

UNSTEADY MHD FREE CONVECTIVE FLOW PAST A MOVING VERTICAL PLATE IN PRESENCE OF HEAT SINK

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Abstract : An attempt is made to study the effects of heat and mass transfer on an unsteady MHD free convective flow past a moving vertical plate embedded in a porous medium in presence of thermal diffusion, chemical reaction and heat sink. The dimensionless governing equations are solved by adopting Laplace transform technique in closed form. The numerical values of the velocity, temperature and concentration fields are studied graphically for different values of the parameters such as magnetic parameter, Soret number, thermal Grashof number, solutal Grashof number, Schmidt number, Prandtl number, chemical reaction parameter and heat sink parameter.

Keywords: Heat and mass transfer; thermal diffusion; Nusselt number; Sherwood number.

Mathematical Subject Classification (2010): 76W05

Nomenclature

B_0	strength of the applied magnetic field
\vec{B}	magnetic induction vector
C^i	species concentration
C_∞^i	species concentration in free stream
C_w^i	species concentration at the plate
C_p	specific heat at constant pressure
D_M	mass diffusion coefficient
\vec{E}	electric field
Gr	Grashof number for heat transfer
Gm	Grashof number for mass transfer
g	acceleration due to gravity
\vec{J}	current density
K^i	rate of first order chemical reaction

K	chemical reaction parameter
k'	permeability of porous medium
K_T	thermal diffusion ratio
M	magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
\vec{q}	fluid velocity
Q	heat sink parameter
Q'	heat sink
Re	Reynolds number
Sr	Soret number
Sh	Sherwood number
T'	fluid temperature
T'_∞	temperature at the free stream
T'_w	temperature at the plate
T_M	mean fluid temperature
t	dimensionless time
μ	coefficient of viscosity
κ	thermal conductivity
ν	kinematic viscosity
σ	electrical conductivity
ρ	fluid density
θ	dimensionless temperature
ϕ	dimensionless concentration
β	coefficient of volume expansion for heat transfer
β^*	coefficient of volume expansion for mass transfer

1. Introduction

Magneto hydrodynamics plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Also, it has applications in the field of stellar and planetary magnetospheres, metrology, Solar physics and in the motion of earth's core. Free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. Free convection effect on a flow past a moving vertical plate embedded in porous medium has been studied by Chaudhary and Jain [7] by adopting Laplace transform technique. The effects of heat and mass transfer on laminar boundary layer flow over a wedge have been investigated by Gebhart and Pera [9]. Hady et al. [10] investigated the effects of temperature dependent viscosity on a fixed convection flow from a vertical plate.

Chamkha [6] studied the unsteady MHD convective heat and mass transfer past a semi-infinite vertical porous moving plate with heat absorption. Jha and Prasad [12] studied the free convection and mass transfer flow of a viscous incompressible fluid past an accelerated vertical plate taking into account the heat source effect. Hossain et al. [11] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Recently Ahmed and Agarwalla [1] studied effect of heat sink on transient MHD mass transfer flow past an accelerated vertical plate with chemical reaction.

The Soret effect arises when the mass flux contains a term that depends on the temperature gradient. In view of importance of Soret effect, many investigations have carried out model studies on the Soret effect in different heat and mass transfer related problems, some of them are Bhavana et al. [5], Ahmed and Das [4], Ahmed and Bhattacharyya [2]. Chemical reaction plays an important role in heat and mass transfer problems. The reaction rate in chemical reaction generally depends on the concentration of the species itself. The reaction is said to be of first order if the reaction rate is directly proportional to the concentration itself as defined in Cussler [8]. In recent years, Kandasamy et al. [13,14] studied the MHD flow taking into account the heat and mass transfer effects under first order homogenous chemical reaction with a heat source. Very recently Ahmed et al. [3] investigated the effects of heat and mass transfer in MHD free convective flow past a moving vertical plate with time dependent plate velocity in porous medium.

The present investigation concerns with the study of unsteady MHD mass transfer flow past an infinite vertical moving plate embedded in a porous medium. The main objective of this work is to study the influence of Heat Sink on the flow and transport characteristics, in presence of thermal diffusion and chemical reaction.

2. Basic equations

The equations governing the flow of a viscous incompressible and electrically conducting fluid in the presence of magnetic field are

Equation of continuity:

$$\vec{\nabla} \cdot \vec{q} = 0 \quad (1)$$

Momentum equation:

$$\rho \left[\frac{\partial \vec{q}}{\partial t'} + (\vec{q} \cdot \vec{\nabla}) \vec{q} \right] = -\vec{\nabla} p + \vec{J} \times \vec{B} + \rho \vec{g} + \mu \nabla^2 \vec{q} - \frac{\mu}{k'} \vec{q} \quad (2)$$

Ohm's law:

$$\vec{J} = \sigma \left[\vec{E} + (\vec{q} \times \vec{B}) \right] \quad (3)$$

Gauss' law of magnetism:

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (4)$$

Energy equation:

$$\rho C_p \left[\frac{\partial T'}{\partial t'} + (\vec{q} \cdot \vec{\nabla}) T' \right] = \kappa \nabla^2 T' + Q' (T'_\infty - T') + \phi \quad (5)$$

Species continuity equation:

$$\frac{\partial C'}{\partial t'} + (\vec{q} \cdot \vec{\nabla}) C' = D_M \nabla^2 C' + K' (C'_\infty - C') + \frac{D_M K_T}{T_M} \nabla^2 T' \quad (6)$$

All the physical quantities are defined in the Nomenclature.

3. Mathematical Analysis

Let us consider a flow of an incompressible viscous electrically conducting fluid past an infinite vertical plate embedded in porous medium. We introduce a Cartesian coordinate system (x', y', z') with X axis along the infinite vertical plate, Y axis normal to the plate and Z axis along the width of the plate. Initially the plate and the fluid were at same temperature T'_∞ with concentration level C'_∞ at all points. At time $t' > 0$ the plate temperature is suddenly raised to T'_w and the concentration level at the plate raised to C'_w . A uniform magnetic field is applied normal to the plate. It is assumed that the plate is suddenly moved with a velocity $\frac{U}{t_0^2} t'^2$ moving upwards at time $t' > 0$.

