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BULK VISCOUS INFLATIONARY UNIVERSE FOR BAROTROPIC FLUID DISTRIBUTION IN BIANCHI TYPE – VIII SPACE TIME

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Abstract: We have investigated bulk viscous inflationary universe for barotropic fluid distribution in Bianchi type–VIII space time. To get the deterministic and realistic model, we assumed barotropic fluid condition, i.e. $p = \gamma\rho;\ 0 \le \gamma \le 1$ where p is isotropic pressure, and ρ is the matter density and a supplementary condition between metric potentials A_1 and A_2 as $A_1 = {A_2}^n$ where n is the constant. The behaviour of the model from physical and geometrical aspects in presence of bulk viscosity is discussed in detail.

Key Words: Inflationary, Bianchi Type-VIII, Bulk Viscous, Barotropic Fluid, General relativity. 7014286576

1. Introduction

Bianchi type space times play a vital role in understanding and description of the early stages of evolution of the universe. Bianchi type-VIII model is one of the important anisotropic cosmological models and hence it is widely studied in general relativity. The anisotropy plays a significant role in the early stage of evolution of the universe and hence the study of anisotropic and homogeneous cosmological models becomes important. Adhav, et al. [3] studied Bianchi type-VIII, IX and II cosmological models within the frame work of $f(R,T)$ theory of gravity for the role of dark energy in the form of wet dark fluid. Bianchi type-VIII cosmological model with quadratic equation of state is studied by Chhajed, et al. [17]. The study of Bianchi type-VIII universe is also important because of familiar solutions like FRW universe with positive curvature, the de Sitter universe, and the Taub-Nut solutions. Argyris, et al. [4] have proposed a study on the influence of noise on chaotic phenomena as arising in the Bianchi types-VIII and IX cosmological models.

Bianchi models have been studied by several authors to achieve better understanding of the observed small amount of anisotropy in the universe. Bianchi type-VIII cosmological model with perfect fluid in theory of gravitation $f(R,T)$ presented by Sancheti and Hatkar [23]. Kuvshinova, et al. [20] have formed non stationary Bianchi type VIII cosmological models with rotation in the framework of general relativity. Inflationary scenario in Bianchi Type-VIII space-time in the presence of massless scalar field with flat potential is investigated by Bali and Swati [9]. Some rotating, time-dependent Bianchi type-VIII cosmologies with heat flow are studied by Bradley and Sviestins [16]. Bianchi Types VIII, IX and II string cosmological models in Brans-Dicke theory of Gravitation are studied by Rao and Santhi [22].

A barotropic fluid is a flow in which the pressure is a function of the density only and in cosmology we often make the assumption $p = \gamma\rho$. The Barotropic equation of state for Bianchi type-VI_0 massive with magnetic field and bulk viscosity studied by Bali, et al. [12]. Recently Singh, et al. [25] derived the Lagrangian formulation with standard and non-standard kinetic terms for barotropic fluid model, and investigated the behaviour of cosmologies in homogeneous and isotropic background with a barotropic fluid. Bali. et al. [13] have investigated Bianchi Type-IX barotropic fluid cosmological model in the frame work of Lyra geometry by considering the dust distribution ($p = 0$) model to get the result in terms of cosmic time. Hernandez, et al. [18] sketched an algorithm to generate exact anisotropic solutions starting from a barotropic and setting an ansatz on the metric functions.

For realistic picture of the universe in terms of cosmic time Bali, and Gupta, [7] solved barotropic perfect fluid condition in Bianchi type IX cosmological models in general relativity by assuming the proportionality constant as unity. Bali and Tinker [10] investigated the Bianchi type-V bulk viscous barotropic fluid cosmological model with variable gravitational constant G and the cosmological constant Λ. Bali and Singh [8] have examined an inflationary scenario in Bianchi Type V space-time for a barotropic fluid distribution with variable bulk viscosity and decaying vacuum energy density, and also observed that the matter density ρ, the coefficient of bulk viscosity ζ and the expansion θ all diverge at $\tau = 0$. Bali and Kumawat [5] have investigated barotropic perfect fluid distribution with heat conduction for Bianchi Type I tilted cosmological model. Barotropic fluid distribution with magnetic field in Lyra geometry for Bianchi type-I cosmological model is investigated by Bali and Vadhwani [11]. The compatibility is based on the functional form of the speed of sound squared, which for barotropic fluid dark energy follows directly from the function for the Equation of state parameter are obtained by Perkovic and Stefanci [21].

