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# A STUDY OF THE BLACK SCHOLES PRICING MODEL UNDER VARYING DIVIDEND YIELDS

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**Abstract:** We have solved the Black-Scholes call option pricing model using an implicit numerical technique under varying dividend condition. On the lines of the (Nuugulu et al. [18]) for the case of equities, which are constantly paying varying irregular dividend yields, in decreasing order under uncertainty, we have incorporated a factor in the Black-Scholes partial differential equation. In this paper we have taken data from NSE sources for Indian equity Axis Bank. The non dividend paying assumption in BS model is relaxed, and an implicit scheme is applied to evaluate call option value through MATLAB. The result is almost verified empirically for real equity data of Axis Bank, except for the hypothetical assumption of the decreasing dividend yield. It is observed that the value of call option increases when dividend yield decreases. In this study we have estimated the volatility of the Axis Bank share price data and incorporated it in Black Scholes option pricing model.

**Key Words:** Black Scholes Option pricing model, Call option, varying dividend condition, implicit numerical scheme.

## 1. Introduction

Determination of option price is an interesting area of research which is relevant to almost every financial industry including mutual funds, where the options are used to create the optimum portfolio. Almost all corporate securities can be interpreted as portfolios of puts and calls on the assets of the firm [4, 8]. The Black–Scholes formula gives no-arbitrage price of a European call or put option derived from a non-dividend-paying equity. An American call as well as put option are financial contracts that give the option holder, the right but not the obligation to buy or sell, a unit of the underlying stock at a fixed strike price, at any time before or at expiry [4]. In order to exercise the right of buying or selling options at an arbitrary strike price, a certain amount of money must be paid. This is known as option premium or option's price.

The fair price of an option contract depends on drift parameter $\mu$, and volatility $\sigma$ along with the factors such as the price of the underlying security, the risk-free interest rate, an arbitrary fixed strike price, time to expiration, and the volatility (dispersion of equity values over the possible exercise time) of the stock price.

For pricing options and futures, first the specification of the price evolution process in terms of geometric Brownian motion for a specific asset is to be done and the price for derivative contracts be formulated by applying Ito's lemma. The option is an instrument that is commonly used in the derivatives markets. Before or on the expiration date, the holder has the right, but not the obligation, to buy or sell a specified amount of the underlying asset, at a specified price, called the strike price. Choice of the strike price is fixed and arbitrary, that can be chosen by the writer / holder of the contract under the interested equity for the contract for the specific period. The holder enters into the contract by carrying limited risk, and the writer bears unlimited risk. There are two types of standard options, one is European and the another is American. American options are more flexible, since they give freedom to exercise at any time on or before the date of expiry, whereas European options cannot be exercised before the expiration date. It is worth mentioning that the option segment of the Indian stock market in works in American style trading.

Classical approach of analysing dynamics of stock markets is based on the well-known efficient market hypothesis, which exploits the martingale property of price movements. In classical asset pricing models, historical price movements cannot be inferred to predict future prices in any significant way, but observing repeated patterns and trends in financial data can help to predict future asset price movements. Basically, the stock movements in financial markets depend on demand and supply chain. These repeated patterns and trends are assumed to be affected by the principles that the markets discount everything, investor psychology is dominant, and the history repeats almost always.

Rana and Ahmad [21] have reported that in case of the out-of-the-money European Call option with dividend paying asset, the option price is very much influenced by strike prices, interest rate, dividends and maturities. But in case of in-the-money and at-the-money options, interest rate and dividend have insignificant effect on option pricing.

Ballester et al. [3] have studied an efficient method for option pricing with discrete dividend payment. The authors have evaluated a numerical solution of the Black–Scholes equation by modeling option price with a discrete dividend payment. Their model solves partial differential equation with two variables: the underlying asset and the time to maturity, and involving the shifted Dirac delta function.

There have been several studies that have extended the no-arbitrage pricing theory to dividend-paying securities with transaction costs in discrete-time markets [6]. It is reported that when dividends are not subject to transaction costs, the no-arbitrage condition is equivalent to a consistent pricing system in the efficient friction case. No-arbitrage conditions are open in general when dividends have transaction costs. The reason for this is that, when an asset pays dividends, the asset price drops by the amount of the dividend. Due to dividends, investors are compensated for price depreciation back in cash. There is therefore no arbitrage opportunity. [6, 7]

There are several recent studies with dividend payments, such as Ballester [3], Rana [21] and others [10,6,3,21,5]. It is also observed that usually most of the companies pay dividend on the basis of their financial performance in irregular mode. Hence study of the Black Scholes model with dividends is a challenging problem for researchers. Several researchers have investigated the option pricing under dividend payments like Rodrigo and Mamon [22], and Korn and Rogers [16].

