Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.15, No.3, September, 2016, 221-238

SORET EFFECT IN MHD FLOW PAST AN INFINITE VERTICAL PLATE MOVING WITH VARIABLE VELOCITY WITH CHEMICAL REACTION AND THERMAL RADIATION

A.H. Sheikh and N. Ahmed
Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India
Email: alihussainmath@gmail.com; saheel_nazib@yahoo.com

Abstract: This paper deals with the problem of transient MHD free convection flow of an electrically conducting viscous incompressible fluid past an infinite vertical isothermal plate moving with variable velocity taking into account the effect of chemical reaction, thermal radiation and thermal diffusion. A uniform magnetic field is applied transversely to the plate and directed to the fluid region. The resultant system of governing equations is solved by Laplace Transform technique in closed form. The expressions for the velocity field, temperature field, concentration field, the coefficient of skin friction at the plate in the direction of flow and the coefficient of heat and mass transfer in terms of Nusselt number and Sherwood number at the plate are obtained in non-dimensional forms. The effects of Soret number, time, thermal Grashof number, Solutal Grashof number, Hartmann number, chemical reaction parameter, Schmidt number and radiation parameter on the flow and transport characteristics are studied through graphs and the result is physically interpreted.
Keywords: MHD, Radiation effect, Thermal diffusion, Heat and Mass transfer, Nusselt number,
MSC 2010 No.: 76D05, 80A20

Nomenclature

B_0	Strength of the applied magnetic field	t	Dimensionless time
$\overline{C}$	Species concentration	t_0	Characteristics time
C_w	Species concentration at the plate	$\overline{T}$	Fluid temperature
C_∞	Species concentration in free stream	T_w	Temperature at the plate
$\overline{C}r$	Rate of first order homogenous chemical reaction	T_∞	Temperature at the free stream
Cr	Non-dimensional chemical reaction parameter	$\overline{u}$	Dimensional velocity
		u	Dimensionless velocity
		u_0	Reference velocity

C_p Specific heat at constant pressure
D Mass diffusion coefficient
D_T Thermal diffusion coefficient
erf Error function
$erfc$ Complementary error function
g Acceleration due to gravity
Gr Grashof number for heat
Gm SolutalGrashof number for mass
κ Thermal conductivity
M Hartmann number
N Radiation parameter
Nu Nusselt number
Pr Prandtl number
q_r The radiative heat flux
Sc Schmidt number
Sr Soret number
Sh Sherwood number
$\bar{t}$ Time
$(\bar{x}, \bar{y})$ Coordinate system
(x, y) Non-dimensional coordinate system

Greek symbols

$\bar{a}$ Mean radiation absorption coefficient
τ Dimensionless skin friction
β Coefficient of volume expansion for heat transfer
$\bar{\beta}$ Coefficient of volume expansion for mass transfer
υ Kinematic viscosity
μ Dynamic viscosity
σ Electric conductivity
ρ Fluid density
θ Non-dimensional temperature
ϕ Non-dimensional concentration

The other symbols have their usual meanings.

1. Introduction

MHD is the science of motion of electrically conducting fluids in presence of magnetic field. MHD generators, MHD pumps and MHD flow meters are some of the numerous examples of MHD principles. Dynamo and motor is a basic example of MHD principle. Free convection problems of electrically conducting fluid in presence of magnetic field have got much importance because of its wide range of applications in Geophysics, Astrophysics, Plasma Physics, Missile technology etc.. MHD principles also find its applications in Medicine and Biology. In case of the development of the subject MHD, the pioneer contributions of several notable authors like Alfven [1], Cowling [4], Shercliff [14] and Crammer and Pai [5] are worthwhile to mention.

Earlier it was considered that, the mass transfer occur only due to concentration gradients. But after the pioneering work of Eckert and Drake [7], researchers believe that, in presence of high temperature gradient, species transportation may also take place. The process of mass transfer that occurs due to the combine effects of concentration as well as temperature gradients is known as thermal diffusion (Soret effect). Study on Soret effect was made by Platten and Chavepeyer [11], who investigated an oscillatory motion in Benard cell. Besides the aforesaid works, some more notable contribution in this regard are made by Jha and Singh [8], Dursunkaya and Worek [6] etc. Recently, Sengupta and Ahmed [13] obtained the closed form solution of the problem to investigate the chemical reaction interaction on unsteady MHD free convection radiative flow past an oscillating plate embedded in porous media with thermal diffusion.

Many processes in engineering as well as in industry occur at very high temperatures and so knowledge of radiation heat transfer becomes very significant. Cess [2] investigated radiation effects on free convective heat transfer flow. The case of unsteady flow in presence of radiation and variable viscosity on MHD flow past a semi-infinite flat plate with an aligned magnetic field was studied by Seddeek [12]. Manivannan et al. [9] investigated the effect of thermal radiation on an isothermal vertical oscillatory plate by considering variable mass diffusion and chemical reaction.

The study of heat and mass transfer problems with chemical reaction is of great practical importance to engineers and scientists, because of its almost universal occurrence in many branches of science and engineering. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role are found in chemical process industries such as food processing and polymer production. Chambre and Young [3] analyzed a first-order chemical reaction in the neighborhood of a horizontal plate.

The prime objective of the present work is to study the effect of thermal diffusion (Soret effect) on the flow and transport characteristics. In addition, we have proposed to focus the influence of time, thermal Grashof number, SolutalGrashof number, Hartmann number, chemical reaction parameter, Schmidt number and radiation parameter on the flow characteristics. Further it may be stated that our present work is a generalization of the work done by Muthucumaraswamy et al. [10] to include the thermal diffusion.

