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HYPERSURFACES OF MATSUMOTO CONFORMAL CHANGED FINSLER SPACES

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Abstact: In the year 1984, Shibata et al. [16] investigated the theory of a change which has been called a β-change of a Finsler metric. On the other hand, in 1985 a systematic study of geometry of hypersurfaces of Finsler spaces was given by Matsumoto.[10] The present paper is devoted to study the condition for a Matsumoto conformal change to be projective and find out when a totally geodesic hypersurface F^{n-1} remains to be a totally geodesic hypersurface ${}^{*}F^{n-1}$ under the projective Matsumoto conformal change.[1] Further, we obtained the condition under which a Finslerian hypersurface given by the projective Matsumoto conformal change are projectively flat.

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1.Introduction

Let (M^n, L) be an n-dimensional Finsler space on a differential manifold M^n, equipped with the fundamental function $L(x, y)$. In 1984, Shibata et.al. [16] introduced the transformation of Finsler metric as

$$\overline{L}(x, y) = f(L, \beta).$$

where $\beta = b_i(x)y^i$ and $b_i(x)$ is a covariant vector in (M^n, L). Here f is positively homogeneous function of degree one in L and β. This change of metric has been called β-change.[8]

The conformal theory of a Finsler spaces has been initiated by Knebelman[9], in 1929 and has been investigated in detail by many authors in the papers ([4],[5],[6],[13],[17],[18] etc.) .

The conformal change has been defined as

$$L(x, y) \rightarrow e^{\sigma(x)}L(x, y)$$

where $\sigma(x)$ is a function of position only and known as conformal factor.

In the present paper, we have studied a transformation which combines above two transformations with Matsumoto metric, which generalizes all the above transformations. In fact, we consider a change of the form

$$L(x,y) \to {}^*L(x,y) = e^{\sigma(x)} \frac{L^2(x,y)}{L(x,y) - \beta(x,y)}. \tag{1}$$

where $\sigma(x)$ is a function of position only and $\beta(x,y) = b_i(x)y^i$ is 1-form in M^n, which we shall call a Matsumoto conformal change. This change generalizes various types of change. When $\beta = 0,$ it reduces to a conformal change. When $\sigma = 0,$ it reduces to Matsumoto change. When $\beta = 0$ and σ is a non-zero constant then it reduces to homothetic change.

In this paper, we have obtained the condition for a Matsumoto conformal change to be projective and find out when a totally geodesic hypersurface ${}^*F^{n-1}$ remains to be a totally geodesic hypersurface F^{n-1} under the projective Matsumoto conformal change. Further, we have obtained the condition under which a Finslerian hypersurfaces given by the projective conformal change are projectively flat.

Throughout the paper we shall confine ourselves to Cartan's connection, and the notations and terminology of the monograph [11] will be used without comment. In the paper monograph of Matsumoto[11] will be quoted by (#).

2. Relation among the quantities of Finsler spaces F^n and ${}^*F^n$

Let (M^n, L) be a Finsler space F^n, where M^n is an n-dimensional differentiable manifold equipped with a fundamental function L, defined by equation (1) has been called Matsumoto conformal change, where $\sigma(x)$ is conformal factor which is function of position only and $\beta(x,y) = b_i(x)y^i$ is 1-form in M^n. A space equipped with fundamental metric ${}^*L(x,y)$ is called conformally changed space ${}^*F^n$.

Differentiating equation (1) with respect to y^i, we obtain the relation among normalized supporting element l_i and ${}^*l_i$:

$${}^*L_i = {}^*l_i = \tau\left(l_i + e^{-\sigma} m_i\right). \tag{2}$$

Throughout the paper, we shall use the notations, symbols and well known relations used in the paper [10].

$$L_i = \frac{\partial L}{\partial y^i} = l_i, \quad L_{ij} = \frac{\partial^2 L}{\partial y^i \partial y^j}, \quad \tau = \frac{{}^*L}{L} = e^{\sigma(x)} \frac{L}{L - \beta}, \quad \text{and}$$

$$m_i = \left(e^{\sigma} - \tau\right) l_i + \tau b_i . \tag{3}$$

The vector m_i is orthogonal to the normalize supporting element l_i.

