function of the torsion scalar T [30]. The $f(T)$ gravity model reconstruction using holographic dark energy studied by Daouda et al. [17]. Mandal and Sahoo have studied into cosmological models featuring particle production within the framework of $f(T)$gravity, the aim is to unravel the current accelerated expansion of the universe [25]. Dagwal investigated tilted two forms of dark energy in $f(T)$ theory of gravity [14]. Jamil et al. have investigated the model of dark energy interacting in $f(T)$ cosmology, considering dark energy to be a perfect fluid and selecting a specific cosmologically viable form $f(T) = \beta\sqrt{T}$ [23]. Yaqi et al. have computed the Quasinormal mode frequencies of a test massless scalar field around static black hole solutions in $f(T)$theory [23]. Dagwal investigated tilted congruence with big rip singularity in $f(T)$ theory of gravity [28]. Chaudhary et al. have investigated the parameter constraints of two dark energy models, the modified Chaplygin-Jacobi gas and modified Chaplygin-Abel gas, within the framework of $f(T)$ gravity model in a non-flat Friedmann-Lemaître-Robertson-Walker universe [11]. Hu et al. [22] have studied the analysis of scalar perturbations and the potential presence of strong coupling issues in $f(T)$gravity around a cosmological background [22, 35]. Sharif and Azeem [35] have explored the actions of the dark energy's state parameter and energy density equation in the setting of $f(T)$ gravity by using anisotropic LRS Bianchi type-I universe model [31, 32]. Bhatti et al. [9] investigate an electromagnetic influence on hyperbolically symmetric sources in $f(T)$ gravity [35, 38]. Bali et al. [3, 4, 5, 6] investigated cosmological models with variable Λ in different Bianchi type space times. The universe is not only expanding but accelerating are investigated by Perlmuter et al. [33] and Riess et al. [34]

This paper focusses on the application of $f(T)$ theory to the Bianchi type-VI cosmological model, enriched by the inclusion of a perfect fluid, in an effort to elucidate the mysteries surrounding of dark energy. We explore the ways in which the perfect fluid influences the cosmic evolution and its potential impact on the characteristics of dark energy. Dark energy, a mysterious component driving the accelerated expansion of the universe, continues to challenge our understanding of fundamental physics. The $f(T)$ theory, an extension of teleparallel gravity, provides an alternative perspective to General Relativity, where T is the torsion scalar. We explore the physical characteristics of the model in the presence of perfect fluid in the framework of $f(T)$ gravity. In section 2, formulation of $f(T)$ theory of gravity. Section 3, Metric and field equations. Section 4, solutions of the field equation. Section 5, evolution of cosmological parameters, section 6, results and discussion and lastly section 7 is the conclusion of overall solutions.

2. Formulation of $f(T)$ Theory of Gravity

In this section, we will give a formal idea of $f(T)$ gravity, where T is referred to as torsion scalar. The $f(T)$ gravity is defined as

$$S_{f(T)} = \int \sqrt{-g}[f(T) + L_m]\, d^4x \tag{1}$$

Where, differential function of the torsion scalar T is given by $f(T)$ and L_m is the representation for matter field Lagrangian.

The set of orthogonal vector fields is associated with metric tensor by the relation

$g_{\emptyset\tau} = \eta_{ij}h^i_\phi h^i_\tau$ with Minkowski metric,

$$\eta_{ij} = diag(1,-1,-1,-1) \tag{2}$$

We find the torsion scalar by using the relation as follows

$$T = S_\rho{}^{\tau\phi}\, T^\rho{}_{\tau\phi} \tag{3}$$

Where the antisymmetric tensor $T^\rho_{\tau\phi}$ gives the following definition of the tensor torsion's component,

$$T^\rho{}_{\tau\phi} = \bar{\Gamma}^\rho{}_{\phi\tau} - \bar{\Gamma}^\rho{}_{\tau\phi} \tag{4}$$