2. Mathematical analysis

Let a two dimensional flow of an incompressible viscous electrically conducting fluid with a suddenly started motion of an infinite isothermal vertical plate with chemical reaction and thermal radiation in presence of Soret effect be presented. The velocity of the plate varies as the square of the time. That is the plate velocity verses time relation is parabolic. We introduce a Cartesian coordinate system $(\bar{x}, \bar{y})$ with $\bar{x}$ axis along the infinite vertical plate and $\bar{y}$ axis normal to the plate. Initially the plate and the fluid were the same temperature $\overline{T}_\infty$ with concentration $\overline{C}_\infty$ at all points. At time $\bar{t} > 0$ the plate temperature is suddenly raised to T_w and the concentration level at the plate gets raised to C_w. A magnetic field of uniform strength B_0 is applied normal to the plate. We assume that the plate is suddenly moved with a velocity $\frac{u_0}{t_0^2}\bar{t}^2$ in its own plane against the gravitational field at time $\bar{t} > 0$. With the foregoing assumptions and under usual boundary layer and Boussinesq approximations, the governing equations are:

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = g\beta\left(\overline{T} - T_\infty\right) + g\bar{\beta}\left(\overline{C} - C_\infty\right) + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho}\bar{u} \tag{1}$$

Energy equation:

$$\rho C_p \frac{\partial \overline{T}}{\partial \bar{t}} = \kappa \frac{\partial^2 \overline{T}}{\partial \bar{y}^2} - \frac{\partial q_r}{\partial \bar{y}} \tag{2}$$

Species continuity equation:

$$\frac{\partial \overline{C}}{\partial \bar{t}} = D \frac{\partial^2 \overline{C}}{\partial \bar{y}^2} - \overline{C}_r \left(\overline{C} - C_\infty \right) + D_T \frac{\partial^2 \overline{T}}{\partial \bar{y}^2} \tag{3}$$

The initial and boundary conditions are:

$$\bar{u} = 0,\ \ \overline{T} = T_\infty,\ \ \overline{C} = C_\infty \text{ for all } \bar{y}, \bar{t} \le 0 \tag{4.1}$$

$$\bar{t} > 0:\ \ \bar{u} = \frac{u_0}{t_0^2} \bar{t}^{\,2},\ \ \overline{T} = T_w,\ \ \overline{C} = C_w \text{ at } \bar{y} = 0 \tag{4.2}$$

$$\bar{u} \to 0,\ \ \overline{T} \to T_\infty,\ \ \overline{C} \to C_\infty \text{ at } \bar{y} \to \infty \tag{4.3}$$

The local radiant for the case of optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial \bar{y}} = -4 \bar{a} \sigma \left(T_\infty^4 - \overline{T}^{\,4} \right) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that $\overline{T}^{\,4}$ may be expressed as a linear function of the temperature. This is accomplished by expanding $\overline{T}^{\,4}$ in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$\overline{T}^{\,4} \cong 4 T_\infty^3 \overline{T} - 3 T_\infty^4 \tag{6}$$

By the use of the equations (5) and (6), the energy equation (2) becomes

$$\rho C_p \frac{\partial \overline{T}}{\partial \bar{t}} = \kappa \frac{\partial^2 \overline{T}}{\partial \bar{y}^2} + 16 \bar{a} \sigma T_\infty^3 \left(T_\infty - \overline{T} \right) \tag{7}$$

To normalize the mathematical model, the following non-dimensional variables and parameters have been introduced.

$$u = \frac{\bar{u}}{u_0},\ y = \frac{\bar{y}}{t_0 u_0},\ t = \frac{\bar{t}}{t_0},\ t_0 = \frac{\upsilon}{u_0^2},\ \theta = \frac{\overline{T} - T_\infty}{T_w - T_\infty},\ \phi = \frac{\overline{C} - C_\infty}{C_w - C_\infty},\ M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2},\ N = \frac{16 \bar{a} \sigma T_\infty^3}{\kappa} \left(\frac{\upsilon}{u_0} \right)^2,$$

$$Gr = \frac{\upsilon \beta g \left(T_w - T_\infty \right)}{u_0^3},\ Gm = \frac{\upsilon \bar{\beta} g \left(C_w - C_\infty \right)}{u_0^3},\ Sc = \frac{\upsilon}{D},\ Pr = \frac{\mu C_p}{\kappa},\ Cr = \frac{\upsilon \overline{C} r}{u_0^2},\ Sr = \frac{D_T \left(T_w - T_\infty \right)}{\upsilon \left(C_w - C_\infty \right)}$$

The dimensionless form of the equations (1), (7) and (3) are:

$$\frac{\partial u}{\partial t} = Gr\theta + Gm\phi + \frac{\partial^2 u}{\partial y^2} - Mu \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{N}{Pr}\theta \tag{9}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - Cr\phi + Sr\frac{\partial^2 \theta}{\partial y^2} \tag{10}$$

The relevant initial and boundary conditions are

$$u = 0,\ \theta = 0,\ \phi = 0 \text{ for all } y, t \le 0 \tag{11.1}$$

$$t > 0 : u = t^2,\ \theta = 1,\ \phi = 1 \text{ at } y = 0 \tag{11.2}$$

$$u \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } y \to \infty \tag{11.3}$$

By taking Laplace transforms of the equations (8), (9) and (10), the following differential equations are obtained:

$$\frac{d^2\bar{u}}{dy^2} - (s + M)\bar{u} = -Gr\bar{\theta} - Gm\bar{\phi} \tag{12}$$

$$\frac{d^2\bar{\theta}}{dy^2} - Pr(s + a)\bar{\theta} = 0 \tag{13}$$

$$\frac{d^2\bar{\phi}}{dy^2} - Sc(s + Cr)\bar{\phi} = -\frac{ScSrPr(s + a)}{s} e^{-y\sqrt{\Pr(s+a)}} \tag{14}$$

$$\text{where } \left.\begin{aligned} \bar{u}(y,s) &= \int_0^\infty u(y,t)e^{-st}dt \\ \bar{\theta}(y,s) &= \int_0^\infty \theta(y,t)e^{-st}dt \\ \bar{\phi}(y,s) &= \int_0^\infty \phi(y,t)e^{-st}dt \end{aligned}\right\} \tag{15}$$

and $a = \dfrac{N}{Pr}$

The corresponding boundary conditions are:

$$\bar{u} = \frac{2}{s^3}, \bar{\theta} = \frac{1}{s}, \phi = \frac{1}{s} \text{ at } y = 0 \tag{16.1}$$

$$\bar{u} \to 0, \bar{\theta} \to 0, \bar{\phi} \to 0 \text{ as } y \to \infty \tag{16.2}$$