Black Scholes PDE can be solved by applying various analytical approaches [4,8] to determine call or put option values V (t, S). Various researchers have also employed some numerical techniques to solve it. The finite difference methods such as explicit method, implicit method, and crank Nicolson method are some of the widely used technique for solving PDE. Various researchers have studied the numerical techniques for estimating option values based on BS PDE. [9, 23,26,11,10,15,2]. The implicit method comparatively gives more accurate results than that of explicit method. The implicit method has more convergent solution.

In the literature, various techniques have also been proposed for the numerical estimation of the option prices, [1, 24, 25] such as the inversion of a Laplace transform method of Geman & Yor [14], the spectral method in Linetsky [17], the analytical approximation of the solution of a pricing PDE in Foschi et al. [12] and the short maturity approximation by Pirjol & Zhu [20].

Wokoma et al. [26] have implementation Crank-Nicolson finite difference method for the valuation of European put option by solving Black-Scholes partial differential equation (PDE) [18, 19]. They concluded that the analytic solution of Black-Scholes PDE [15] and numerical solution via CN method are relatively close. Emmanuel et al. [11] have studied finite difference methods for options pricing. These methods are suitable to solve partial differential equations and provide a general numerical solution to option valuation problems. The stability and accuracy of the methods have also been discussed, and it is concluded that the Crank-Nicolson method is more accurate and converges faster than the implicit method.

Motivated by the above survey and research work, we have constructed a modified Black Scholes Model with a varying dividend with decreasing yield. The aim of our paper is to analyse the effect of decreasing dividend yield on the Call option value by implicit method.

## 2. Fundamentals of Black Scholes Model

Let us consider a portfolio consisting of a long position in the stock S and a short position in the option V (S, t) which depends on time t. Hence the portfolio is defined as

$$\pi = V(S,t) - \Delta S \tag{1}$$

Applying Ito's Lemma for the option value V(S, t), we have,

$$d\,V\,(S,t) = \frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\,dS^2 + \frac{\partial V}{\partial S}dS + \frac{\partial V}{\partial t}dt \tag{2}$$

Here we consider that $W$ be the Wiener process of the random variable S, with drift parameter $\mu$, and stock volatility $\sigma$.

dS = $\mu S dt + \sigma S dW$ (3)

and

$dS^2 = \sigma^2 S^2\, dt,$ (4)

So, the equation (3) reduces to

$$dV(S,t) \;=\; \frac{\partial V}{\partial t}dt \;+\; \frac{\partial V}{\partial S}(\mu S dt + \sigma S dW\,) + \frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2\, dt \quad (5)$$

The change in the portfolio value is:

$$d\pi \;=\; dV(S,t) - \;\Delta dS \quad (6)$$

Putting the values of dV and dS from equations (5) and (3) into (6), we have

$$d\pi = \frac{\partial V}{\partial S}\sigma S \mathrm{dW} \;-\; \Delta(\,\mu S dt + \sigma S dW) + (\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2)\, dt \quad (2.5)$$

In order to eliminate $dW$ term let us consider $\Delta = \frac{\partial V}{\partial S}$ , we obtain,

$$d\pi = (\,\frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2 + \;\frac{\partial V}{\partial t}\;)\, dt \quad (7)$$

If the stock pays a continuous decreasing dividend yield . the stock price dynamics under risk free interest rate $r$ for Indian equities context becomes

dS = $\sigma S dW$ +$(r-p)S dt$ (8)

So, the portfolio value change should include the dividend yield:

$$d\pi = (\frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2 + \frac{\partial V}{\partial t} \;+\; \frac{\partial V}{\partial S}(r-p)S\,)\, dt \quad (9)$$

For the portfolio to be risk-free:

$d\pi = r\pi dt$ (2.9)

Substituting portfolio amount as $V - \frac{\partial V}{\partial S}S$, We get:

$$(\frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2 \;+ \frac{\partial V}{\partial t} \;+\; \frac{\partial V}{\partial S}(r-p)S\,)\, dt = \;\; r\,(V - \frac{\partial V}{\partial S}S)dt \quad (10)$$

Equating the coefficients of dt, we have

$$rV \;-\; r\frac{\partial V}{\partial S}S \;=\; \frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2 + \frac{\partial V}{\partial S}(r-p)S + \frac{\partial V}{\partial t} \quad (11)$$

Rearrange the terms to get the Black-Scholes PDE with a continuous decreasing dividend yield p, we have

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S}(r-p)S + \frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2 - \mathrm{rV} = 0 \quad (12)$$

which is the Black-Scholes partial differential equation for option price with continuous assumed decreasing dividend yield $p$. Now we will develop the numerical scheme for solving this PDE (12).