Where the component of the Weitzenb$\ddot{o}$ck connection are defined as

$$\bar{\Gamma}^\rho{}_{\phi\tau} = h^\rho_i \partial_\tau h^i_\phi \tag{5}$$

and antisymmetric tensor is

$$S_\rho{}^{\tau\phi} = \frac{1}{2}\left(K^{\tau\phi}{}_\rho + \delta^\tau_\rho T^{\alpha\phi}{}_\alpha - \delta^\phi_\rho T^{\alpha\tau}{}_\alpha\right) \tag{6}$$

Where the contortion tensor is

$$K^{\tau\phi}{}_\rho = -\frac{1}{2}\left(T^{\tau\phi}{}_\rho - T^{\phi\tau}{}_\rho - T^{\tau\phi}_\rho\right) \tag{7}$$

The teleparallel theory of gravity's modified field equation is attained by pursuing variation on the action of equation (1) with reference to tetrad field.

$$\left[\frac{1}{\sqrt{-g}}\partial_\tau\left(\sqrt{-g}h^\rho_i S^{\tau\phi}_\rho\right) - h^\alpha_i T^\rho{}_{\tau\alpha} S^{\phi\tau}_\rho\right] f_T + h^\rho_i S^{\tau\phi}_\rho \partial_\tau(T) f_{TT} + \frac{1}{4}h^\phi_i f(T) = \frac{1}{2}k^2 h^\rho_i T^\phi_\rho \tag{8}$$

Where, $k^2 = 8\pi G, f_T = \frac{df}{dT}, f_{TT} = \frac{d^2 f}{dT^2}$, while T^ϕ_ρ is the energy momentum tensor for Dark Energy with perfect cosmic fluid.

$$T^\phi_\rho = (\rho + p)u_\rho u^\phi + p g^\phi_\rho \tag{9}$$

Where ρ and p are the energy density and matter pressure.

3. Metric and $f(T)$ Field Equation

We have considered the metric of the space time of Bianchi type-VI is in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2qx}dy^2 + C^2e^{2qx}dz^2 \tag{10}$$

Where A, B, C are the function of cosmic time t only and q is constant

The field equation (8) in $f(T)$ theory of gravity for the space time Bianchi type-VI using the energy momentum tensor in eq. (9) as

$$\left[\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{2\dot{B}\dot{C}}{BC}\right]f_T+\left[\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right]\dot{T}f_{TT}-\frac{f}{2}=-k^2p \tag{11}$$

$$\left[\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{2\dot{A}\dot{C}}{AC}\right]f_T+\left[\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right]\dot{T}f_{TT}-\frac{f}{2}=-k^2p \tag{12}$$

$$\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}+\frac{2\dot{A}\dot{B}}{AB}\right]f_T+\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right]\dot{T}f_{TT}-\frac{f}{2}=-k^2p \tag{13}$$

$$\left[2\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right)-q\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)\right]f_T-\frac{f}{2}=k^2\rho \tag{14}$$

The torsion scalar T is obtained by using Eq's (3-7) as

$$T=2\left[\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right]+\frac{3q^2}{A^2} \tag{15}$$

And $$\dot{T}=2\left\{\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)+\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\left(\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\right\}-\frac{6q^2}{A^2}\frac{\dot{A}}{A} \tag{16}$$

Here, the dot over a field variable represents the differentiation with respect to time t.

The directional Hubble parameters in the direction of the x, y and z-axis respectively are

$$H_1=\frac{\dot{A}}{A}\ ,\ H_2=\frac{\dot{B}}{B}\ and\ H_3=\frac{\dot{C}}{C} \tag{17}$$

The average scale factor and spatial volume as

$$V=a^3=ABC \tag{18}$$

The average Hubble parameter, which expresses the volumetric expansion rate of the universe is given by

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{c}\right) \tag{19}$$

The mean anisotropy parameter is given by

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2=\frac{1}{3}\left(\frac{H_1^2+H_2^2+H_3^2}{H^2}-3\right) \tag{20}$$

The expansion scalar θ and the shear scalar σ of the fluid are defined as

$\theta=3H$ and

$$\sigma^2=\frac{3}{2}\Delta H^2 \tag{21}$$

4. Solution of the Field Equation

The equations from (11) to (14) are four field equations with five unknowns A,B,C,p,ρ.

So, to find a determinate solution we take one additional constraint. Consider that the shear scalar is proportional to the expansion scalar.