The solutions of the equations (12), (13) and (14) subject to the boundary conditions (16.1) and (16.2) are as follows:

$$\begin{aligned}\bar{u} = {} & \frac{2}{s^3}e^{-y\sqrt{P_3}} - \frac{B}{s(s-c)}\left(e^{-y\sqrt{P_3}} - e^{-y\sqrt{P_1}}\right) - \frac{E}{s(s-d)}\left(e^{-y\sqrt{P_3}} - e^{-y\sqrt{P_2}}\right) \\ & - \frac{AE(s+a)}{s(s-b)(s-d)}\left(e^{-y\sqrt{P_3}} - e^{-y\sqrt{P_2}}\right) + \frac{AF(s+a)}{s(s-b)(s-c)}\left(e^{-y\sqrt{P_3}} - e^{-y\sqrt{P_1}}\right)\end{aligned} \tag{17}$$

$$\bar{\theta} = \frac{1}{s}e^{-y\sqrt{P_1}} \tag{18}$$

$$\bar{\phi} = \frac{1}{s}e^{-y\sqrt{P_2}} + \frac{A(s+a)}{s(s-b)}\left(e^{-y\sqrt{P_2}} - e^{-y\sqrt{P_1}}\right) \tag{19}$$

where $a, b, c, d, P_1, P_2, P_3, A, B, E, and\ F$ are defined in the Appendix.

Taking inverse Laplace transforms of the equations (17), (18) and (19), we get

$$u = 2g - A_4 f_1 - A_5 f_2 - A_6 f_3 + A_7 f_4 + A_8 f_5 - A_9(f_6 - f_7) - A_{10}(f_8 - f_9) - A_{11} f_{10} \tag{20}$$

$$\theta = f_1 \tag{21}$$

$$\phi = A_1 f_1 + A_2 f_2 + A_3(f_4 - f_3) \tag{22}$$

where
$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, f_1, f_2, f_3, f_4, f_5, f_6, f_7. f_8, f_9, f_{10}\ and\ g$ are defined in Appendix.

3. Skin friction

The coefficient of skin friction τ at the plate in the direction of the flow is given by

$$\tau = -\left[\frac{\partial u}{\partial y}\right]_{y=0} = 2\bar{g} - A_4 h_1 - A_5 h_2 - A_6 h_3 + A_7 h_4 + A_8 h_5 - A_9(h_6 - h_7) - A_{10}(h_8 - h_9) - A_{11} h_{10} \tag{23}$$

where $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, \bar{g}\ and\ h_{10}$ are defined in the Appendix.

4. Nusselt number

The coefficient of rate of heat transfer at the plate in term of Nusselt number is specified by

$$Nu = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0} = h_1 \tag{24}$$

5. Sherwood number

The coefficient of rate of mass transfer at the plate in terms of Sherwood number in is given by

$$Sh = -\left[\frac{\partial \phi}{\partial y}\right]_{y=0} = A_1 h_1 + A_2 h_2 + A_3 (h_4 - h_3) \tag{25}$$

6. Results and discussion

In order to get a physical view of the problem, numerical computations from the analytical solutions are carried out for the velocity field (*u*), temperature field (θ), concentration field (ϕ) and skin friction (τ), Nusselt number (*Nu*) and Sherwood number (*Sh*). The influence of the Soret number (*Sr*), time parameter (*t*), thermal Grashof number (*Gr*), SolutalGrashof number (*Gm*), Hartmann number (*M*), chemical reaction parameter (*Cr*), Schmidt number (*Sc*) and radiation parameter (*N*) on the flow and, heat and mass transfer characteristics have been depicted graphically. Throughout our investigation, the value of Prandtl number (*Pr*) is chosen as 0.71 which represent air at 20^0 temperatures and 1 atmospheric pressure. The value of Schmidt number (*Sc*) is taken as 0.60 which represent water vapour. The values of the other parameters namely radiation parameter, thermal Grashof number, chemical reaction parameter, SolutalGrashof number, Hartmann number and time parameter are chosen arbitrary.

The figures 1-4 exhibit the behaviour of the velocity field (*u*) due to variation of *Sr*, *t*, *Gr* and*Gm*. These figures show that the flow is accelerated for the increasing values of *Sr*, *t*, *Gr* and *Gm*. This phenomenon indicates that the thermal diffusion accelerates the flow. Further it is evident that a rise in concentration buoyancy force causes a comprehensive increase in the fluid velocity as mark from figure 4. It is inferred from Figure 3 that there is a substantial growth in the fluid velocity due to thermal buoyancy force. All the figures 1-4 exhibit a common fact that the fluid velocity first increases in a thin layer adjacent to the plate and there after it falls asymptotically at a distance far away from the plate.This establishes a phenomenon that the buoyancy effect is significant near the plate, and the effect gets nullified as we move away from the plate.

The figures 5, 6 and 7 demonstrate the behaviour of the velocity field under the influence of *M*, *Cr* and *Sc*. These figures show that the flow is retarded for increasing values of *M*, *Cr* and *Sc*. That is, an increase in magnetic field intensity leads to a fall in fluid velocity.

This phenomenonagrees with the expectations, that the magnetic field exerts a retarding force on the flow. Further we recall that an increase in Schmidt number means a decrease in mass diffusivity. This observation has an excellent agreement to the physical fact that the fluid moves freely as it becomes less dense due to high mass diffusivity.