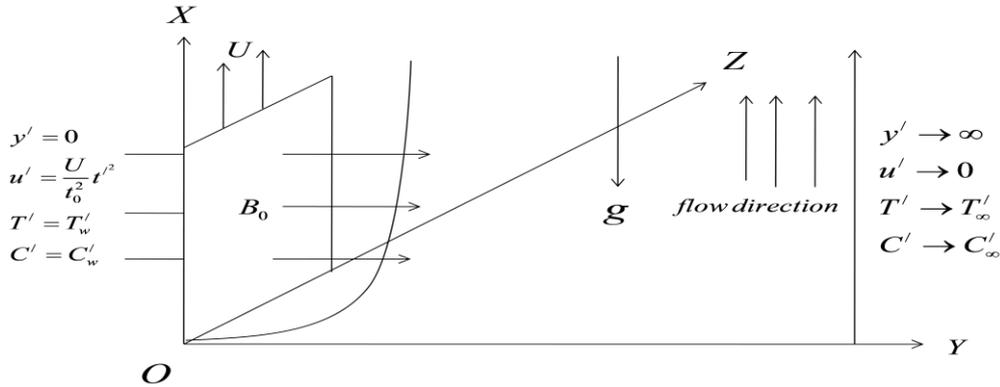


Figure 1: Flow configuration of the problem.

Our investigation is restricted to the following assumptions:

- (i) All the fluid properties are considered constants except the influence of the variation in density in the buoyancy force term.

- (ii) Viscous dissipation and Ohmic dissipation of energy are neglected.
- (iii) Magnetic Reynolds number is assumed to be small enough to neglect the induced magnetic field.
- (iv) Induced electric field is neglected.

The governing equations of motion are

Mass continuity equation:

$$\frac{\partial u'}{\partial x'} = 0 \quad (7)$$

MHD momentum equation:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu}{k'} u' \quad (8)$$

Energy equation:

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} - Q'(T' - T'_\infty) \quad (9)$$

Species continuity equation:

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} - K'(C' - C'_\infty) + \frac{D_M K_T}{T_M} \frac{\partial^2 T'}{\partial y'^2} \quad (10)$$

Initial and boundary conditions for the flow problems:

$$\left. \begin{aligned} t' \leq 0 : u' = 0, T' = T'_\infty, C' = C'_\infty & \quad \text{for all } y' \\ t' > 0 : u' = \frac{U}{t_0^2} t'^2, T' = T'_w, C' = C'_w & \quad \text{at } y' = 0 \\ u' \rightarrow 0, T' = T'_\infty, C' = C'_\infty & \quad \text{as } y' \rightarrow \infty \end{aligned} \right\} \quad (11)$$

We introduce the following non-dimensional variables and parameters:

$$\begin{aligned}
u &= \frac{u'}{U}, t = \frac{t'}{t_0}, y = \frac{y'}{Ut_0}, Gr = \frac{g\beta v(T_w' - T_\infty')}{U^3}, Gm = \frac{g\beta^* v(C_w' - C_\infty')}{U^3} \\
Pr &= \frac{\mu C_p}{\kappa}, Sc = \frac{\nu}{D_M}, Sr = \frac{D_M K_T (T_w' - T_\infty')}{\nu T_M (C_w' - C_\infty')}, M = \frac{\sigma B_0^2 \nu}{\rho U^2}, \alpha = \frac{U^2 k'}{\nu^2} \\
Q &= \frac{\nu t_0 Q'}{\kappa}, Re = \frac{U^2 t_0}{\nu}, K = t_0 K', \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}, \phi = \frac{C' - C_\infty'}{C_w' - C_\infty'}
\end{aligned}$$

The non-dimensional form of the equations (8) to (10) are:

$$Re \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Re^2 (Gr\theta + Gm\phi) - Re^2 u\xi \quad (12)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Re Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{Q\theta}{Pr} \quad (13)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc Re} \frac{\partial^2 \phi}{\partial y^2} - K\phi + \frac{Sr}{Re} \frac{\partial^2 \theta}{\partial y^2} \quad (14)$$

The corresponding initial and boundary conditions are:

$$\left. \begin{aligned}
t \leq 0: u = 0, \theta = 0, \phi = 0 & \quad \text{for all } y \\
t > 0: u = t^2, \theta = 1, \phi = 1 & \quad \text{at } y = 0 \\
u \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 & \quad \text{as } y \rightarrow \infty
\end{aligned} \right\} \quad (15)$$

4. Method of Solution

Taking Laplace transform of the equations (12) to (14), the following equations are obtained:

$$\frac{d^2 \bar{u}}{dy^2} - (Re s + Re^2 \xi) \bar{u} = -Re^2 (Gr\theta + Gm\phi) \quad (16)$$

$$\frac{d^2 \bar{\theta}}{dy^2} - Re(Q + sPr) \bar{\theta} = 0 \quad (17)$$

$$\frac{d^2 \bar{\phi}}{dy^2} - Sc Re (s + K) \bar{\phi} = -\frac{Sc Re Sr (Q + sPr)}{s} e^{-y\sqrt{Re(Q+sPr)}} \quad (18)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} \bar{u} &= \frac{2}{s^3}, \bar{\theta} = \frac{1}{s}, \bar{\phi} = \frac{1}{s} & \text{at } y=0 \\ \bar{u} &= 0, \bar{\theta} = 0, \bar{\phi} = 0 & \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (19)$$

Equations (16), (17) and (18) are second order ordinary differential equations which are solved subject to the boundary conditions (19). The following solutions are obtained for $\bar{\theta}$, $\bar{\phi}$ and \bar{u}

$$\bar{\theta} = \frac{1}{s} e^{-y\sqrt{\text{Re}(Q+sPr)}} \quad (20)$$

$$\bar{\phi} = \left[\frac{1}{s} + \frac{\text{ScSr}(Q+sPr)}{s\{Q+sPr-\text{Sc}(s+K)\}} \right] e^{-y\sqrt{\text{ScRe}(s+K)}} - \frac{1}{s} \left[\frac{\text{ScSr}(Q+sPr)}{\{Q+sPr-\text{Sc}(s+K)\}} \right] e^{-y\sqrt{\text{Re}(Q+sPr)}} \quad (21)$$

$$\bar{u} = \left[\begin{aligned} &\frac{2}{s^3} + \frac{\text{ReGr}}{s\{Q+sPr-(s+\text{Re}\xi)\}} + \frac{\text{ReGm}}{s\{\text{Sc}(s+K)-(s+\text{Re}\xi)\}} \\ &+ \frac{\text{ScSrGmRe}(Q+sPr)}{s\{Q+sPr-\text{Sc}(s+K)\}\{\text{Sc}(s+K)-(s+\text{Re}\xi)\}} \\ &- \frac{\text{ScSrGmRe}(Q+sPr)}{s\{Q+sPr-\text{Sc}(s+K)\}\{Q+sPr-(s+\text{Re}\xi)\}} \end{aligned} \right] e^{-y\sqrt{\text{Re}s+\text{Re}^2\xi}} \quad (22)$$

$$\begin{aligned} & - \frac{\text{ReGr}}{s\{Q+sPr-(s+\text{Re}\xi)\}} e^{-y\sqrt{\text{Re}(Q+sPr)}} - \frac{\text{ReGm}}{s\{\text{Sc}(s+K)-(s+\text{Re}\xi)\}} e^{-y\sqrt{\text{ScRe}(s+K)}} \\ & - \frac{\text{ScSrGmRe}(Q+sPr)}{s\{Q+sPr-\text{Sc}(s+K)\}\{\text{Sc}(s+K)-(s+\text{Re}\xi)\}} e^{-y\sqrt{\text{ScRe}(s+K)}} \\ & + \frac{\text{ScSrGmRe}(Q+sPr)}{s\{Q+sPr-\text{Sc}(s+K)\}\{Q+sPr-(s+\text{Re}\xi)\}} e^{-y\sqrt{\text{Re}(Q+sPr)}} \end{aligned}$$