**3. Implicit method with underlying assumption about dividend**

When pricing financial derivatives using numerical methods, such as the finite difference method (FDM) for solving the Black-Scholes partial differential equation (PDE), we divide the computational domain into a grid of both asset price (S) and time (t). This grid uses discrete steps, denoted as ΔS for asset price and Δt for time. In this study we have estimated volatility $\sigma$ for real data of Axis Bank security, which is a part of Nifty-50.

The pay-off is known at the end points of the grid, corresponding to the expiration time (T) of the option, with closing value of the underlying security. So, we apply backward iteration on this grid to find the call option price at earlier times (from T to initial time). Here, along with the existing assumptions, the Black-Scholes partial differential equation has the dependence of option price on multiple factors such as equity value, time value, drift parameter (μ), volatility ($\sigma$), risk free interest rate ( $r$), together with incorporating the dividend yields ($p$). By using implicit, we can iteratively calculate these prices across the grid. This method allows us to approximate option prices effectively, considering the underlying equity's price dynamics over time.

In numerical method for solving the Black-Scholes PDE, we discretize the asset price S and time $t$ into grids. For the underlying equity price, the grid points are $S_k = k.\Delta S$ where $k = 0,1,2, \dots, M$ and ΔS is the step size. Similarly, for time, the grid points are $t_i = i.\Delta t$ where $i = 0,1,2, \dots., N$ and $\Delta t$ is the time step. The option price $V(t, S)$ is represented on this grid as $V_i^k$ = V $(t_i, S_k)$.

**3.1 Initial (final) and Boundary Conditions for the Axis Bank security:**

For the European call option, now we incorporate the initial and boundary conditions. The value at the final time $t = T$ can be calculated from the definition of the call option. If at the final time stock price (S) is greater than the strike price (K), i.e. $(S > K)$ then the call option will be worth for holder, in terms of loss to writer with adjusted premium, $(S - K)$ , as the buyer can buy the stock for arbitrary fixed strike price K, and sell it instantly when the stock price jumps upside S, and hence the profit is gained. But if at the final time, the stock price (S) is less than the strike price (K) i.e. $(S < K)$ then the buyer will not exercise the option, and it becomes worthless. So, at t = T the option value known, that is $V(S, T) = max(S - K, 0)$. This is called the final condition.

For the boundary conditions, we consider the value of $V(S, t)$ at S = 0 and S → ∞. If S = 0 then from Equation (1) the payoff must also be zero. Hence the boundary condition when $S = 0$ is $V(0, t) = 0$. But, when S → ∞ , it is more likely that the option will be exercised and the value will be $S - K$. As S → ∞ ,exercise price does not have any impact on the option value, so the option value is equivalent to

$V(S, t) = S - Ke^{-r(T-t)}$ as $S \to \infty$

which is the right-side boundary condition.

**3.2 Implicit Finite Difference Scheme:**

For time and Space discretization we have divided time interval [0, T] into N equally length subintervals of length Δt. The underlying asset will assume prices in the interval [0, ∞). However, an artificial limit $S_{max}$ is introduced which is normally two or three times bigger than the strike price K. This underlying asset interval [0, $S_{max}$] is also divided into M equally sized subintervals having length ΔS. Therefore, the space [0, T] × [0, $S_{max}$] is approximated by grids

$(i.\Delta t, k.\Delta S) \approx [0, N\Delta t] \times [0, M\Delta S]$

where i = 0,1, 2,…,N and k = 0,1,2,…,M.

Now to formulate an implicit difference scheme for the Black-Scholes partial differential equation in a continuous decreasing dividend yield case:

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S}(r-p)S + \frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2 - rV = 0 \quad (12(i))$$

The equation (12(i)), is discretized using the following formulae.