We solve the above nonlinear equations with the help of a relation in metric potential as

$$A = (BC)^n \tag{22}$$

Where n is real number.

By using equations (11) and (12)

$$\left[\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\right] f_T + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\dot{T} f_{TT} = 0 \tag{23}$$

and by using equations (12) and (13)

$$\left[\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\right] f_T + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\dot{T} f_{TT} = 0 \tag{24}$$

By using (22) we have

$$\frac{\dot{A}}{A} = n\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{25}$$

Let us take the form of $f(T)$ gravity model as

$$f(T) = \lambda T \tag{26}$$

Where λ is free model parameter.

Equation (24) by using equations (25) and (26)

$$\frac{(\dot{B}C - \dot{C}B)^{\cdot}}{(\dot{B}C - \dot{C}B)} = -n\frac{(\dot{B}C + \dot{C}B)}{BC} \tag{27}$$

On integrating above, we get

$$C^2\left(\frac{B}{C}\right)^{\cdot} = \frac{c_1}{(BC)^n} \tag{28}$$

Where c_1 is the integrating constant

For $B^2 = rs$, and $C^2 = \frac{r}{s}$ equation (28) becomes

$$\frac{\dot{s}}{s} = \frac{c_1}{r^{(n+1)}} \tag{29}$$

By using equation (23) and (24) we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} = 0 \tag{30}$$

Now again by using equation (25) and (30) we get,

$$(1-n)\frac{\ddot{C}}{C} - n\frac{\ddot{B}}{B} - n(n-1)\frac{\dot{C}^2}{C^2} - n^2\frac{\dot{B}^2}{B^2} - (2n^2 + n - 1)\frac{\dot{B}}{B}\frac{\dot{C}}{C} = 0 \tag{31}$$

Above equation can be rewritten as

$$(1-2n)\frac{\ddot{r}}{r} - \frac{\ddot{s}}{s} + n(1-2n)\frac{\dot{r}^2}{r^2} + \frac{\dot{s}^2}{s^2} - (n+1)\frac{\dot{r}}{r}\frac{\dot{s}}{s} = 0 \tag{32}$$

With the help of equation (29), equation (32) becomes

$$r\ddot{r} + n\dot{r}^2 = 0 \tag{33}$$

The equation (33) by using $\dot{r} = f(r)$ can be written as

$$\frac{d}{dr}f^2 + \frac{2n}{r}f^2 = 0 \tag{34}$$

On integrating, we get

$$r = (n+1)^{\frac{1}{(n+1)}}(c_2 t + c_3)^{\frac{1}{(n+1)}} \tag{35}$$

Where c_2 and c_3 are the constant of integration.

With the help of equation (35), equation (29) becomes

$$s = c_4(c_2 t + c_3)^{\frac{c_1}{c_2(n+1)}} \tag{36}$$

Where c_4 is the integrating constant.

Therefore, the equation (15) the torsion scalar T and $f(T)$

$$T = \frac{(4n+1){c_2}^2 - {c_1}^2}{2[(n+1)(c_2 t + c_3)]^2} + \frac{3q^2}{[(n+1)(c_2 t + c_3)]^{\frac{2n}{(n+1)}}} \tag{37}$$

$$f(T) = \lambda\left[\frac{(4n+1){c_2}^2 - {c_1}^2}{2[(n+1)(c_2 t + c_3)]^2} + \frac{3q^2}{[(n+1)(c_2 t + c_3)]^{\frac{2n}{(n+1)}}}\right] \tag{38}$$

From the equations (22), (35) and (36), the metric (10) reduces to the form

$$ds^2 = -dt^2 + [(n+1)(c_2 t + c_3)]^{\frac{2n}{(n+1)}}\, dx^2 + c_4(n+1)^{\frac{1}{(n+1)}}(c_2 t + c_3)^{\frac{c_1 + c_2}{c_2(n+1)}}\, dy^2 + \frac{1}{c_4}(n+1)^{\frac{1}{(n+1)}}(c_2 t + c_3)^{\frac{c_2 - c_1}{c_2(n+1)}}\, dz^2 \tag{39}$$

5. Evolution of Cosmological Parameters

In this section, we will discuss some physical and geometrical parameters to validate the cosmological model (39). Such as the pressure, energy density, spatial volume, mean Hubble parameter, expansion scalar, shear scalar and anisotropic parameter.