An inflationary stage is a very general property of the solutions concluded by Belinskii and the concept of quantum 'creation' of the universe. Belinskii et al. [15] concluded that inflationary stages are an unavoidable property of a large class of solutions which indicated that the concept of spontaneous creation of the universe with a subsequent inflationary stage probably does not require the ful-fillment of any special requirement. The theory and phenomenology of reheating and preheating after inflation is reviewed by Bassett, et al. [14] and provided a unified discussion of both the gravitational and non-gravitational features of multi-field inflation. Sharma and Poonia [24] have constructed the inflationary universe in Bianchi type-IX under framework of the effect of bulk viscosity and flat potential. For inflationary universe Adhav, et al. [2] have considered a flat region in which potential V is constant and investigated axially symmetric Bianchi type-IX space-time in the presence of mass less scalar field with a flat potential.

Inflation is the extremely rapid exponential expansion of the early universe by a factor of at least 10^{78} in volume driven by a negative pressure vacuum energy density. Bali, R., and Goyal, R. [6] discussed about Inflationary scenario in Bianchi Type-V space-time with variable bulk viscosity and dark energy in radiation dominated phase for exact cosmological solutions of the Einstein gravitational equations with non-interacting combination of a classical scalar field and isotropic radiation as a source. An inflationary universe of a flat region in which potential V is constant for Bianchi type-III inflationary universe in the presence of mass less scalar field with a flat potential is obtained by Katore [19]. Predictions for the anisotropy of the microwave background produced by gravitational waves generated by ordinary exponential inflation are presented by Abbott and Wise [1]. Following the inflationary period, the universe continued to expand but at a slower rate. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmology.

In this paper, we have investigated bulk viscous inflationary universe for barotropic fluid distribution in bianchi type–VIII space time using the special condition $A_1 = {A_2}^n$ between metric potentials, n being a constant. To get the deterministic and realistic model, we have assumed barotropic fluid condition $p = \gamma\rho; 0 \le \gamma \le 1$ where p is the isotropic pressure, and ρ is matter density.The behaviour of the model from physical and geometrical aspects in presence of bulk viscosity is discussed in detail.

2. The Metric and Field Equations

We consider Bianchi type-VIII line element in form

$$ds^2 = dt^2 - {A_2}^2dx^2 - {A_1}^2dy^2 - \left[{A_1}^2 sinh^2y + {A_2}^2 cosh^2y\right]dz^2 + 2{A_2}^2 \cosh y\, dxdz \tag{1}$$

Where A_1 and A_2 are metric potentials and functions of 't' alone.

In case of gravity minimally coupled to a scalar field $V(\phi)$, as given by Stein- Schabes, we have

$$S = \int \sqrt{-g}\, [R - \tfrac{1}{2} g^{ij}\partial_{ij}\phi\partial_{ij}\phi - V(\phi)]d^4x \tag{2}$$

The Einstein's field equations (in the gravitational unit $8\pi G = c = 1$) in case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R_{ij} - \tfrac{1}{2}Rg_{ij} = -T_{ij} \tag{3}$$

The energy momentum tensor (T_{ij}) for scalar field in presence of viscosity is given by

$$T_{ij} = (\rho + p)v_iv_j - pg_{ij} + \partial_i\phi\partial_j\phi - \left[\tfrac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right]g_{ij} + \xi\theta\left(g_{ij} + v_iv_j\right) \tag{4}$$

Where V is the effective potential, ϕ is Higgs field, ξ is the coefficient of bulk viscosity and θ is the expansion in the model.

Also v^i, the unit time like vector satisfying the following condition: $v_iv^i = 1$

The co-moving coordinate system is chosen as

$v^1 = 0 = v^2 = v^3,\ v^4 = 1, v_4 = 1$

The energy conservation law coincides with the equation of motion for ϕ and we have

$$\left(1/\sqrt{-g}\right)\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) = -dV/d\phi \tag{5}$$

Where scalar field ϕ is the function of t-alone.