Figure 8 represents the behaviour of the fluid temperature distribution against the variation of radiation parameter. It is seen that the temperature drops due to thermal radiation.

The variation of concentration field under the effectof the Soret number is presented in Figure 9. This Figure shows that,there is a substantial increase in the concentration level of the fluid under the effect of thermal diffusion.

It is inferred from the figure 10 that the skin friction decreases as Schmidt number increases. But the reverse trend of behaviour is observed for the Soret number in Figure 11. In other words the viscous drag on the plate is reduced for increasing Schmidt number or for decreasing the mass diffusivity of the fluid.

The influence of the radiation on the Nusselt number is illustrated in Figure 12. It is observed that the Nusselt number increases with radiation. This establishes the fact that the rate of heat transfer from the plate to the fluid rises under the radiation effect.

The figure 13 depicts how the Soret number influences the Sherwood number on the plate. This figure prevails that the rate of mass transfer from the plate to the fluid is reduced under the effect of thermal diffusion.

7. Conclusions

(i) The fluid velocity is accelerated due to increase in Soret number, thermal Grashof number andSolutalGrashof number; whereas reverse phenomenon are observed under the influence of magnetic field, chemical reaction parameter or Schmidt number. Further it seems that the fluid velocity increases as time progresses.

(ii) The temperature of the plate decreases under the thermal radiation effect.

(iii) The concentration level of the fluid rises due to thermal diffusion.

(iv) The viscous drag on the plate decreases with the increase of Schmidt number whilst this trend gets reversed under the effect of Soret number.

(v) The rate of heat transfer on the plate increases with the increasing values of the radiation parameter.

(vi) The rate of mass transfer from the plate to the fluid gets decreased under the influence of thermal diffusion.

(vii) The fluid temperature and the species concentration decrease steadily with the increasing distance from the plate surface.

(viii) The heat and mass flux decrease steadily as time progresses.

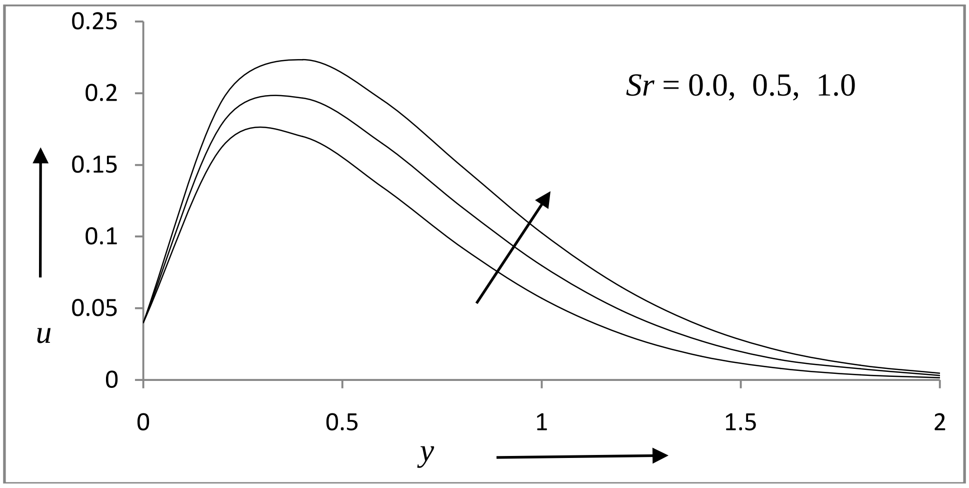

Figure 1: Velocity field u against y, for N=5, Pr=0.71, Gr=2, Cr=30, Sc=0.6, Gm=5, M=1, t=0.2.

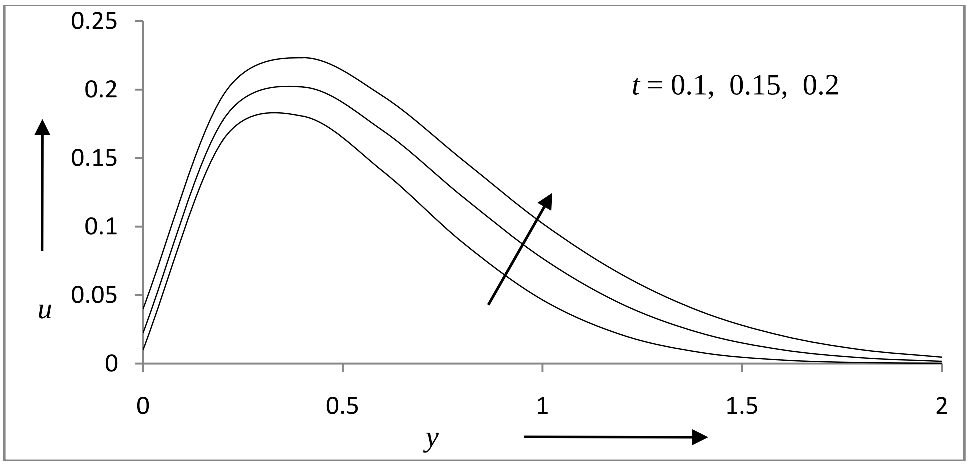

Figure 2: Velocity field u against y, for N=5, Pr=0.71, Gr=5, Cr=30, Sc=0.6, Sr=1, M=1, Gm=5.