For the model (39) by using equation (11), we find the pressure of the universe as

$$p = \frac{\lambda}{8\pi G}\left[\frac{(4n-1){c_2}^2 + {c_1}^2}{4[(n+1)(c_2 t + c_3)]^2} + \frac{3q^2}{2[(n+1)(c_2 t + c_3)]^{\frac{2n}{(n+1)}}}\right] \tag{40}$$

Similarly, by using equation (14), we find the energy density of the universe as

$$\rho = \frac{\lambda}{8\pi G}\left[\frac{(4n+1){c_2}^2 - {c_1}^2}{4[(n+1)(c_2 t + c_3)]^2} - \frac{q c_1}{(n+1)(c_2 t + c_3)} - \frac{3q^2}{2[(n+1)(c_2 t + c_3)]^{\frac{2n}{(n+1)}}}\right] \tag{41}$$

The Spatial Volume becomes

$$V = (n+1)(c_2 t + c_3) \tag{42}$$

The Mean Hubble's Parameter

$$H = \frac{1}{3}\left(\frac{c_2}{c_2 t + c_3}\right) \tag{43}$$

The scalar expansion θ and shear scalar σ^2 are respectively given as,

$$\theta = \frac{c_2}{c_2 t + c_3} \tag{44}$$

$$\sigma = \frac{1}{2\sqrt{3}}\left(\frac{\sqrt{(2n-1)^2 {c_2}^2 + 3{c_1}^2}}{(n+1)(c_2 t + c_3)}\right) \tag{45}$$

The Anisotropy Parameter, by using equation (20) and (43)

$$\Delta = \frac{(2n-1)^2 {c_2}^2 + 3{c_1}^2}{2{c_2}^2 (n+1)^2} \tag{46}$$

6. Results and Discussion

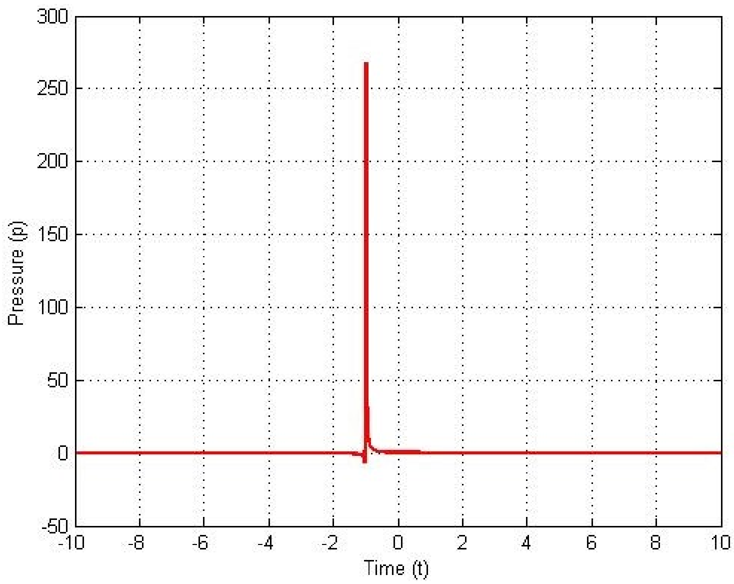

Fig. 1 Pressure (p) versus cosmic time (t) for the particular values of constants $G = 1, q = 1, \lambda = 1, n = 2, c_1 = c_2 = c_3 = 0.1$

Figure 1 shows the evolution of Pressure (p) versus cosmic time (t) by taking the particular values of constants $G = 1, q = 1, \lambda = 1, n = 2, c_1 = c_2 = c_3 = 0.1$. The pressure of the dark energy decreases monotonically with respect to time and tends to a constant value to words zero in the large time limit. The behavior of the Pressure (p) over cosmic time (t) reflects the dynamic nature of the Dark Energy in the universe. Initially, the pressure is very high, indicates a strong repulsive force. Whenever time in progress, the pressure at a great rate increase towards zero, implying that the repulsive force vanishes and stable. This indicates that the influence of dark energy reduces over to time for a more stable universe.