Again, differentiating equation (2) with respect to y^j we obtain

$$^{*}L_{ij} = \frac{\tau}{L}\left\{\left(2 - e^{-\sigma}\tau\right)h_{ij} + 2e^{-2\sigma} m_i m_j\right\}, \tag{4}$$

where

$$\frac{\partial \tau}{\partial y^j} = \frac{m_j}{L-\beta} = e^{-\sigma}\,\frac{\tau}{L} m_j \quad \text{and} \quad \frac{\partial m_i}{\partial y^j} = \frac{1}{L}\left\{\left(e^{\sigma-\tau}\right)h_{ij} + \left(e^{-\sigma} m_i - l_i\right)m_j\right\}.$$

If we denote $g_{ij}(x, y)$ the fundamental tensor $\dfrac{\partial^2 L^2}{2\partial y^i \partial y^j}$ and put

$$h_{ij} = g_{ij} - l_i l_j . \tag{5}$$

Then in virtue of (5) and (2) the equation (4) is rewritten as the relation between the fundamental tensors

$$^{*}g_{ij} = \tau^2\left\{\left(2 - \tau e^{-\sigma}\right)g_{ij} + \left(\tau e^{-\sigma} - 1\right)l_i l_j + 3e^{-2\sigma} m_i m_j + e^{-n}\left(l_i m_j + l_j m_i\right)\right\}. \tag{6}$$

Reciprocal of $^{*}g^{ij}$ of $^{*}g_{ij}$ has been work out using the fact that $^{*}g^{jk}\,{}^{*}g_{ij} = \delta_i^k$.

$$^{*}g^{ij} = \frac{1}{\tau^2 \mu}\left\{A g^{ij} - B\left(l^i m^j + m^i l^j\right) - 2e^{-\sigma} m^i m^j - C l^i l^j\right\}, \tag{7}$$

where $\mu = AB,\ A = 2m^2 e^{-\sigma} + 2e^{\sigma} - \tau,\ \ B = 2 - \tau e^{-\sigma}$, and

$$C = \left(\tau e^{\sigma} - 1\right)\left(A + m^2 e^{-\sigma}\right)$$

Next we obtain easily the relation between $^{*}L_{ijk}$ and L_{ijk} by differentiating equation (5) with respect to y^k and using the relation

$\dfrac{\partial h_{ij}}{\partial y^k} = 2C_{ijk} - \dfrac{1}{L}(l_i h_{jk} + l_j h_{ik})$, we obtain

$$^{*}L_{ijk} = \frac{2\tau}{L^2}\{\left(2 - \tau e^{-\sigma}\right)C_{ijk} + e^{-2\sigma}\left(e^{\sigma} - \tau\right)\mathrm{e}_{ijk} h_{ij} m_k$$
$$-\frac{1}{2}\mathrm{e}_{ijk}\left(2 - \tau e^{-\sigma}\right)h_{ij} l_k + e^{-2\sigma}\mathrm{e}_{ijk} l_i m_j m_k$$

$$+3e^{-2\sigma}m_i m_j m_k\} \tag{8}$$

where the symbol $\mathfrak{S}_{(ijk)}$ denote the sum of cyclic permutation of indices i, j and k. Meaning there by

$\mathfrak{S}_{(ijk)} h_{ij} m_k = h_{ij} m_k + h_{jk} m_i + h_{ki} m_j$

From the equation (5) we have

$$\frac{\partial h_{ij}}{\partial y^k} = 2C_{ijk} - \frac{1}{L}(l_i h_{jk} + l_j h_{ik}), \tag{9}$$

where $C_{ijk} = \frac{1}{2}\frac{\partial g_{ij}}{\partial y^k}$ and $\frac{\partial l_i}{\partial y^j} = \frac{h_{ij}}{L}$. Also, from above equation we get

$$L_{ijk} = \frac{2}{L} C_{ijk} - \frac{1}{L^2}(h_{ij} l_k + h_{jk} l_i + h_{ki} l_j) \tag{10}$$