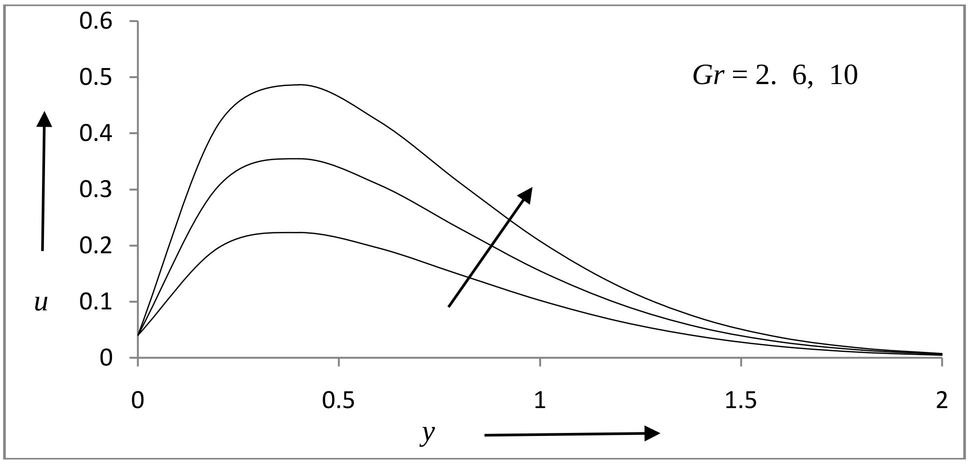

Figure 3: Velocity field u against y, for N=5, Pr=0.71, Gm=5, Cr=30, Sc=0.6, Sr=1, M=1, t=0.2.

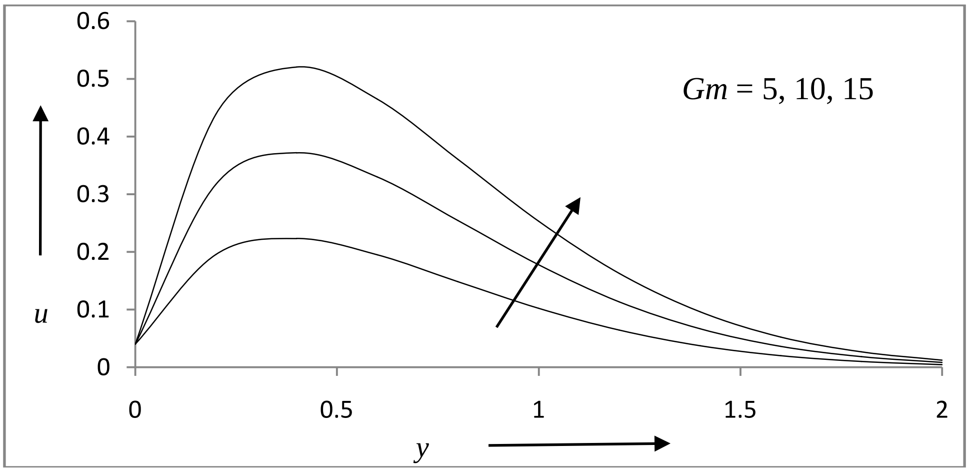

Figure 4: Velocity field u against y, for N=5, Pr=0.71, Gr=2, Cr=30, Sc=0.6, Sr=1, M=1, t=0.2.

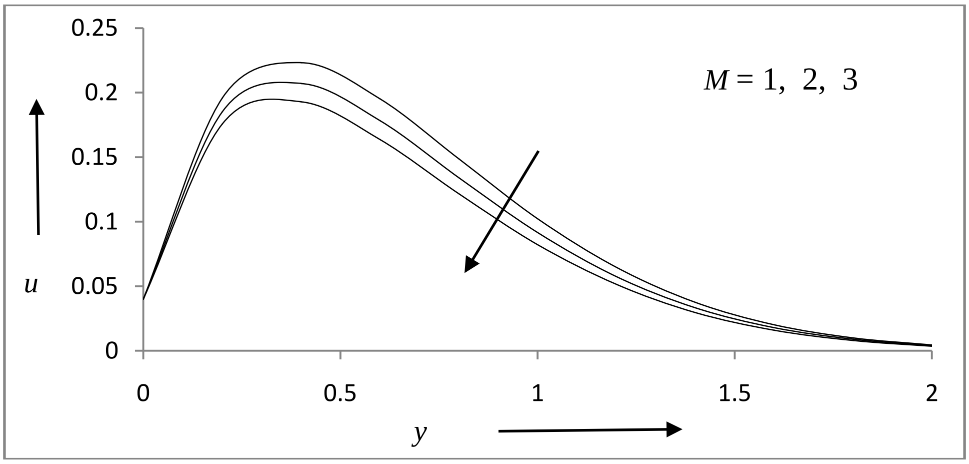

Figure 5: Velocity field *u* against *y*, for *N*=5, *Pr*=0.71, *Gr*=2, *Cr*=30, *Sc*=0.6, *Sr*=1, *Gm*=1, *t*=0.2.

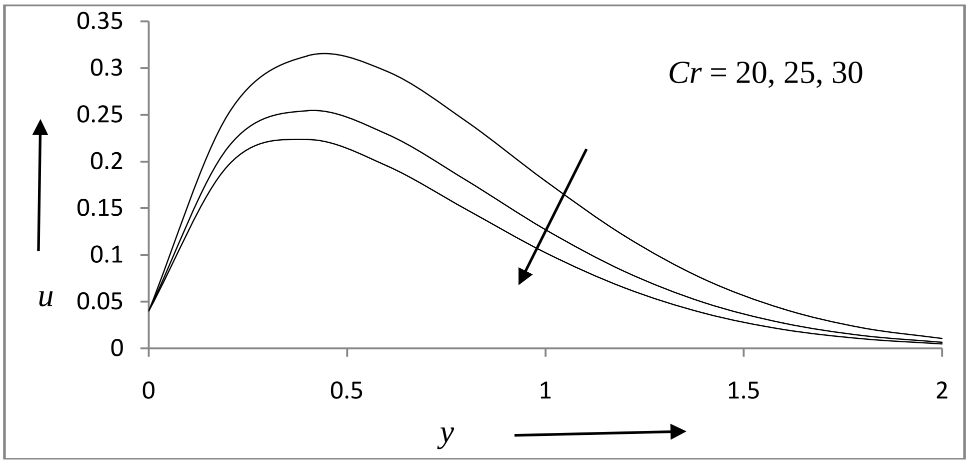

Figure 6: Velocity field *u* against *y*, for *N*=5, *Pr*=0.71, *Gr*=2, *Gm*=5, *Sc*=0.6, *Sr*=1, *M*=1, *t* =0.2.