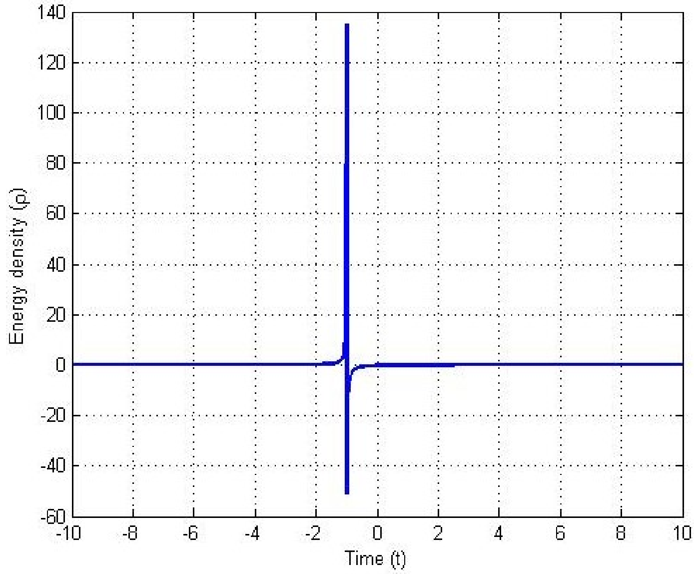

Fig. 2 Energy density (ρ) versus cosmic time (t) for the particular values of constants $G = 1, q = 1, \lambda = 1, n = 2, c_1 = c_2 = c_3 = 0.1$

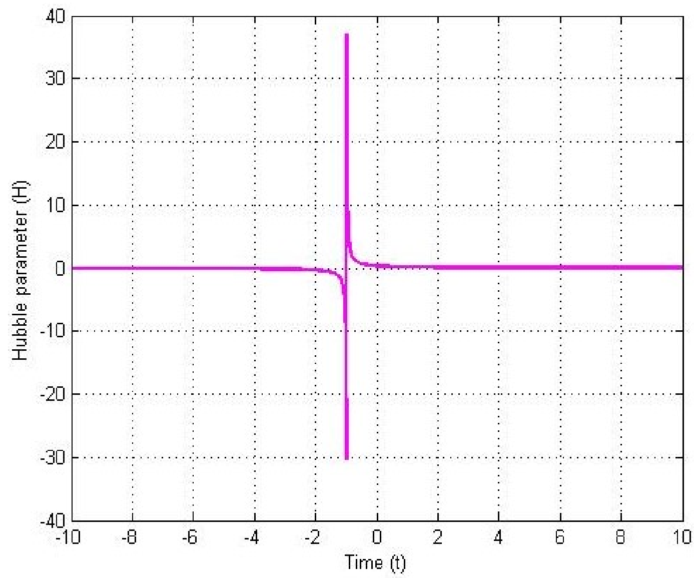

Fig. 3 Hubble parameter (H) versus cosmic time (t) for the particular values of constants $c_2 = c_3 = 0.1$

Figure 2 shows the evolution of Energy density (ρ) versus cosmic time (t) by taking the particular values of constants $G = 1, q = 1, \lambda = 1, n = 2, c_1 = c_2 = c_3 = 0.1$. The Energy density of the dark energy decreases monotonically with respect to time and tends to a constant value to words zero in the large time limit. Initially, the Energy density (ρ) is very high and decreases very fast indicates a dynamic early universe where the density of dark energy plays an important role. The energy density stabilizes near to zero, indicating a long term steady where the impact on the expansion of the universe remains constant.

Figure 3 shows the evolution of Hubble parameter (H) versus cosmic time (t) by taking the particular values of constants $c_2 = c_3 = 0.1$. It is clear that the value of Hubble parameter (H) is very high at the early time of universe and is decreases very rapidly at late time, and it is constant to zero when the time is increases.