Taking inverse Laplace transform of equations (20), (21) and (22), we obtain

$$\theta(y,t) = f_1 \quad (23)$$

$$\phi(y,t) = f_2 + \frac{a_4}{a_5} \left[a_6 e^{a_5 t} (f_3 - f_4) + a_1 (f_1 - f_2) \right] \quad (24)$$

$$\begin{aligned}
u(y,t) = & b_1 f_1 + b_2 f_2 - b_3 e^{a_3 t} f_3 + b_4 e^{a_4 t} f_4 + \left(\frac{y^2 Re + 4a_7 t^2}{4a_7} - b_5 \right) f_5 + b_6 e^{a_6 t} f_6 + b_7 e^{a_7 t} f_7 + b_8 e^{a_8 t} f_8 + b_9 e^{a_9 t} f_9 \\
& - b_{10} e^{a_{10} t} f_{10} + \frac{y\sqrt{Re}(1-4a_7 t)}{8a_7^{3/2}} f_{11} - \frac{y}{2} \sqrt{\frac{Ret}{\pi}} \frac{1}{a_7} e^{-\left(\frac{y^2 Re + 4a_7 t^2}{4t}\right)}
\end{aligned} \tag{25}$$

5. Skin friction

The skin friction at the plate is given by

$$\begin{aligned}
\tau = & \left[\frac{\partial u}{\partial y} \right]_{y=0} \\
= & -b_1 g_1 - b_2 g_2 + b_3 e^{a_3 t} g_3 - b_4 e^{a_4 t} g_4 - (t^2 - b_5) g_5 - b_6 e^{a_6 t} g_6 - b_7 e^{a_7 t} g_7 - b_8 e^{a_8 t} g_8 - b_9 e^{a_9 t} g_9 + b_{10} e^{a_{10} t} g_{10} \\
& + \frac{\sqrt{Re}(1-4a_7 t)}{4a_7^{3/2}} \operatorname{erfc}(\sqrt{a_7 t}) - \frac{1}{2} \sqrt{\frac{Ret}{\pi}} \frac{1}{a_7} e^{-a_7 t}
\end{aligned} \tag{26}$$

6. Rate of heat transfer

Rate of heat transfer in terms of Nusselt number is given by

$$\begin{aligned}
Nu = & - \left[\frac{\partial \theta}{\partial y} \right]_{y=0} \\
= & \sqrt{\frac{a_2}{\pi t}} e^{-a_1 t} + \sqrt{a_1 a_2} \operatorname{erfc}(\sqrt{a_1 t})
\end{aligned} \tag{27}$$

7. Rate of mass transfer

Rate of mass transfer in terms of Sherwood number is given by

$$\begin{aligned}
Sh = & - \left[\frac{\partial \phi}{\partial y} \right]_{y=0} \\
= & g_2 + \frac{a_4}{a_5} \left[a_6 e^{a_6 t} (g_3 - g_4) + a_1 (g_1 - g_2) \right]
\end{aligned} \tag{28}$$

Where

$$\begin{aligned} \xi &= M + \frac{1}{\alpha}; a_1 = \frac{Q}{Pr}; a_2 = RePr; a_3 = ScRe; a_4 = \frac{ScRePr}{Pr - Sc}; a_5 = \frac{ScK - Q}{Pr - Sc}; \\ a_6 &= a_1 + a_5; a_7 = Re\xi; a_8 = \frac{ReGr}{Pr - 1}; a_9 = \frac{a_7 - Q}{Pr - 1}; a_{10} = \frac{ReGm}{Sc - 1}; a_{11} = \frac{a_7 - ScK}{Sc - 1}; \\ a_{12} &= \frac{a_1}{a_5 a_{11}}; a_{13} = \frac{a_6}{a_5 (a_5 - a_{11})}; a_{14} = \frac{a_1 + a_{11}}{a_{11} (a_{11} - a_5)}; a_{15} = \frac{a_1}{a_5 a_9}; a_{16} = \frac{a_6}{a_5 (a_5 - a_9)}; \\ a_{17} &= \frac{a_1 + a_9}{a_9 (a_9 - a_5)}; b_1 = \frac{a_8}{a_9} + a_4 a_8 a_{15}; b_2 = \frac{a_{10}}{a_{11}} - a_4 a_{10} a_{12}; b_3 = a_4 a_{10} a_{13}; b_4 = a_4 a_8 a_{16}; \\ b_5 &= \frac{a_8}{a_9} - \frac{a_{10}}{a_{11}} + a_4 a_{10} a_{12} - a_4 a_8 a_{15}; b_6 = \frac{a_8}{a_9} - a_4 a_8 a_{17}; b_7 = \frac{a_{10}}{a_{11}} + a_4 a_{10} a_{14}; \\ b_8 &= a_4 a_{10} a_{13} - a_4 a_8 a_{16}; b_9 = a_4 a_8 a_{17} - \frac{a_8}{a_9}; b_{10} = \frac{a_{10}}{a_{11}} + a_4 a_{10} a_{14} \end{aligned}$$

$$\begin{aligned} f_1 &= f(a_1, y, a_2, t); f_2 = f(K, y, a_3, t); f_3 = f(K + a_5, y, a_3, t); \\ f_4 &= f(a_6, y, a_2, t); f_5 = f(a_7, y, Re, t); f_6 = f(a_7 + a_9, y, Re, t); f_7 = f(a_7 + a_{11}, y, Re, t); \\ f_8 &= f(a_5 + a_7, y, Re, t); f_9 = f(a_1 + a_9, y, a_2, t); f_{10} = f(K + a_{11}, y, a_3, t); f_{11} = \bar{f}(a_7, y, Re, t). \end{aligned}$$

$$f(x, y, z, t) = \frac{1}{2} \left[e^{y\sqrt{xz}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{z}{t} + \sqrt{xt}} \right) + e^{-y\sqrt{xz}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{z}{t} - \sqrt{xt}} \right) \right]$$

$$\bar{f}(x, y, z, t) = \frac{1}{2} \left[e^{y\sqrt{xz}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{z}{t} + \sqrt{xt}} \right) - e^{-y\sqrt{xz}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{z}{t} - \sqrt{xt}} \right) \right]$$

8. Results and Discussion

In order to get a clear insight of the physical problem, numerical calculations have been carried out for non-dimensional velocity field, temperature field, concentration field, skin friction, Nusselt number and Sherwood number at the plate for different values of the physical parameters involved and these are demonstrated in graphs. The values of Prandtl number Pr are chosen as 0.71 and 7.0 which represents air and water respectively at 20°C temperature and 1 atmospheric pressure. The values of Schmidt number Sc are taken as 0.22, 0.30 and 0.60 which correspond to H_2 , He and H_2O respectively with air as the diffusing medium and the other parameters are chosen arbitrarily.