In the virtue of equation (9) we obtain the relation between C_{ijk} and ${}^*C_{ijk}$ from (8)

$$ {}^*C_{ijk} = \frac{\tau^2}{L}\left\{LBC_{ijk} + (4-3\tau)(h_{ij} m_k + h_{jk} m_i + h_{ki} m_j) + 12 m_i m_j m_k\right\}. \tag{11}$$

Now, we consider the relation between the Cartan's connection F^i_{jk} and ${}^*F^i_{jk}$ in the paper as [11]

$$D^i_{jk} = {}^*F^i_{jk} - F^i_{jk} \tag{12}$$

Transvecting above equation by y^j and using $F^i_{jk} y^j = G^i_k$, and $D^i_{jk} y^j = D^i_k$, we get

$$ {}^*G^i_k = G^i_k + D^i_k \tag{13}$$

where the subscript '0' denote the contraction by the supporting element y^j.

Further, transvecting (13)by y^k and using $G^i_k y^k = G^i$, and $D^i_k y^k = D^i$, we get

$$ {}^*G^i = G^i + D^i \tag{14}$$

These relations used in latter articles.

3. Relation between projective change and Matsumoto conformal change

For two Finsler spaces $F^n = (M^n, L)$ and $^*F^n = (M^n, {}^*L)$ if any geodesic of F^n is also a geodesic of $^*F^n$ and vice-versa, then the change $L \to {}^*L$ is called projective change. A geodesic of F^n is given by a system of differential equations

$$\frac{d^2 y^i}{dt^2} + 2G^i(x, y) = y^i, \quad \text{where } y^i = \frac{dx^i}{dt}.$$

and $G^i(x, y)$ are second degree positively homogeneous function in y^i. We are now in a position to find condition under which Matsumoto conformal change is projective. Let us consider Euler's-Lagrange differential equation of a geodesic in the form,

$$B_i = \frac{\partial L}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial y^i}\right) = 0.$$

Similar Euler-Lagrenge's differential equation for $^*L$, will be

$$^*B_i = \frac{\partial\, {}^*L}{\partial x^i} - \frac{d}{dt}\left({}^*L_i\right) = 0. \tag{15}$$

Differentiating $^*L = \tau L$ with respect to x^i, we have

$$\frac{\partial\, {}^*L}{\partial x^i} = \tau\left\{\left(2 - \tau e^{-\sigma}\right)\frac{\partial L}{\partial x^i} + L\frac{\partial \sigma}{\partial x^i} + \tau e^{-\sigma}\frac{\partial \beta}{\partial x^i}\right\}.$$

Differentiating equation (2) with respect to t , we have

$$\frac{d}{dt}\left({}^*L_i\right) = \tau\left(2 - \tau e^{-\sigma}\right)\frac{dL_i}{dt} + \left\{\left(1 - \tau e^{-\sigma}\right)l_i + \tau b_i\right\}\frac{d\tau}{dt}.$$

Substituting above values in equation (15), we get

$$^*B_i = \tau\left(2 - \tau\rho^{-\sigma}\right)B_i + \tau^2\rho^{-\sigma}\frac{\partial \beta}{\partial x^i} - \left\{\left(1 - \tau\rho^{-\sigma}\right)l_i + \tau b_i\right\}\frac{d\tau}{dt} = 0. \tag{16}$$

which leads to

$$^*B_i = e^{\sigma(x)}B_i + A_i = 0. \tag{17}$$

where, is a covariant vector.

Thus we have,

Theorem 3.1*Matsumoto conformal change (1) is projective iff covariant vector A_i given by equation (17) vanishes identically.*

4. Hypersurface of a projective Matsumoto conformal changed space

A hypersurface M^{n-1} of a underlying smooth manifold M^n may be parametrically represented by the equation $x^i = x^i(u^\alpha)$, where u^α are Gaussian co-ordinate on M^{n-1} [3] and α varies from 1 to n-1. Here we shall assume that the matrix consisting of the projection factor $B^i_\alpha = \dfrac{\partial x^i}{\partial u^\alpha}$ is of rank n-1.