7. Conclusion

We have studied the Bianchi type-VI cosmological model in the presence of perfect fluid in $f(T)$ theory of gravity, in which the dark energy in the universe is present. Here we take a theory of gravitation with torsion scalar T, so called teleparallel equivalent of general relativity, and its extension, so called $f(T)$ theory of gravity. The aim of aur work is to obtain solutions by using cosmological model Bianchi type-VI. In this work we have investigated the dark energy as a perfect fluid with assist of $f(T)$ theory of gravity. Here it has been assumed that $f(T) = \lambda T$. From equation (40) and figure 1, the pressure p continuously constant for late time and approaching to 0 as $t \to \infty$. From equation (41) and figure 2, the energy density ρ approaching to 0 as $t \to \infty$. From the equation (44) it is observed that the scalar expansion factor θ is decreasing function of 't' and approach to zero as t tends to infinite. Since $\lim_{t\to\infty} \frac{\sigma}{\theta} = constant$, the model is not isotropic for the future large value of t. Our model (39) starts with a big-bang at $t = 0$ and the expansion in the model increases as time increases. For this model the spatial volume $V \to \infty$ as $t \to \infty$. The torsion scalar T decreases when the cosmic time t increases and it is zero when t is infinite. The Anisotropic parameter of the expansion is found to be constant. The evolution of Hubble parameter (H) with respect to cosmic time (t) by figure 3, indicates that the value of Hubble parameter (H) is very high at the early time of universe and is decreases very rapidly at late time, and it is constant to zero when the time is increases. From all this it is clear that the universe is accelerated expansion in the later phase of the evolution.

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References

[1] Akarsu and Kilinc (2010). locally rotationally symmetric Bianchi type-I cosmological models with a constant deceleration parameter in the presence of anisotropic dark energy and a perfect fluid. Astrophys. Space Sci. **326**, 315.

[2] Astier, P. et al. (2006). The Supernovae Legacy Survey: Measurement of, and w from the first-year data set. Astronomy and Astrophys **447**(1), 31-48.

[3] Bali, R. (2022). Creation field cosmological models with dust distribution with time dependent cosmological term (Λ) in Bianchi Type II space-time, Int. J. Geom. Methods in Modern Phys. **19**, 14.

[4] Bali, R., Pradhan, A. and Hassan, A. (2008). Bianchi type VI_0 magnetized barotropic bulk viscous fluid massive string universe in general relativity, Int. J. Theor. Phys. **47**, 2594.

[5] Bali, R., Singh, P. and Singh, J.P. (2012). Bianchi type V viscous fluid cosmological models with decaying vacuum energy, Astrophys. and Space-Science, **341**, 701.

[6] Bali, R., Tinker, S. and Singh, P. (2010). Bianchi Type III cosmological models for barotropic perfect fluid distribution with variable G and Λ, Int. J. Theor. Phys. **49**, 1431 (2010).

[7] Bamba, K. et al. (2011). Equation of state for dark energy in $f(T)$gravity. Journal of cosmology and Astroparticle Physics, 021.

[8] Bengochea, G.R. et al. (2009). Dark energy as the cosmic speed-up. Physical review D, **79**(12), 124019.

[9] Bhatti, M.Z. et al. (2022). Studied an electromagnetic influence on hyperbolically symmetric sources in $f(T)$ gravity. Eur. Phys. J.C **82**, 340.

[10] Carroll, S.M. et al. (2004). Is cosmic speed-up due to new gravitational physics? Physical Review D, **70**(4), 043528.

[11] Chaudhary, H. et al. (2023). Constraints on the parameters of modified Chaplygin-Jacobi and modified Chaplygin-Abel gases in $f(T)$gravity.

[12] Chirde, V.R. et al. (2015). Accelerating universe, dark energy and exponential $f(T)$gravity. The African Review of Physics, 10.

[13] Chirde, V.R. et al. (2020). Bianchi type-I cosmological model with perfect fluid and string in $f(T)$ theory of gravitation. Int. J. of modern physics D, **29**(08), 2050054.

[14] Dagwal, V.J. (2022). investigated tilted congruence with big rip singularity in $f(T)$ theory of gravity. Int. J. of Geometric Methods in modern Physics **19**(8).