The Einstein field equation (3) for the metric (1) and energy momentum tensor (4) leads to the following system of equations

$$\left(2\ddot{A_1}/A_1\right) + \left(\dot{A_1}^2/A_1^2\right) - \left(1/A_1^2\right) - \left(3A_2^2/4A_1^4\right) = -\left(\dot{\phi}^2/2\right) - V(\phi) - (p - \xi\theta) \tag{6}$$

$$\left(\ddot{A_1}/A_1\right) + \left(\ddot{A_2}/A_2\right) + \left(\dot{A_1}\dot{A_2}/A_1A_2\right) + A_2^2/4A_1^4 = -\left(\dot{\phi}^2/2\right) - V(\phi) - (p - \xi\theta) \tag{7}$$

$$\left(2\dot{A_1}\dot{A_2}/A_1A_2\right) + \left(\dot{A_1}^2/A_1^2\right) - \left(1/A_1^2\right) - \left(A_2^2/4A_1^4\right) = \rho + \dot{\phi}^2/2 - V(\phi) \tag{8}$$

The equation (5) for scalar field (ϕ) leads to

$$\ddot{\phi} + \left[2\dot{A_1}/A_1 + \dot{A_2}/A_2\right]\dot{\phi} = -dV/d\phi \tag{9}$$

Where the over head symbol dot$(\cdot)$ with A_1 and A_2 indicates derivative with respect to time t.

3. Solution of Field Equations

We are interested in inflationary solution so flat region is considered. Thus we have $V(\phi)$ is constant.

i.e. $V(\phi) = K$ (10)

From equations (9) and (10), we get

$$\ddot{\phi} + \left[2\dot{A_1}/A_1 + \dot{A_2}/A_2\right]\dot{\phi} = 0 \tag{11}$$

above equation leads to

$$\dot{\phi} = {}^{l}\!/_{A_1{}^2 A_2} \tag{12}$$

Where l is constant of integration.

The average scale factor (R) for line element (1) is given by

$$R^3 = A_1{}^2 A_2 \tag{13}$$

By equation (7) and (8), we have

$$\left(\frac{\ddot{A_1}}{A_1}\right) + \left(\frac{\ddot{A_2}}{A_2}\right) + \left(\frac{3\dot{A_1}\dot{A_2}}{A_1 A_2}\right) + \left(\frac{\dot{A_1}^2}{A_1{}^2}\right) - \left(\frac{1}{A_1{}^2}\right) = \rho - 2K - (p - \xi\theta) \tag{14}$$

The above equation (14) has $A_1, A_2, \phi, V, p, \rho, \xi$ and θ unknown parameters. To obtain the deterministic solution, we assume the following conditions:

(i) Barotropic fluid condition

$$p = \gamma\rho; 0 \le \gamma \le 1$$

$$\theta = 3H, \rho = 3H^2, \xi = \rho^{1/2} \tag{15}$$

(ii) Shear(σ) is proportional to the expansion (θ), which leads to

$$A_1 = A_2{}^n \tag{16}$$

Now equations (14), (15) and (16) together lead to

$$\ddot{A_2} + \left[2n - \left(\frac{1-\gamma}{3} + \frac{1}{\sqrt{3}}\right)\frac{(2n+1)^2}{(n+1)}\right] \frac{\dot{A_2}^2}{A_2} = \frac{1}{(n+1)}\left[A_2{}^{1-2n} - 2kA_2\right] \tag{17}$$

Which leads to

$$2\ddot{A_2} + \left({}^{a}\!/_{A_2}\right)\dot{A_2}^2 = b_1 A_2{}^{1-2n} - b_2 A_2 \tag{18}$$

Where $a = 2\left[2n - \left(\frac{1-\gamma}{3} + \frac{1}{\sqrt{3}}\right)\frac{(2n+1)^2}{(n+1)}\right]$, $b_1 = \frac{2}{(n+1)}$, and $b_2 = \frac{4k}{(n+1)}$ (19)

To get solution, we assume that $\dot{A_2} = f$. Thus equation (18) leads to

$$\frac{df^2}{dA_2} + \left({}^{a}\!/_{A_2}\right) f^2 = b_1 A_2{}^{1-2n} + b_2 A_2 \tag{20}$$

Which on integration equation (20) leads to

$$f^2 = \left(\frac{b_1}{a - 2n + 2}\right) A_2{}^{-2(n-1)} - \left(\frac{b_2}{a+2}\right) A_2{}^2 + b_3 A_2{}^{-a} \tag{21}$$

Where b_3 is constant of integration.

$$f = \dot{A}_2 = {dA_2}/{dt} = \left[\left({b_1}/{a - 2n + 2}\right) A_2{}^{-2(n-1)} - \left({b_2}/{a + 2}\right) A_2{}^{2} + b_3 A_2{}^{-a}\right]^{1/2} \quad (22)$$

After suitable transformation of co-ordinates, the metric (1) leads to the form

$$ds^2 = \left({1}/{(a_1 T^{-2(n-1)} - a_2 T^2 + b_3 T^{-a})}\right) dT^2 - T^2 dX^2 - T^{2n} dY^2 - [T^{2n} sinh^2 Y + T^2 cosh^2 Y] dZ^2 + 2T^2 \cosh Y \, dXdZ \quad (23)$$

Where $a_1 = \frac{b_1}{(a-2n+2)}$, $a_2 = \frac{b_2}{(a+2)}$, $x = X$, $y = Y$, $z = Z$ and $A_2 = T$.