To introduce a Finsler structure in M^{n-1}, the supporting element y^i at a point (u^α) of M^{n-1} is assumed to be tangential to M^{n-1}, so that we may write

$$y^i = B^i_\alpha(u)v^\alpha. \tag{18}$$

Thus v^α is thought of as supporting element of M^{n-1} at the point (u^α).

Since the function ${}^*L(u,v) = L\{x(u), y(u,v)\}$ gives rise to a Finsler metric of M^{n-1}, we get a (n-1)dimensional Finsler space $F^{n-1} = \{M^{n-1}, {}^*L(u,v)\}$.

The fundamental function ${}^*L(u,v)$ of this Finslerian hypersurfaces F^{n-1} of F^n is called the induced metric of F^{n-1}

$$B^i_{\alpha\beta} = \frac{\partial \beta^i_\alpha}{\partial u^\beta} = \frac{\partial x^i}{\partial u^\alpha \partial u^\beta}, \quad B^i_{0\beta} = v^\alpha B^i_{\alpha\beta}. \tag{19}$$

At each point (u^α) of F^{n-1}, the unit normal vector $N^i(u,v)$ is defined by

$$(i) \quad g_{ij}B^i_\alpha N^j = 0, \quad (ii) \quad g_{ij}N^i N^j = 1. \tag{20}$$

If (B^α_i, N_i) is the inverse of the matrix (B^i_α, N^i), then

$$B^i_\alpha B^\beta_i = \delta^\beta_\alpha, \quad B^i_\alpha N_i = 0, \quad N^i N_i = 1 \text{ and } B^i_\alpha B^\alpha_j + N^i N_j = \delta^i_j.$$

$$\because \quad B^\alpha_i = g^{\alpha\beta} g_{ij}, \quad N_i = g_{ij}N^j. \tag{21}$$

For induced Cartan's connection $C\Gamma = (F^\alpha_\beta\gamma, N^\beta_\alpha, C^\alpha_{\beta\gamma})$ on F^{n-1}, the normal curvature vector H_α is given by,

$$H_\alpha = N_i(B^i_{0\alpha} + N^i_j B^j_\alpha). \tag{22}$$

Consider a Finslerian hypersurface $F^{n-1} = \{M^{n-1}, {}^*L(u,v)\}$ of the F^n and another Finsler hypersurface ${}^*F^{n-1} = \{M^{n-1}, {}^*L(u,v)\}$ of the ${}^*F^n$ given by the Matsumoto conformal change. Let N^i be the unit vector at each point of F^{n-1} and (B_i^α, N_i) be the inverse of the matrix of (B_α^i, N^i). The function B_α^i may be considered as component of (n-1) linearly independent tangent vectors of F^{n-1} and they are invariant under Matsumoto conformal change. Thus we shall show that a unit normal vector ${}^*N^i(u,v)$ of F^{n-1} is uniquely determined by

$$(i)\quad {}^*g_{ij}B_\alpha^i\ {}^*N^j = 0, \quad (ii)\quad {}^*g_{ij}\ {}^*N^i\ {}^*N^j = 1 \tag{23}$$

Contracting (6) by N^iN^j and paying attention to the fact that $l_iN^i = 0$ and using equation (20) we have

$${}^*g_{ij}N^iN^j = \tau^2\left\{\left(2-\tau e^{-\sigma}\right)+3e^{-2\sigma}\tau^2\left(b_iN^i\right)^2\right\}$$

It can also be written as

$${}^*g_{ij}\left\{\pm\frac{N^i}{\tau\sqrt{\left(2-\tau e^{-\sigma}\right)+3e^{-2\sigma}\tau^2\left(b_iN^i\right)^2}}\right\} \tag{24}$$

$$\times\left\{\pm\frac{N^j}{\tau\sqrt{\left(2-\tau e^{-\sigma}\right)+3e^{-2\sigma}\tau^2\left(b_iN^i\right)^2}}\right\} = 1.$$