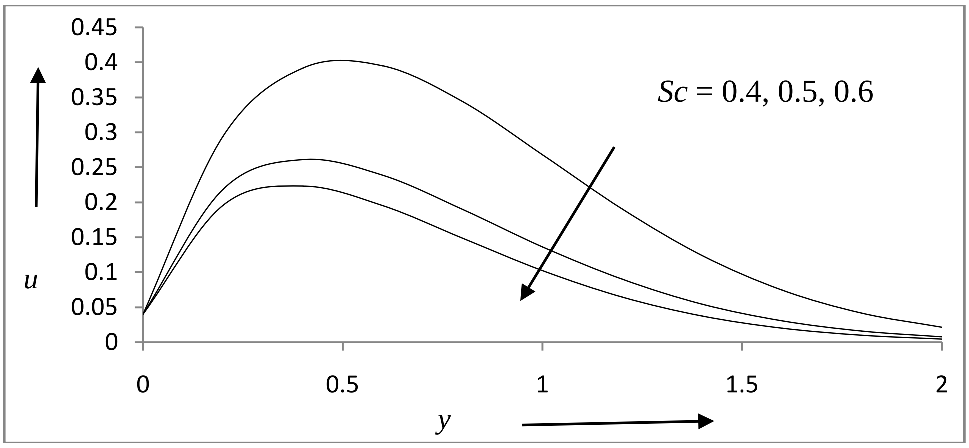

Figure 7: Velocity field u against y, for N=5, Pr=0.71, Gr=2, Gm=5, Cr=30, Sr=1, M=1, t =0.2.

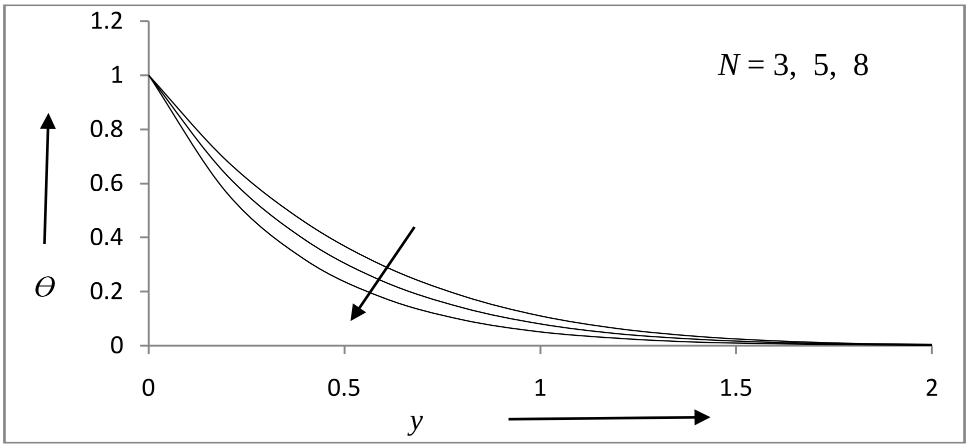

Figure 8: Temperature field θ against y, for Cr=2, Pr=0.71, Gr=2, Gm=5, Sc=0.6, Sr=1, M=1, t =0.2.

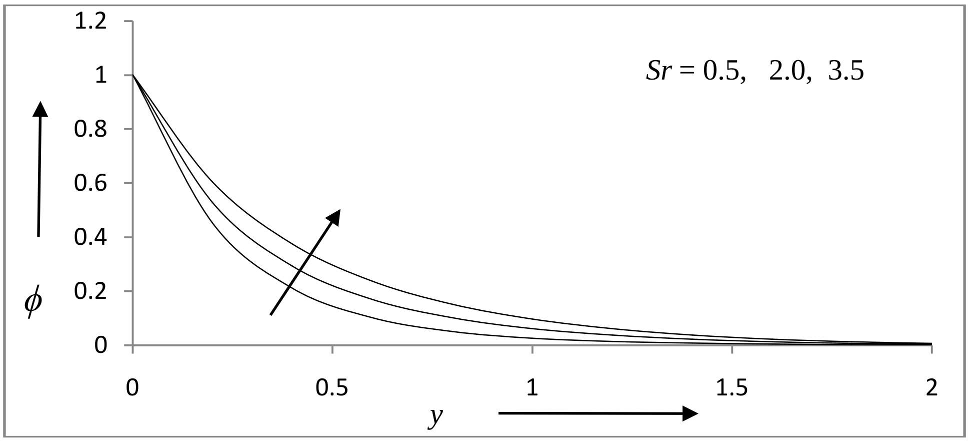

Figure 9: Concentration field ϕ against y, for N=5, Pr=0.71, Gr=2, Gm=5, Sc=0.6, Cr=30, M=1, t =0.2.

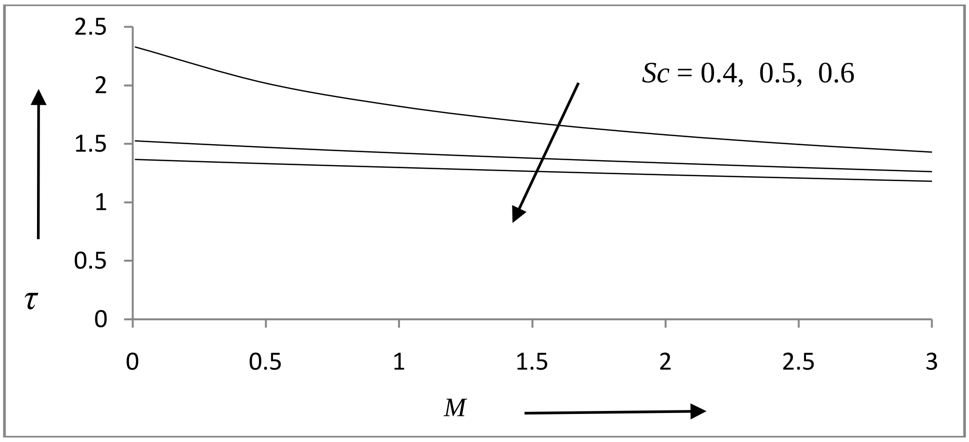

Figure 10: Skin friction τ against M, for N=5, Pr=0.71, Gr=2, Gm=5, Cr=30, Sr=1, t =0.2.