[15] Dagwal, V.J. (2023). investigated tilted two forms of dark energy in $f(T)$ theory of gravity. Indian J Phys.

[16] Daniel, S.F. et al. (2008). large-scale structure as a probe of gravitational slip, Phys. Rev. D. **77**, 103513.

[17] Daouda, et al. (2012). Reconstruction of $f(T)$ gravity according to holographic dark energy. The European Physical Journal C, **72**(2), 1-6.

[18] Dent, J.B. et al. (2011). $f(T)$ gravity mimicking dynamical dark energy: background and Perturbation Analysis. Journal of cosmology and Astroparticle Physics, (01) 009.

[19] Eisenstein, D.J. et al. (2005). Detection of the baryon acoustic peak in the large-scale Correlation function of SDSS luminous red galaxies. The Astrophys. J. **633**(2), 560.

[20] Filippenko, A.V., Riess, A.G. (1998). Results from the High-z Supernova Search Team, Phys. Rep. **307**, 31-44.

[21] Harko, T. et al. (2011). $f(R,T)$ gravity. *Physical Review D*, **84**(2), 024020.

[22] Hu, Y.M. et al. (2023). The effective field theory approach to the strong coupling issue in $f(T)$ gravity. J. Cosmol. Astropart. Phys. (07), 060.

[23] Jamil, M. et al. (2012). Phase space analysis of interacting dark energy in $f(T)$ cosmology. Central European Journal of Physics, **10**(5), 1065-1071.

[24] Koivisto, T. and Mota, D.F. (2006). Dark energy anisitropic stress and large-scale structure Formation, Phys. Rev. D. **73**, 083502.

[25] Mandal, S., Sahoo, P.K. (2020). On the temporal evolution of particle production in $f(T)$ gravity. Mod. Phys. Lett. A **35**(40), 2050328.

[26] Nadathur, S., Percival, W.J. et al. (2020). Tesing Low-Redshift cosmic acceleration with large-scale structure, Phys. Rev. Lett. **124**, 221301.

[27] Nojiri, S.I. and Odintsov, S.D. (2003b). Modified gravity with negative and positive powers of curvature: Unification of inflation and cosmic acceleration. Physical Review D, **68**(12), 23512.

[28] Odintsov, S.D. et al. (2020). Inflationary attractors in $f(T)$ gravity. Physics Letters B, **807**, 135576.

[29] Pawar, D.D. and Solanke, Y.S. (2014). An exact Kantowski-Sachs anisotropic dark energy cosmological models in Brans Dicke theory of gravitation. Int. J. Theor. Phys. **53**(9), 3052.

[30] Pawar, D.D. et al. (2014). Magnetized dark energy cosmological models with time dependent cosmological term in Lyra geometry. Bulg. J. Phys. **41**, 60.

[31] Pawar, D.D. et al. (2016). Role of constant deceleration parameter in cosmological model Filled with dark energy in $f(R,T)$ theory. Bulg. J. of Phys, **43**(2), 148-155.

[32] Pawar, D.D., Ghungarwar, N.G., Gaikwad, P.S. (2024). Perfect fluid with heat flow in $f(T)$ theory of gravity. East Eur. J. Phys. **1**, 102-111.

[33] Perlmutter, S. et al. (1999). Measurements of Ω and Λ from 42 High-redshift supernovae, Astrophys. J. **517**, 565-586.

[34] Riess, A.G. et al. (2004). Type Ia Supernovae Discoveries at $z > 1$ from the Hubbel Space Telescope: Evidence for past Deceleration and Constraints on dark energy evolution, Astrophys. J. **607**, 665-687.

[35] Sharif and Azeem (2012). Cosmological evolution for dark energy models in $f(T)$ gravity. Astrophysics and space science, **342**(2), 521-530.

[36] Spergel, D.N. et al. (2003). First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of cosmological parameters, Astrophys. J. Suppl. Ser. **148**, 175-194.

[37] Spergel, D.N. et al. (2007). Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for cosmology. The Astrophys. J. Suppl. Ser. **170**(2), 377.

[38] Zhao, Y. et al. (2022). Quasinormal modes of black holes in $f(T)$ gravity. J. Cosmol. Astropart. Phys. (10), 087.