Since,

$${}^*N^i = \frac{N^i}{\tau\sqrt{\left(2-\tau e^{-\sigma}\right)+3e^{-2\sigma}\tau^2\left(b_iN^i\right)^2}}. \tag{25}$$

Substituting the values from equations (16) and (25) in the equation (i) of (23) we have

$$\tau^2\left\{\left(2-\tau e^{-\sigma}\right)g_{ij}+\left(\tau e^{-\sigma}-1\right)l_il_j+3m_im_je^{-2\sigma}+\tau e^{-\sigma}\left(l_im_j+l_jm_i\right)\right\}$$
$$\times\frac{B_\alpha^iN^j}{\tau\sqrt{\left(2-\tau e^{-\sigma}\right)+3e^{-2\sigma}\tau^2\left(b_iN^i\right)^2}} = 0.$$

Contracting above equations by v^α and using $y^i = B_\alpha^i v^\alpha$, we get

$$\tau e^{-\sigma}\left\{3\tau e^{-\sigma}\left(b_i-l_i\right)+4l_i\right\}B_\alpha^i = 0.$$

In the virtue of (20) above equation becomes

$$\left\{3\tau^2 e^{-2\sigma}\left(b_i - l_i\right) + 4\tau e^{-\sigma} l_i\right\} \frac{B^i_\alpha N^j b_j}{\tau\sqrt{\left(2-\tau e^{-\sigma}\right) + 3e^{-2\sigma}\tau^2\left(b_i N^i\right)^2}} \tag{26}$$

$$= 0.$$

which implies that

$$3\tau e^{-\sigma}\left(\beta - L\right) + 4L = 0.$$

Thus, we have

$$L = 0 \quad \left(\because \quad \tau e^{-\sigma}\left(L - \beta\right) = L\right),$$

which contradicts the assumption that $L > 0$. So that (25) gives $b_i N^i = 0$. Therefore equation (25) can be rewritten as

$$^*N^i = \frac{N^i}{\tau\sqrt{\left(2-\tau e^{-\sigma}\right)}} \tag{27}$$

Thus we have

Proposition 4.1 *If* $\{(B^\alpha_i N^i), \alpha = 1,2,3,....,(n-1)\}$ *be the field of linear frame of the Finsler space* F^n *and we consider* $(B^\alpha_i N^i), \alpha = 1,2,3,...,(n-1)\}$ *as a linear frame of the Finsler space* $^*F^n$ *such that (4.10) holds good along* $^*F^{n-1}$ *then* b_i *is tangential to both the hypersurfaces* F^{n-1} *and* $^*F^{n-1}$.

Now, we are going to examine, under what condition Matsumoto conformal change of the metric is projective also.

The quantities $^*B^\alpha_i$ are uniquely has been defined [10] along $^*F^{n-1}$ by

$$^*B^\alpha_i = {^*g^{\alpha\beta}}\ {^*g_{ij}} B^j_\beta$$

where $^*g^{\alpha\beta}$ is the inverse matrix of $^*g_{\alpha\beta}$.

Let $(^*B^\alpha_i, {^*N_i})$ be the inverse matrix of $(B^i_\alpha, {^*N^i})$, then

$$B^i_\alpha\ {^*B^\beta_i} = \delta^\beta_\alpha, \quad B^i_\alpha\ {^*N_i} = 0, \quad {^*N^i}\ {^*N_i} = 1.$$