The backward differencing for the time derivative with respect to t:

$$\frac{\partial V}{\partial t} \approx \frac{V_i^k - V_{i-1}^k}{\Delta t} \quad (13)$$

The central differencing for the first derivative with respect to S is

$$\frac{\partial V}{\partial S} \approx \frac{V_i^{k+1} - V_i^{k-1}}{2\Delta S} \quad (14)$$

The central differencing for the second derivative for S is

$$\frac{\partial^2 V}{\partial S^2} \approx \frac{V_i^{k+1} - 2V_i^k + V_i^{k-1}}{\Delta S^2} \quad (15)$$

Now, substituting these values into the modified Black-Scholes PDE including dividends, and rearrange terms for the implicit method:

$$\frac{V_i^k - V_{i-1}^k}{\Delta t} + (r-p)S_k\frac{V_i^{k+1} - V_i^{k-1}}{2\Delta S} + \frac{1}{2}\sigma^2 S_k^2\, \frac{V_i^{k+1} - 2V_i^k + V_i^{k-1}}{\Delta S^2} - rV_i^k = 0 \quad (16)$$

Here, the decreasing dividend yield have been assumed as, $= \sum_{n=1}^{k}\frac{d}{n}$ . For base dividend yield $d$, the equation (16) takes the following form

$$V_i^k\left(1 + r\Delta t + \frac{\sigma^2 S_k^2 \Delta t}{\Delta S^2}\right) + \Delta t\left(-\frac{(r-d)S_k}{2\Delta S} - \frac{\sigma^2 S_k^2}{2\Delta S^2}\right)V_i^{k-1} + \Delta t\left(\frac{(r-d)S_k}{2\Delta S} - \frac{\sigma^2 S_k^2}{2\Delta S^2}\right)V_i^{k+1} = V_{i-1}^k \quad (17)$$

If the dividend yield is $\frac{d}{2}$ , we have,

$$V_i^k\left(1+r\Delta t+\frac{\sigma^2 S_k^2\Delta t}{\Delta S^2}\right)+\Delta t\left(-\frac{(r-d/2)S_k}{2\Delta S}-\frac{\sigma^2 S_k^2}{2\Delta S^2}\right)V_i^{k-1}+\Delta t\left(\frac{(r-d/2)S_k}{2\Delta S}-\frac{\sigma^2 S_k^2}{2\Delta S^2}\right)V_i^{k+1}=V_{i-1}^k \quad (3.7)$$

In case of dividend yield $\frac{d}{3}$, we have,

$$V_i^k\left(1+r\Delta t+\frac{\sigma^2 S_k^2\Delta t}{\Delta S^2}\right)+\Delta t\left(-\frac{(r-d/3)S_k}{2\Delta S}-\frac{\sigma^2 S_k^2}{2\Delta S^2}\right)V_i^{k-1}+\Delta t\left(\frac{(r-d/3)S_k}{2\Delta S}-\frac{\sigma^2 S_k^2}{2\Delta S^2}\right)V_i^{k+1}=V_{i-1}^k \quad (18)$$

This gives the system of equation of the form:

$$-\alpha_k V_i^{k-1}+\beta_k V_i^k-\gamma_k V_i^{k+1}=V_{i-1}^k \quad (19)$$

where the coefficients are given as

$$\alpha_k=\Delta t\left(\frac{\sigma^2 S_k^2}{2\Delta S^2}+\frac{(r-p)S_k}{2\Delta S}\right)$$

$$\beta_k=1+r\Delta t+\frac{\sigma^2 S_k^2\Delta t}{\Delta S^2}$$

$$\gamma_k=\Delta t\left(\frac{\sigma^2 S_k^2}{2\Delta S^2}-\frac{(r-p)S_k}{2\Delta S}\right)$$

Now solving the equations (17), (18) and (19) and performing the time dependent graphical analysis of call option price with estimated volatility of real data of Axis Bank security, through MATLAB for every dividend case under consideration, we obtain the numerical results reported below.

## 4. Call Option Pricing Results

As an application of BS option pricing model, we obtain the call option pricing solution using MATLAB software. by considering the equation (17) for the European call option with the parameters of Axis Bank security as, K = 1050, $r = 0.065, T = \frac{1}{12}$ years, $S\_max = 3150$ and $\sigma = 0.2$ (Axis Bank volatility). The dividend yield is assumed to be decreasing, which is believed to be dependent on the performance of equity.

