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CYLINDRICALLY SYMMETRIC COSMOLOGICAL MODEL WITH VARIABLE Λ FOR DUST DISTRIBUTION IN GENERAL RELATIVITY

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Abstract: Cylindrically symmetric cosmological model with variable cosmological constant (Λ**)** for dust distribution is investigated. We find that the matter density is initially large but decreases with time & tends to a finite quantity for large values of time. The model also satisfies reality condition $\rho > 0$ given by Ellis [9]. The model starts with a big bang at T = 0 and the expansion in the model decreases with time. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, hence the model does not approach isotropy for large values of T. However ifcothT = $\frac{\alpha}{l}$then the model isotropizes. The cosmological constant Λ (vacuum energy density) decreases with time and finally tend to zero for large values of T. The model also has point type singularity at T = 0 (MacCallum [11])

Key Words & Phrases: Cylindrically Symmetric, Cosmological, Variable Λ, dust distribution.

1. Introduction

After formulation of field equations in general relativity in 1915, Einstein applied these for the study of universe. The universe at that time was supposed to be static, homogeneous and isotropic. But the later works done by Hubble and Humason indicated that universe is not static but is expanding that is time – dependent. Friedmann [10] in his investigation pointed out that for time dependent cosmological model, there is no need of cosmological constant in Einstein's field equation. Einstein added cosmological constant into his field equations to make pressure and density positive for static universes. However, Abers and Lee [1] predicted that during an early exponential phase, the vacuum energy is treated as large cosmological constant which is expected by Grand Unified Field Theory (GUT) (Sakharov [13])). Therefore, the present day observations of smallness of cosmological constant ($\Lambda \approx 10^{-122}$) support to assume that Λ is time dependent. Several cosmological models in which Λ decays with time have been investigated by number of authors viz. Beesham [8], Abdussattar and Vishwakarma [2], Singh and Chaubey [14], Bali et al. [2-7], Singh and Baghel [15]. Recently, Bali and Singh [7] investigated cosmological model with variable Λin Bianchi Type I space time.

In this paper, we have investigated cosmological model with variable Λ taking $\Lambda \approx e^{-2\alpha t}$ considering cylindrically symmetric space time. To get the deterministic solution in terms of cosmic time t, we have also assumed that $\frac{A_4}{A} = \alpha$ (constant). The physical and geometrical aspects of the model are also discussed.

2. Metric and Field Equations

We consider the cylindrically symmetric metric given by Marder [12] in the form as

$$ds^2 = A^2 (dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2 \tag{1}$$

where A, B, C are metric potentials and are functions of t-alone.

The energy momentum tensor T_i^j for perfect fluid is given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j$$

Einstein field equation is given by

$$R_{ij} - \frac{1}{2} R g_{ij} - \wedge g_{ij} = -8\pi T_i^j \tag{2}$$

where Λ is cosmological constant and depends on time.

The Einstein field equation (2) for the metric (1) leads to

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC}\right] + \Lambda = 8\pi p \tag{3}$$

$$\frac{1}{A^2}\left[-\frac{C_{44}}{C} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2}\right] + \Lambda = 8\pi p \tag{4}$$

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2}\right] + \Lambda = 8\pi p \tag{5}$$

$$\frac{1}{A^2}\left[\frac{B_4 C_4}{BC} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC}\right] - \Lambda = 8\pi\rho \tag{6}$$

where ρ is the matter density, p the isotropic pressure and v^i the flow vector satisfying

$v^1 = 0 = v^2 = v^3$, $v_i v^j = -1$ i.e. $v^4 = \frac{1}{A}$, $v_4 = -A$

3. Solution of Field equations

From equations (4) and (5), we have

$$\frac{B_{44}}{B} = \frac{C_{44}}{C} \tag{7}$$

which leads to

$(CB_4 - BC_4)_4 = 0$

$(CB_4 - BC_4) = k$

which gives

$(\frac{B}{C})_4 = \frac{k}{C^2}$ (8)

To get the result in terms of time t , we assume $\frac{A_4}{A} = \alpha$ = constant

which leads to

A = $\beta e^{\alpha t}$ (9)

Equation (4) after using p = 0 , $\frac{A_4}{A} = \alpha$, $\Lambda = \gamma e^{-2\alpha t}$, leads to

$\frac{C_{44}}{C} + (\frac{A_4}{A})_4 = \Lambda A^2$

which leads to

$\frac{C_{44}}{C} = \gamma\beta^2 = c_2$ (constant) (10)

To find the solution, we put

C_4 = f(C)