Figures 2-6 demonstrate the variation of velocity field u versus normal coordinate y under the effects of magnetic parameter M , Soret number Sr , Schmidt number Sc , chemical reaction parameter K and heat sink parameter Q . Figure 2 shows that the fluid velocity is retarded for increasing magnetic parameter. This phenomenon is consistent to the physical fact that the presence of magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against direction of the flow when the magnetic field is applied normal to the fluid flow. As a consequence of this type of

resistive force tends to slow down the fluid flow. It is observed from Figure 3 that the flow is accelerated under the effect of thermal diffusion. Figure 4 clearly shows that there is a substantial fall in the fluid velocity when the Schmidt number Sc is increased. That is to say that the fluid velocity increases significantly due to high mass diffusivity. This observation is consistent with the fact that an increase in Sc means a decrease in molecular diffusivity. It is noticed from the figure 5 that fluid velocity decreases due to increasing values of heat sink parameter. In other words the heat absorption leads the rate of heat transfer of the fluid to fall significantly. The chemical reaction has some contribution in reducing the fluid velocity which are reflected in Figure 6. All the figures uniquely establish the fact that the velocity first increases in a thin layer adjacent to the plate and thereafter it decreases asymptotically to its value $u=0$ as $y \rightarrow \infty$.

The variation of temperature field θ against y under the influence of heat sink Q and Prandtl number Pr are exhibited in Figures 7-8. It is seen from the figure 7 that the temperature decreases for increasing values of Prandtl number which indicates that thermal boundary layer thickness is reduced under the effect of Prandtl number. It is noticed from the figure 8 that there is a substantial fall in temperature for increasing heat sink within the fluid region.

Figures 9-11 illustrates the concentration ϕ versus y under the effects of Schmidt number Sc , chemical reaction parameter K and Reynolds number Re . Figure 9 shows that the concentration level is decreased for increasing Schmidt number. This is consistent with the fact that an increase in Sc means a decrease of molecular diffusivity which results in a fall in the thickness of the concentration boundary layer. It is seen from the figure 10 and figure 11 that there is a comprehensive fall in the concentration level of the fluid due to chemical reaction and increasing Reynolds number which indicates a reduction in the thickness of the concentration boundary layer.

Figures 12-15 highlight the influence of the magnetic field, Soret effect, mass diffusivity, chemical reaction, heat absorbing source and time t on the viscous drag at the moving plate. All the figures except figure 12 uniquely establish the fact that skin friction falls asymptotically as time progresses. As the effect of magnetic field on τ (Fig 12) is concerned it is seen that for small values of t the viscous drag falls for high intensity magnetic field, whilst for moderate value of t this behaviour takes a reverse turn. In other words the internal friction due to viscosity gets enhanced under the effect of the applied magnetic field. The same figure further simulates that the effect of the magnetic field gets almost nullified as time progresses to a large extent. Figures 13 and 14 show that the thermal diffusion and heat sink contribute in increasing the viscous drag at the plate whereas a reverse phenomenon is observed in Figure 15 due to the increasing values of Schmidt number.

The effects of Prandtl number and heat sink on the rate of heat transfer versus time are demonstrated in Figures 16-17. It is seen from Figures 16 that Prandtl number enhances the rate of heat transfer. On the contrary the rate diminishes under the influence of heat sink which is reflected in Figure 17.

Figures 18-21 display the variations in the rate of mass transfer due to thermal diffusion, heat sink, Reynolds number and Prandtl number. The rate of mass transfer falls with the increase in values of Soret number is conveyed in Figure 18. Similar type of behaviour is observed in Figure 21 due to Prandtl number. The mass transfer rate is found to be rising with the increase of Reynolds number and heat absorption parameter in Figures 19 and 20 respectively.

9. Comparison

It should be mentioned that in the absence of heat sink (i.e., $Q = 0$) the validity of our result is reflected by comparing Figure 2 and Figure 2 of Ahmed et al. [3] of the present work. Both the figures indicate a decrease in velocity under the influence of the parameter M . Moreover, the velocity profiles for both the figures are almost identical, thereby showing an excellent agreement between the results of the present work and that obtained by Ahmed et al. [3].

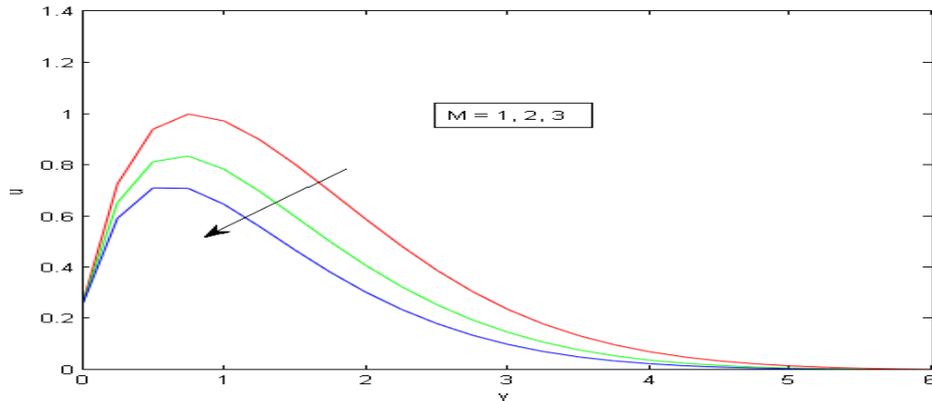


Figure 2 : Velocity u versus y for variations in M when $\alpha = 1; Q = 1; Re = 1; Sc = 0.3; Sr = 0.2; Pr = 0.71; K = 1; Gr = 4; Gm = 4; t = 0.5$