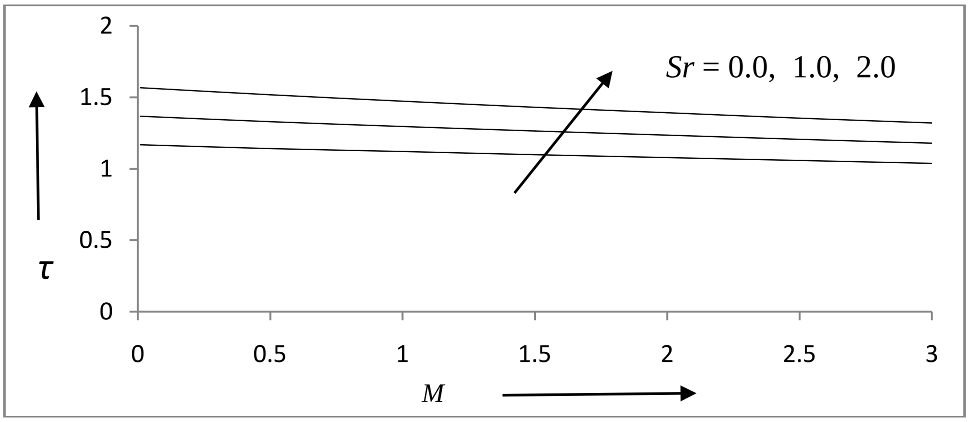

Figure 11: Skin friction τ against M, for N=5, Pr=0.71, Gr=2, Gm=5, $Cr=30$, Sc=0.6, t =0.2.

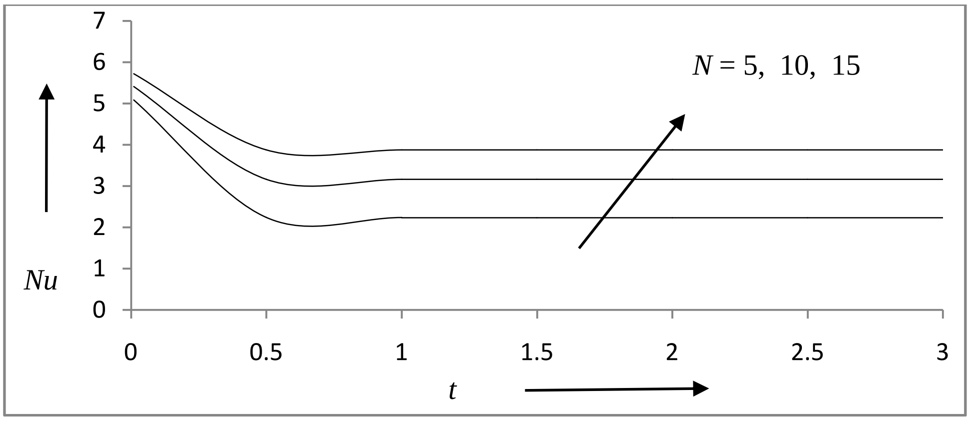

Figure 12: Nusselt number Nu against t, for M=1, Pr=0.71, Gr=2, Gm=5, Sc=0.6, Sr=1, Cr=30.

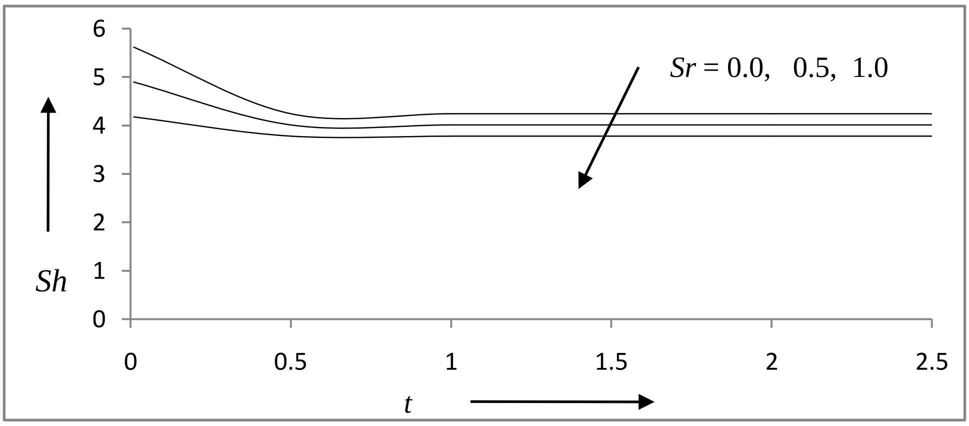

Figure 13: Sherwood number Sh against t, for N=5, Pr=0.71, Gr=2, Gm=5, Sc=0.6, M=1, Cr=30.

Acknowledgment

The authors are thankful to the Referee for valuable comments and suggestions.

References

[1] Alfven, H. (1942). Discovery of Alfven Waves, *Nature*, **150**, 405-406.

[2] Cess, R.D. (1966). Radiative effects on free convective heat transfer flow, *Int. J. Heat Mass Trans.*, **9**, 269-277.

[3] Chambre, P.L. and Young, I.D. (1958). On the diffusion of a chemically reactive species in a laminar boundary layer flow, *The Physics of Fluids*, **1**, 48-54.

[4] Cowling, T.G. (1957). *Magnetohydrodynamics*, Willey Inter Science, New York.