Thus $C_{44} = \frac{df}{dt} = \frac{df}{dC}\frac{dC}{dt} = ff'$

Now equation (10) leads to

$f\, df = c_2 C\, dC$

$f^2 = c_2C^2 + c_3$

where c_2 , c_3 are constant

which leads

$$\frac{dC}{\sqrt{c_2C^2 + c_3}} = dt$$

$\frac{dC}{\sqrt{C^2+l^2}} = \sqrt{c_2}dt$ (11)

where $\frac{c_3}{c_2} = l^2$ (12)

Thus, we have

$$\sinh^{-1}\frac{C}{l} = at + b$$

Where $\sqrt{c_2} = a$

Thus $C = l\sinh(at + b)$ (13)

Equation (5) after using p = 0 for dust distribution leads to

$$\frac{B_{44}}{B} + (\frac{A_4}{A})_4 = \Lambda A^2 \tag{14}$$

using $\frac{A_4}{A} = \alpha$ and $\lambda = \gamma e^{-2\alpha t}$ in equation (14), we have $\frac{B_{44}}{B} = \gamma\beta = l$

Which leads to

$$B = l \sinh(at + b) \tag{15}$$

Where l is arbitrary constant.

Thus metric (1) leads to the form

$$ds^2 = \beta^2 e^{2\alpha t}(dx^2 - dt^2) + l^2 sinh^2(at + b)(dy^2 + dz^2)$$

which leads to

$$ds^2 = \beta^2 e^{2\alpha\left(\frac{T-a}{b}\right)}\left(dX^2 - \frac{dT^2}{a^2}\right) + l^2 sinh^2 T(dY^2 + dZ^2) \tag{16}$$

after using the transformation , at+b = T , x =X , y =Y , z =Z

4. Some physical and geometrical properties

The matter density $(\rho)$for the model (16) is given by

$$\rho = \frac{1}{8\pi\beta^2 e^{2\alpha(\frac{T-a}{b})}}[2\alpha a \coth T + a^2 coth^2 T] - \frac{\gamma}{8\pi}e^{-2\alpha(\frac{T-a}{b})} \tag{17}$$

The reality condition $\rho > 0$ Ellis (9) is satisfied.

The expansion $(\theta)$is given by

$$\theta = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}(v^i\sqrt{-g})$$

$$= \frac{1}{A}(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C})$$

$$= \frac{1}{\beta e^{\alpha(\frac{T-a}{b})}}[\alpha + 2a \coth T] \tag{18}$$

Shear tensor(σ_{ij}) is given by

$$\sigma_{ij} = \frac{1}{2}(v_{i;j} + v_{j;i}) + \frac{1}{2}(\dot{v}_i v_j + \dot{v}_j v_i) - \frac{1}{3}\theta(g_{ij} + v_i v_j)$$

Thus, we have

$$\sigma_1^1 = g^{11}\sigma_{11} = \frac{1}{3A}[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}]$$

$$= \frac{1}{3\beta e^{\alpha(\frac{T-a}{b})}}[2\alpha - 2l\coth T] \tag{19}$$

$$\sigma_2^2 = g^{22}\sigma_{22} = \frac{1}{3A}[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}]$$

$$= \frac{1}{3\beta e^{\alpha(\frac{T-a}{b})}}[l\coth T - \alpha] \tag{20}$$

$$\sigma_3^3 = g^{33}\sigma_{33} = \frac{1}{3A}[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}]$$

$$= \frac{1}{3\beta e^{\alpha(\frac{T-a}{b})}}[l\coth T - \alpha] \tag{21}$$

$$\sigma_4^4 = g^{44}\sigma_{44} = 0 \tag{22}$$

Now shear(σ) is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ji} = \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2]$$

$$= \frac{1}{3\beta^2 e^{2\alpha(\frac{T-a}{b})}}[(\alpha - l\coth T)^2]$$

Thus,

$$\sigma = \frac{1}{\sqrt{3}\beta e^{\alpha(\frac{T-a}{b})}}[(\alpha - l\coth T)] \tag{23}$$

Now,

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\left(\frac{\alpha - l\coth\ T}{\alpha + 2\alpha\coth\ T}\right) \tag{24}$$

$$\neq 0$$

But if $\coth T = \frac{\alpha}{l}$ then

$$\frac{\sigma}{\theta} = 0 \tag{25}$$

5. Conclusion

Cylindrically symmetric cosmological model with variable cosmological constant (Λ) for dust distribution is investigated. We find that the matter density is initially large but decreases with time & tends to a finite quantity for large values of T. The model also satisfies reality condition $\rho > 0$ given by Ellis [9]. The model starts with a big bang at T = 0 and the expansion in the model decreases with time. Since $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$, hence the model does not approach isotropy for large values of T. However if cothT = $\frac{\alpha}{l}$ then the model isotropizes . The cosmological constant Λ (vacuum energy density) decreases with time and finally tend to zero for large values of T. The model also has Point Type singularity at T = 0 (MacCallum [11])

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