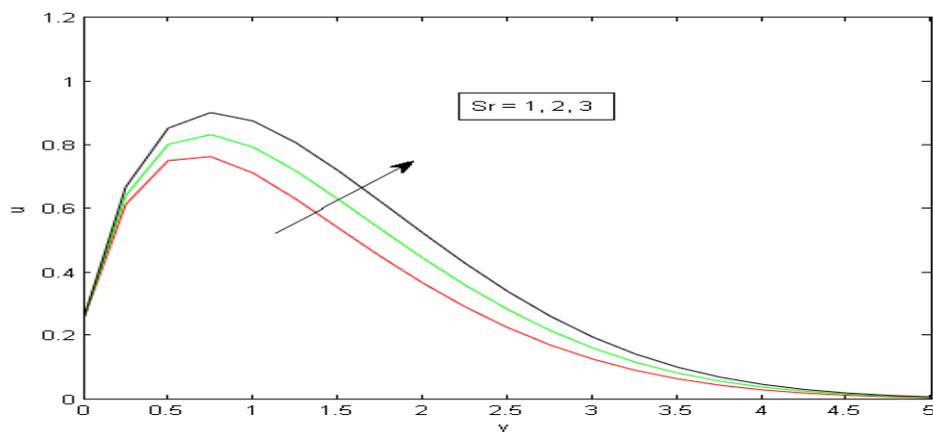


Figure 3: Velocity u versus y for variations in Sr when $\alpha = 1; Q = 1; Re = 1; Sc = 0.3; M = 3; Pr = 0.71; K = 1; Gr = 4; Gm = 4; t = 0.5$

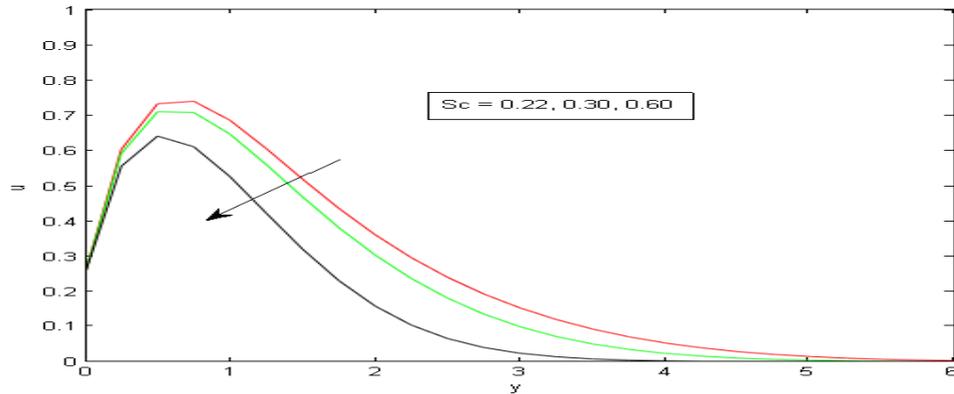


Figure 4 : Velocity u versus y for variations in Sc when $\alpha = 1; Q = 1; Re = 1; Sr = 0.2; M = 3; Pr = 0.71; K = 1; Gr = 4; Gm = 4; t = 0.5$

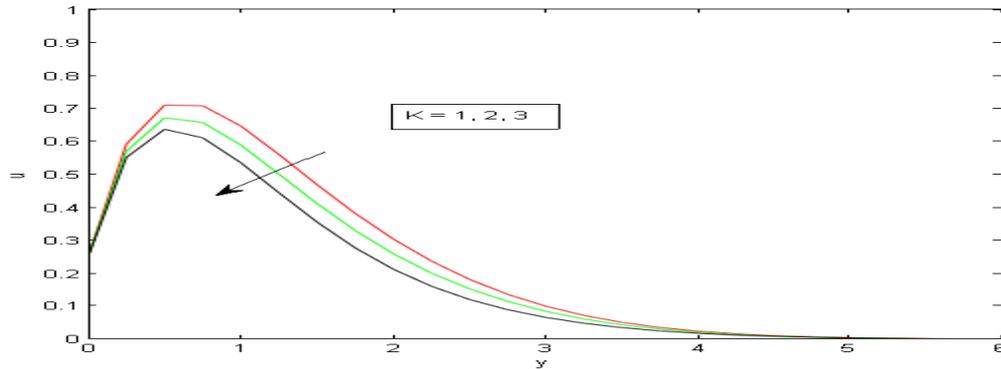


Figure 5 : Velocity u versus y for variations in K when $\alpha = 1; Q = 1; Re = 1; Sr = 0.2; M = 3; Pr = 0.71; Sc = 0.3; Gr = 4; Gm = 4; t = 0.5$

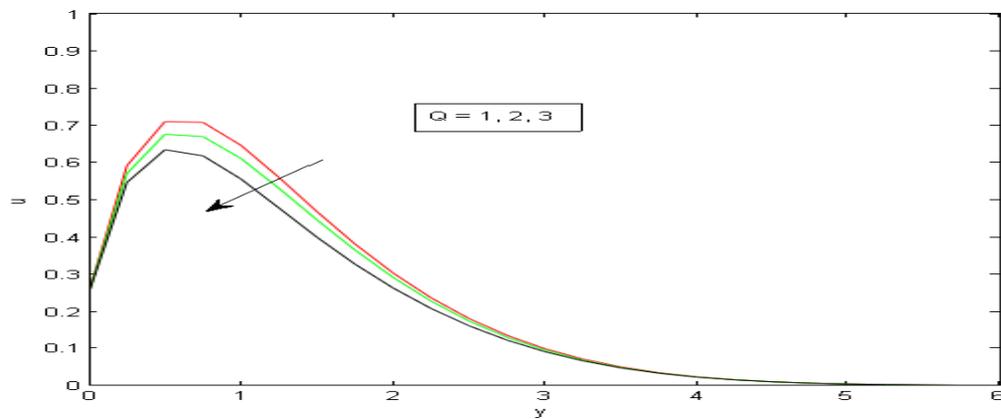


Figure 6 : Velocity u versus y for variations in Q when $\alpha = 1; Sc = 0.3; Re = 1; Sr = 0.2; M = 3; Pr = 0.71; K = 1; Gr = 4; Gm = 4; t = 0.5$

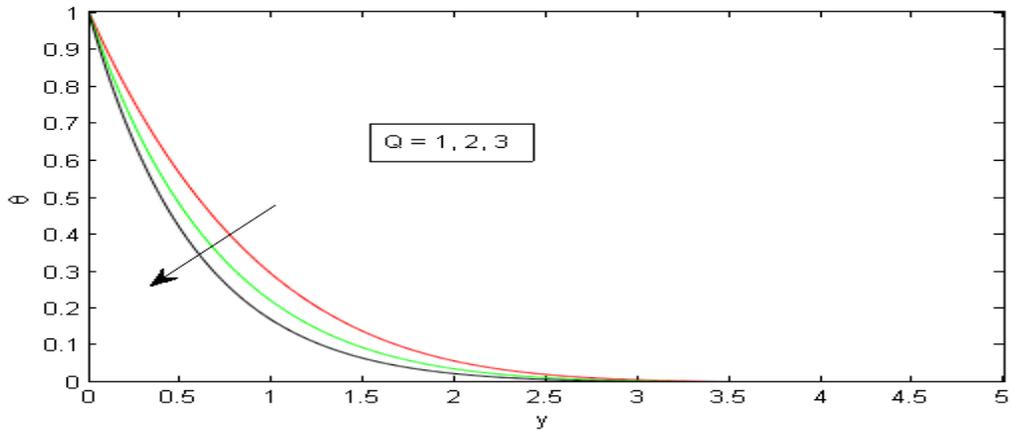


Figure 7 : Temperature θ versus y for variations in Q .