In the virtue of $B^i_\alpha\ {^*B^\alpha_j} + {^*N^i}\ {^*N_j} = \delta^i_j$ and $^*N_i = {^*g_{ij}}\ {^*N^j}$ the equations (6) and (27) give

$$ {}^{*}N_i = {}^{*}g_{ij}\ {}^{*}N^j = \frac{N^j\ {}^{*}g_{ij}}{\tau\sqrt{2-\tau e^{-\sigma}}} = \tau\ \sqrt{2-\tau e^{-\sigma}}\ N_i\ , $$

$$ {}^{*}N_i = \tau\ \sqrt{2-\tau e^{-\sigma}}\ N_i. \tag{28} $$

Differentiating equation (14) with respect to y^j and using $D^i_j = \dfrac{\partial D^i}{\partial y^j}$ and $N^i_j = \dfrac{\partial G^i}{\partial y^j}$, we get

$$ D^i_j = {}^{*}N^i_j - N^i_j. \tag{29} $$

Further, contracting above equation by $N_i B^j_\alpha$ we have

$$ N_i D^i_j B^j_\alpha = {}^{*}N^i_j N_i B^j_\alpha - N^i_j N_i B^j_\alpha, $$

$\Rightarrow$

$$ N_i D^i_j B^j_\alpha = 0. \tag{30} $$

If each geodesic of $F^{(n-1)}$ with respect to the induced metric is also a geodesic of $F^{(n)}$, then $F^{(n-1)}$ is called totally geodesic[10]. A totally geodesic hypersurfaces is characterized by [2, 7, 12, 14] $H_\alpha = 0$.

From (22), (28) and (30) we have

$$ {}^{*}H_\alpha = {}^{*}N_i({}^{*}B^i_{0\alpha} + {}^{*}N^i_j\ {}^{*}B^j_\alpha), $$

$$ = {}^{*}N_i\ {}^{*}B^i_{0\alpha} + {}^{*}N_i D^i_j\ {}^{*}B^j_\alpha + {}^{*}N_i\ N^i_j\ {}^{*}B^j_\alpha, $$

$$ {}^{*}H_\alpha = \tau\sqrt{2-\tau e^{-\sigma}} H_\alpha + N_i D^i_j\ {}^{*}B^j_\alpha. \tag{31} $$

In view of equation (31) above equation gives

$$ {}^{*}H_\alpha = \tau\sqrt{2-\tau e^{-\sigma}} H_\alpha \tag{32} $$

Thus we have

Theorem 4.1 *A hypersurface F^{n-1} of a Finsler space $F^n (n > 3)$ is totally geodesic iff the hypersurface ${}^{*}F^{(n-1)}$ of the space ${}^{*}F^n$ obtained from F^n by a projective Matsumoto conformal change, is also totally geodesic.*

5.Hypersurfaces of projectively flat Finsler spaces

In this section, we shall consider a projective Matsumoto conformal change with the Berwald connection $B\Gamma$ of $F^n = (M^n, L)$ and $B\Gamma$ on ${}^*F^n = ({}^*M^n, L)$. In the theory of projective changes in the Finsler spaces we have two essential projective invariants, one is the Weyle torsion W_{ij}^h and other is the Douglas tensor D_{ijk}^h, so the under projective Matsumoto conformal change, we get ${}^*W_{ij}^h = W_{ij}^h$ and ${}^*D_{ijk}^h = D_{ijk}^h$.

Now we are concerned with a projectively flat Finsler spaces defined as follows if there exist a projective change $L \to {}^*L$ of a Finsler space $F^n = (M^n, L)$ such that the Finsler space ${}^*F^n = ({}^*M^n, L)$ is a locally Minkowski space then F^n is called projectively flat Finsler space. Following two theorems [15] are well-known:

Theorem 5.1*A Finsler space* $F^n (n > 2)$ *is projectively flat iff* $W_{ijk}^h = 0$ *and* $D_{ijk}^h = 0$.

Theorem 5.2*A Finsler space* $F^n (n > 3)$ *is projectively flat then the totally geodesic hypersurfaces* F^{n-1} *is also projectively flat.*

Thus from theorem (18), theorem (5.1) and (5.2), we have

Theorem5.3 *Let* $F^n (n > 3)$ *be a projectively flat Finsler space. If the hypersurfaces* F^{n-1} *is totally geodesic, then the hypersurfaces* ${}^*F^{(n-1)}$ *of the space* ${}^*F^n$ *obtained from* F^n *by a projective Matsumoto conformal change, is also projectively flat.*

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