**Case:1** For assumed dividend yield $p = (r-d) = 0.1$, we have obtained the graph presented in fig.1.

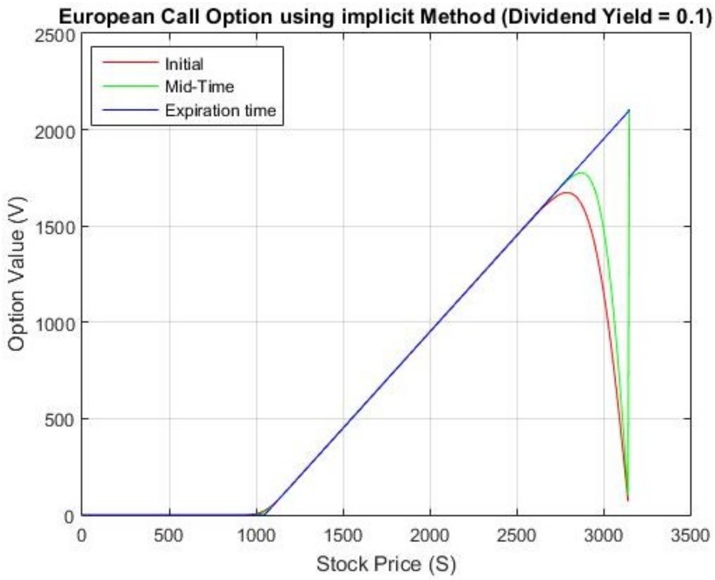


**Figure 1**

**Case:2** For assumed dividend yield $p = (r - d/2) = 0.05$, we have obtained the graph presented in fig.2.

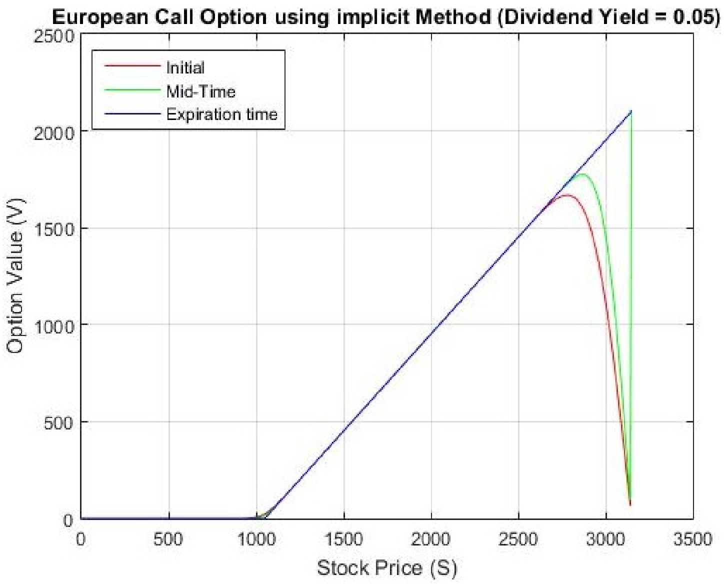


**Figure 2**

**Case:3** For assumed dividend yield $p = (r - d/3) = 0.03$, we have obtained the graph presented in fig.3.

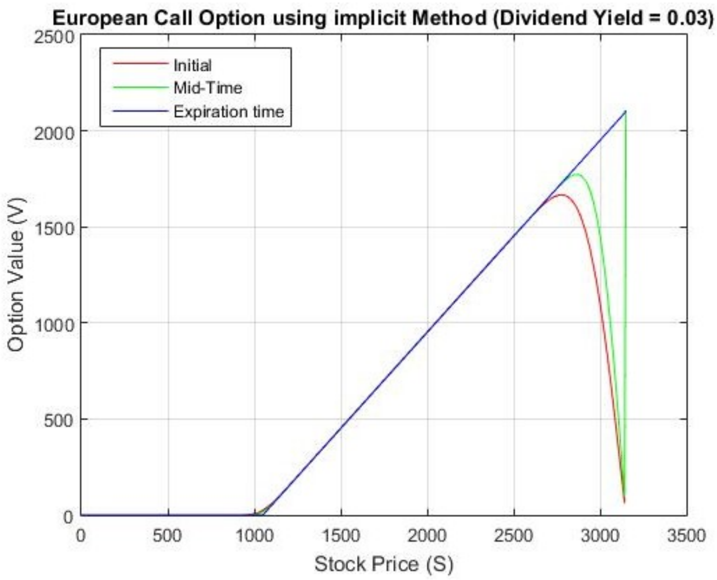


**Figure 3**

## 5. Results and Conclusions

The boundaries of the computational domain are specified by $S = 0$ and $S = S_{max}$. where $S_{max}$ is the maximum possible value of the underlying asset's price. At these boundaries and at the final time $t = T$, the terminal and boundary conditions are known. Actually, we have the option prices known at these points, which we use to compute the values at all other interior grid points through the implicit numerical method. Using this approach the option price can be computed at every grid point, taking into consideration the known values at the grid boundaries and terminal time under volatility obtained for real Axis Bank security. The call option graphs, show that the option price fluctuates significantly due to the dividend yields. In case the dividend yield decreases, a sharp increase in call option price is reflected. Under the decreasing dividend yield situation, the model supports the holder of the contract.

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