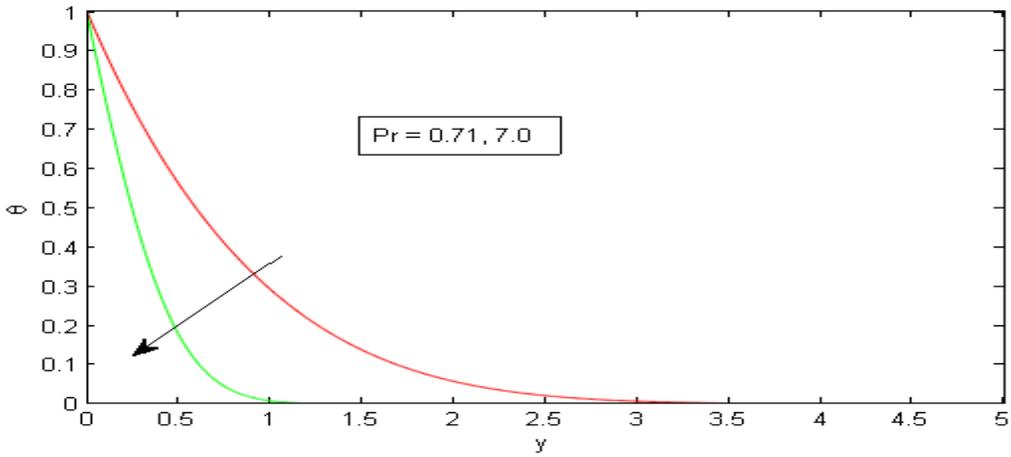


Figure 8: Temperature θ versus y for variations in Pr

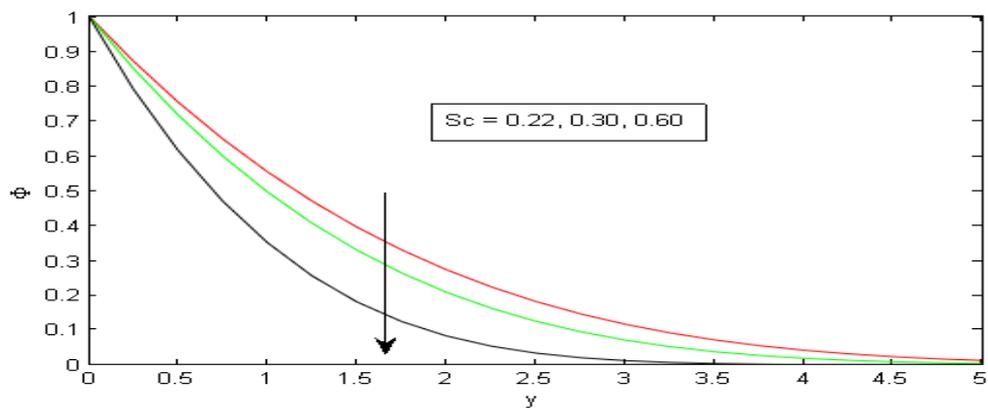


Figure 9 : Concentration ϕ versus y for variations in Sc when
 $Q = 1; Re = 1; Sr = 0.2; Pr = 0.71; K = 1; t = 0.5$

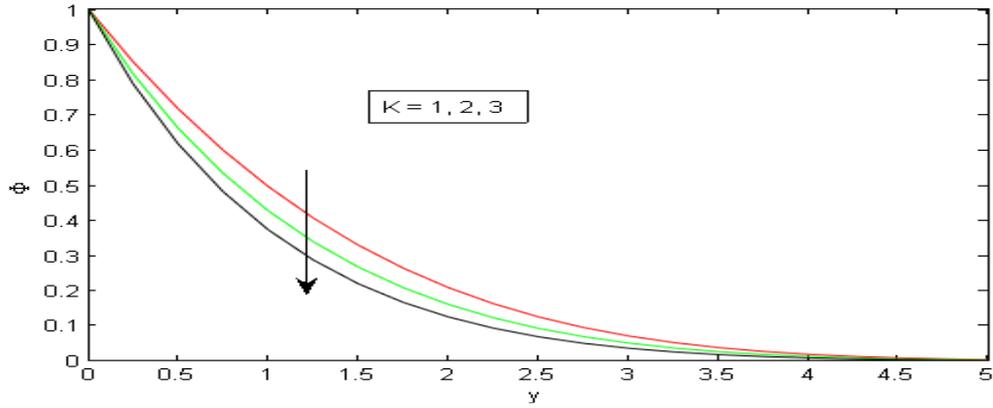


Figure 10 : Concentration ϕ versus y for variations in K when
 $Q = 1; Re = 1; Sr = 0.2; Pr = 0.71; Sc = 0.3; t = 0.5$

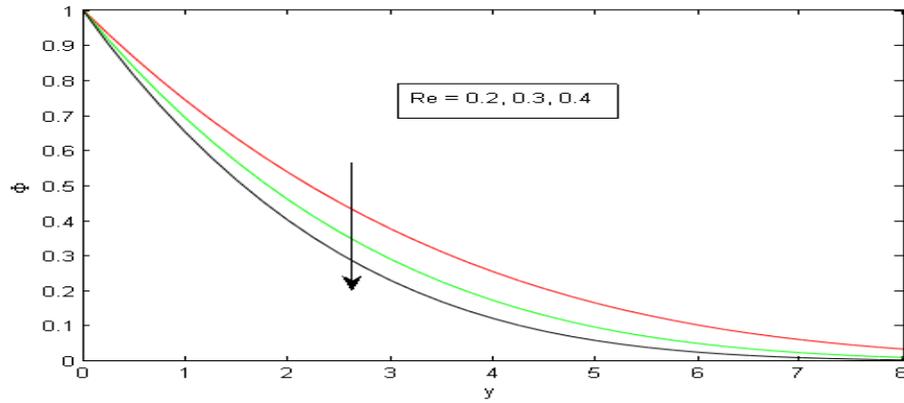


Figure 11 : Concentration ϕ versus y for variations in Re when
 $Q = 1; K = 1; Sr = 0.2; Pr = 0.71; Sc = 0.3; t = 0.5$