[5] Crammer, K.P. and Pai, S.L. (1973). *Magneto Fluid Dynamics for Engineers and Applied Physicist*, Mc Grow-Hill Book Co. New York.

[6] Dursunkaya, Z. and Worek, W.M. (1992). Diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, *International Journal of heat and mass transfer*, **35**, 2060-2065.

[7] Eckert ERG and Drake, R.M (1972). *Analysis of heat and mass transfer*, Hemisphere Pub. Corp., Washington, D.C

[8] Jha, B.K. and Singh, A.K. (1990). Soret effects on free-convection and mass transfer flow in the Stokes problem for a infinite vertical plate, *Astrophysics and Space Sci.*,**173(**2), 251-255.

[9] Manivannan, K., Muthucumaraswamy, R. and Thangaraj, V. (2009). The effect of thermal radiation on an isothermal vertical oscillating plate by considering variable mass diffusion and chemical reaction, *Thermal Science*. **13**(2), 155-162.

[10] Muthucumaraswamy, R. and Sivakumar, P. (2016). MHD flow past a parabolic flow past an infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction, *Int. J. Applied Mechanics and Engineering*, **21**, 95-105.

[11] Platten, K., Chavepeyer, G. (1973). Oscillatory motion in Benard cell, *J. Fluid Mech.*, **60**, 305-319.

[12] Seddeek, M.A. (2002). The unsteady flow in presence of radiation and variable viscosity on a MHD flow past a semi-infinite flat plate with an aligned magnetic field, *Int. J. Heat Mass Transfer*, **45**, 931-935.

[13] Sengupta, S. and Ahmed, N. (2015). Chemical reaction interaction on unsteady MHD free convective radiative flow past an oscillating plate embedded in porous media with thermal diffusion, *Advances in Applied Science Research*, **6**(7), 87-104.

[14] Shercliff, J.A. (1965). *A text book of Magnetohydrodynamics*, Pergomon Press, London.

Appendix

$$a=\frac{N}{Pr},\ b=\frac{ScCr-N}{Pr-Sc},\ c=\frac{N-M}{1-Pr},\ d=\frac{ScCr-M}{1-Sc},\ P_1=Pr(s+a),\ P_2=Sc(s+Cr),\ P_3=s+M,$$

$$A=\frac{ScSrPr}{Pr-Sc},\ B=\frac{Gr}{1-Pr},\ E=\frac{Gm}{1-Sc},\ F=\frac{Gm}{1-Pr},\ A_1=\frac{Aa}{b},\ A_2=1-\frac{Aa}{b},\ A_3=\frac{A(a+b)}{b}e^{bt},$$

$$A_4=\frac{B}{c}+\frac{aAF}{bc},\ A_5=\frac{E}{d}-\frac{aAE}{bd},\ A_6=\frac{AF(a+b)}{b(b-c)}e^{bt},\ A_7=\frac{AE(a+b)}{b(b-d)}e^{bt},A_8=\frac{B}{c}+\frac{E}{d}-\frac{aAE}{bd}+\frac{aAF}{bc},$$

$$A_9=\left(\frac{B}{c}-\frac{(a+b)AF}{c(c-b)}\right)e^{ct},\ A_{10}=\left(\frac{E}{d}+\frac{(a+d)AE}{d(d-b)}\right)e^{dt},\ A_{11}=\frac{A(a+b)}{b}\left(\frac{E}{b-d}-\frac{F}{b-c}\right)e^{bt},$$

$$f_1=f(\mathrm{Pr},a,0,y,t),\ f_2=f(Sc,Cr,0,y,t),\ f_3=f(\mathrm{Pr},a,b,y,t),\ f_4=f(Sc,Cr,b,y,t),$$

$$f_5=f(1,M,0,y,t),\ f_6=f(1,M,c,y,t),\ f_7=f(\mathrm{Pr},a,c,y,t),\ f_8=f(1,M,d,y,t),$$

$$f_9=f(Sc,Cr,d,y,t),\ f_{10}=f(1,M,b,y,t),\ g=g(M,y,t),\ h_1=h(\mathrm{Pr},a,o,t),$$

$$h_2=h(Sc,Cr,0,t),\ h_3=h(\mathrm{Pr},a,b,t),\ h_4=h(Sc,Cr,b,t),\ h_5=h(1,M,0,t),\ h_6=h(1,M,c,t),$$

$$h_7=h(\mathrm{Pr},a,c,t),\ h_8=h(1,M,d,t),\ h_9=h(Sc,Cr,d,t),\ h_{10}=h(1,M,b,t)$$

and the functions $f,\ g,\ h,\ \bar{g}$ are defined by

$$f(\mathrm{Pr},a,b,y,t)=\frac{1}{2}\left[e^{y\sqrt{\mathrm{Pr}(a+b)}}erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}+\sqrt{(a+b)t}\right)+e^{-y\sqrt{\mathrm{Pr}(a+b)}}erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}-\sqrt{(a+b)t}\right)\right],$$

$$g(M,y,t)=\frac{\left(\frac{y^2}{4t}+Mt\right)t}{4M}\left\{e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)+e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)\right\}$$

$$-\frac{y(1-4Mt)}{16M^{\frac{3}{2}}}\left\{e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)-e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)\right\}-\frac{y}{4M}\sqrt{\frac{t}{\pi}}\,e^{-\left(\frac{y}{4t}+Mt\right)},$$

$$h(\mathrm{Pr},a,b,t)=\sqrt{\mathrm{Pr}(a+b)}\,erf\left(\sqrt{(a+b)t}\right)+\sqrt{\frac{\mathrm{Pr}}{\pi t}}\,e^{-(a+b)t},$$

$$\bar{g}(M,t)=\frac{t^2}{2}h_5-\frac{(1-4Mt)}{8M^{\frac{3}{2}}}erf\left(\sqrt{Mt}\right)+\frac{1}{4\pi}\sqrt{\frac{t}{\pi}}\,e^{-Mt}.$$