4. Physical and Geometrical Aspects

The physical and geometrical aspects for the model (23) are given as follows:

The rate of Higgs field(ϕ) is

$$\phi = l \int \left({dT}/{T^{2n+1} [a_1 T^{-2(n-1)} - a_2 T^2 + b_3 T^{-a}]^{1/2}} \right) + L \quad (24)$$

Where L is constant of integration.

The spatial volume(R^3) is

$$R^3 = A_2{}^{2n+1} = T^{2n+1} \quad (25)$$

The directional Hubble parameter(H_x, H_y, H_z) are

$$H_x = H_y = {\dot{A}_1}/{A_1} = {n\dot{A}_2}/{A_2} = n \left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right) \right]^{1/2}$$

and $H_z = {\dot{A}_2}/{A_2} = \left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right) \right]^{1/2}$

The average Hubble parameter (H) is found to be

$$H = \left[{(2n + 1)}/{3} \right] \left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right) \right]^{1/2} \quad (26)$$

The expansion(θ) is

$$\theta = 3H = (2n+1)\left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right)\right]^{1/2} \tag{27}$$

The deceleration parameter(q) is

$$q = -1 + \frac{3}{2(2n+1)}\left[\frac{\left\{{2na_1}/{T^{2n}}\right\}+\left\{{(a+2)b_3}/{T^{(a+2)}}\right\}}{\left\{{a_1}/{T^{2n}}\right\}-a_2+\left\{{b_3}/{T^{-(a+2)}}\right\}}\right] \tag{28}$$

The anisotropic parameter(Δ) is found to be

$$\Delta = \frac{1}{3}\sum_{i=3}^{3}\left[\frac{H_i}{H}-1\right]^2 = {2(n-1)^2}/{(2n+1)^2} \tag{29}$$

The shear scalar (σ) for the model is

$$\sigma^2 = \left({3}/{2}\right)\Delta H^2$$

$$\sigma = \left(n - {1}/{\sqrt{3}}\right)\left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right)\right] \tag{30}$$

Now $\quad {\sigma}/{\theta} = {(n-1)}/{\sqrt{3}(2n+2)} \neq 0(\text{constant})$

The energy density ρ is given by Barotropic fluid condition (15), we haves

$$\rho = 3H^2 = \left\{{(2n+1)^2}/{3}\right\}\left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right)\right] \tag{31}$$

The pressure p is given by Barotropic fluid condition (15), we have

$$p = \gamma\rho = \gamma\left\{{(2n+1)^2}/{3}\right\}\left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right)\right] \tag{32}$$

And the bulk coefficient ξ is given by equation (15)

$$\xi = \rho^{1/2} = \left\{2n + {1}/{\sqrt{3}}\right\}\left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right)\right]^{1/2} \tag{33}$$

Now $(p - \xi\theta) = \left(\gamma - \frac{1}{3}\right)(2n+1)^2\left[\left({a_1}/{T^{2n}}\right) - a_2 + \left({b_3}/{T^{(a+2)}}\right)\right] \quad (34)$

5. Conclusion

The Spatial volume R^3 increases as time increases for $n > -1/2$. Thus the inflationary scenario exists in Bianchi type-VIII space time with viscous fluid distribution for $n > -1/2$.

The model (23) starts expanding with Big-bang at $T = 0$. It is observed that the expansion decreases as time increases and approaches to constant as $T \to \infty$.The Hubble parameter decreases as time increases. Also the energy density and pressure of the model are initially large.

The rate of Higgs field is initially large, but decreases as time increases and ultimately tends to constant. Since $\lim_{T\to\infty} \sigma/\theta \neq 0$, hence the anisotropy is maintained throughout.There is a Point Type singularity in the model at $T = 0$ when n > 0.

When $T \to \infty$, the deceleration parameter (q) tends to -1, so the model represent accelerating phase of the universe.

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