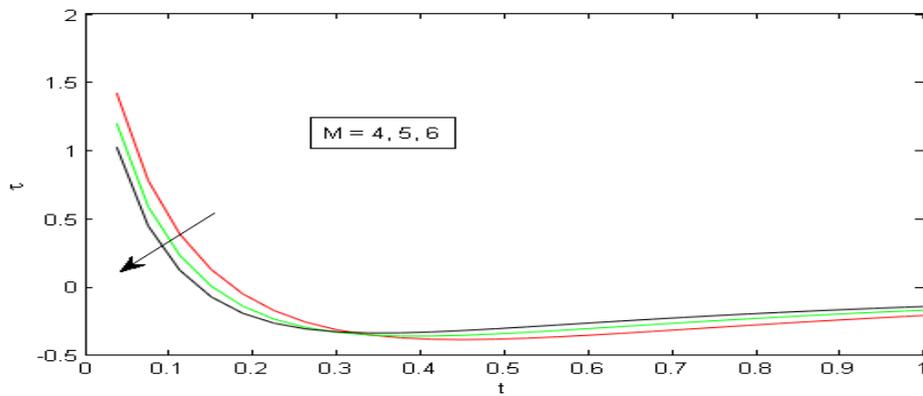


Figure 12 : Skin friction τ versus t for variations in M when
 $\alpha = 1; Q = 1; Re = 1; Sr = 0.2; Sc = 0.3; Pr = 0.71; K = 1; Gr = 5; Gm = 5; t = 0.5$

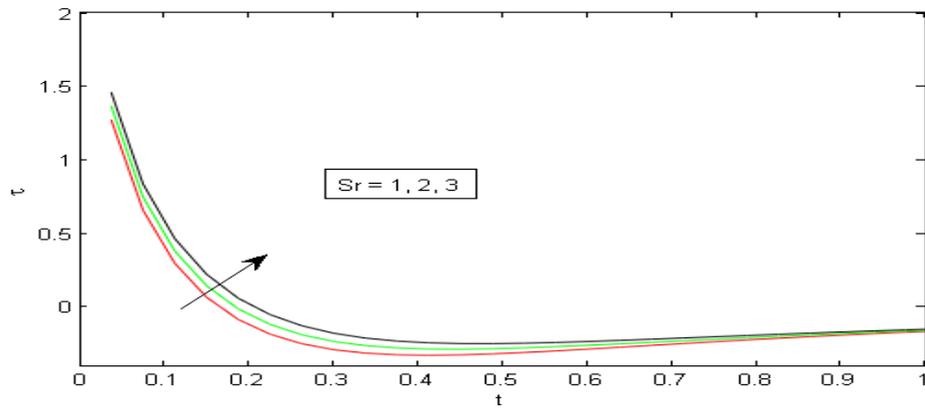


Figure 13 : Skin friction τ versus t for variations in Sr when $\alpha = 1; Q = 1; Re = 1; M = 5; Sc = 0.3; Pr = 0.71; K = 1; Gr = 5; Gm = 5; t = 0.5$

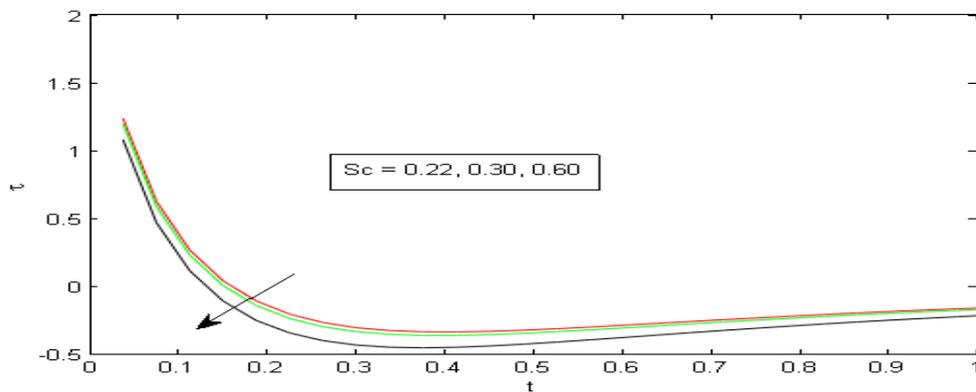


Figure 14 : Skin friction τ versus t for variations in Sc when $\alpha = 1; Q = 1; Re = 1; M = 3; Sr = 0.2; Pr = 0.71; K = 1; Gr = 5; Gm = 5; t = 0.5$

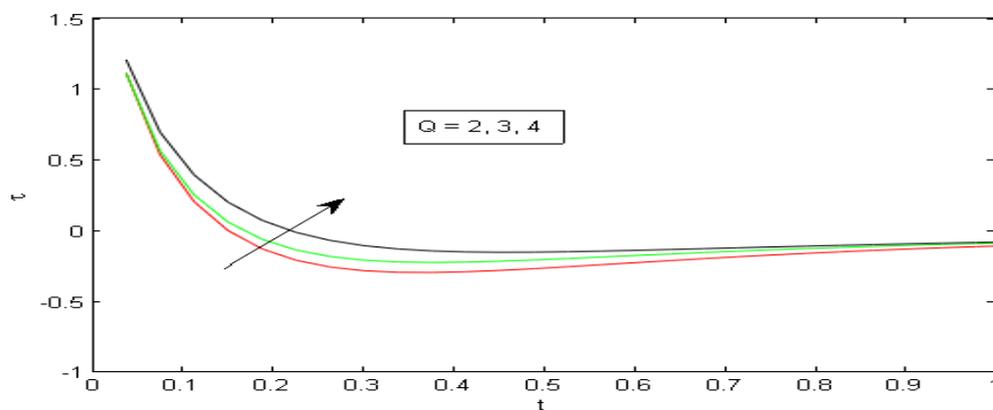


Figure 15 : Skin friction τ versus t for variations in Q when $\alpha = 1; K = 1; Re = 1; M = 3; Sr = 0.2; Pr = 0.71; Sc = 0.3; Gr = 5; Gm = 5; t = 0.5$

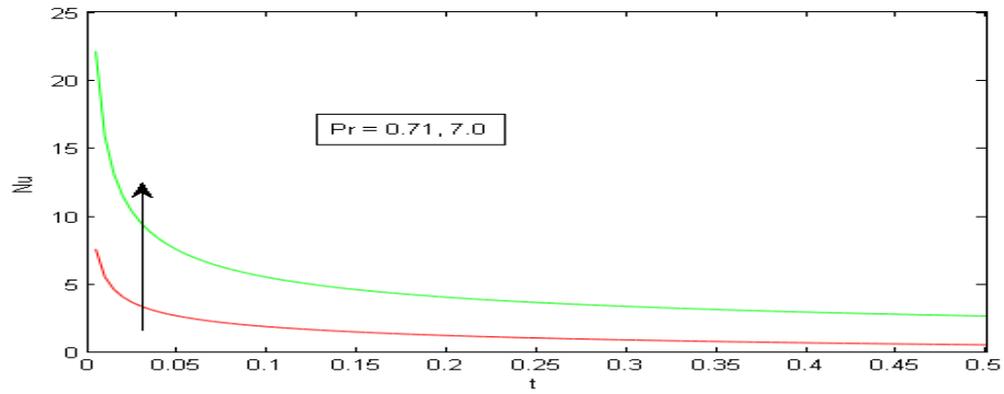


Figure 16: Nusselt number Nu versus t for variations in Pr .

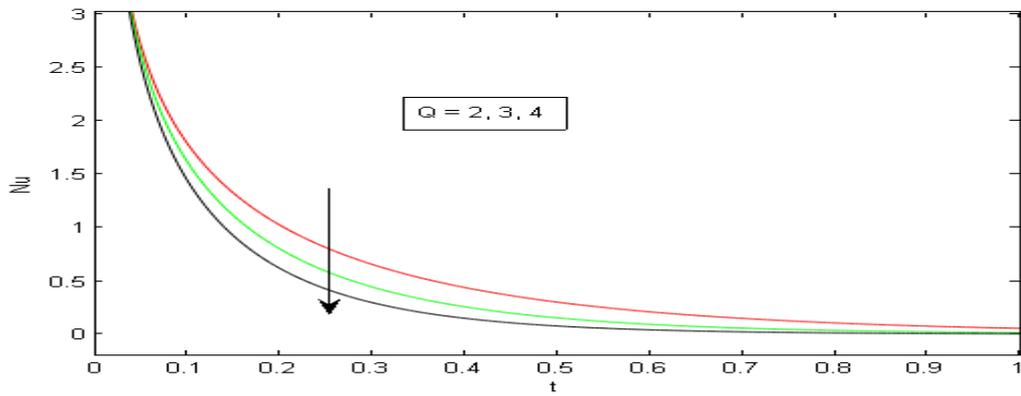


Figure 17 : Nusselt number Nu versus t for variations in Q .

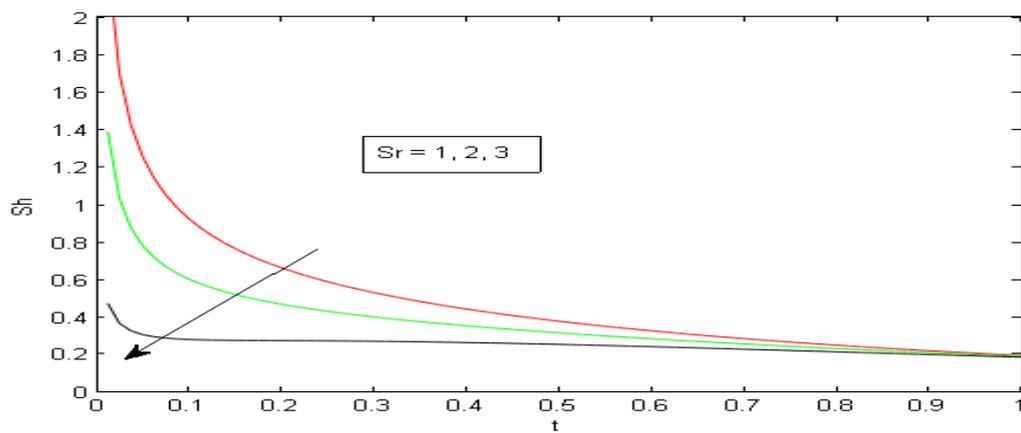


Figure 18 : Sherwood number Sh versus t for variations in Sr when $\alpha = 1; Q = 1; Re = 1; Pr = 0.71; K = 1$

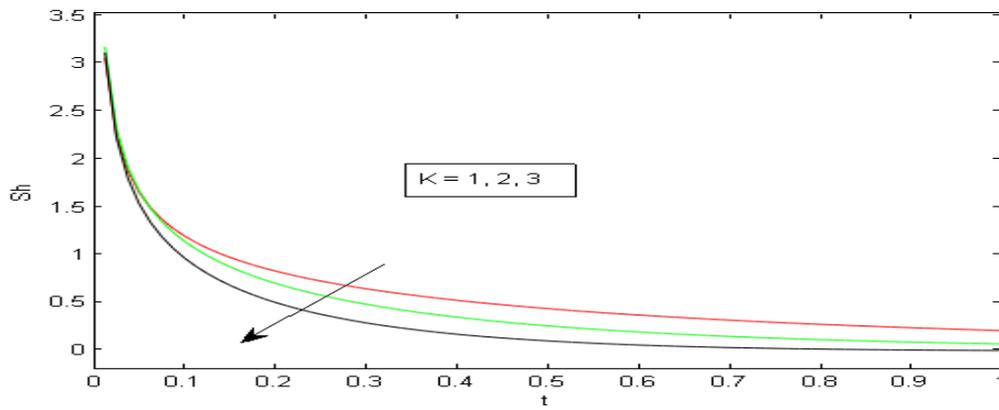


Figure 19 : : Sherwood number Sh versus t for variations in Q when $\alpha = 1; Sr = 0.2; Re = 1; Pr = 0.71; Q = 1$

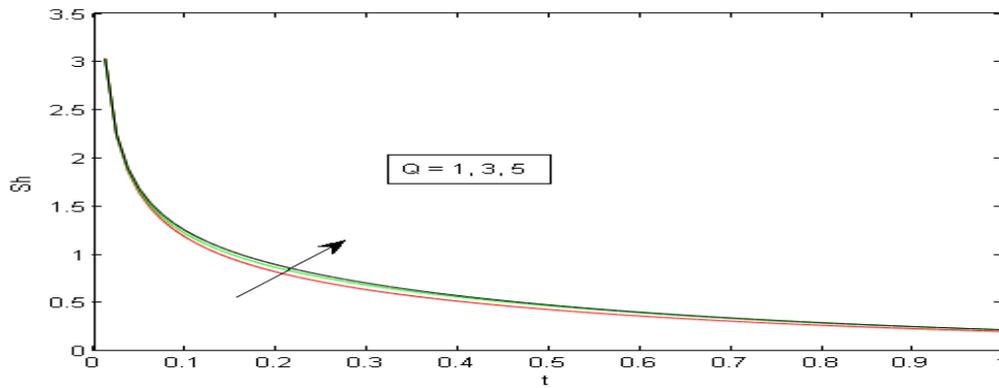


Figure 20 : Sherwood number Sh versus t for variations in Re when $\alpha = 1; Sr = 0.2; Re = 1; Pr = 0.71; K = 1$

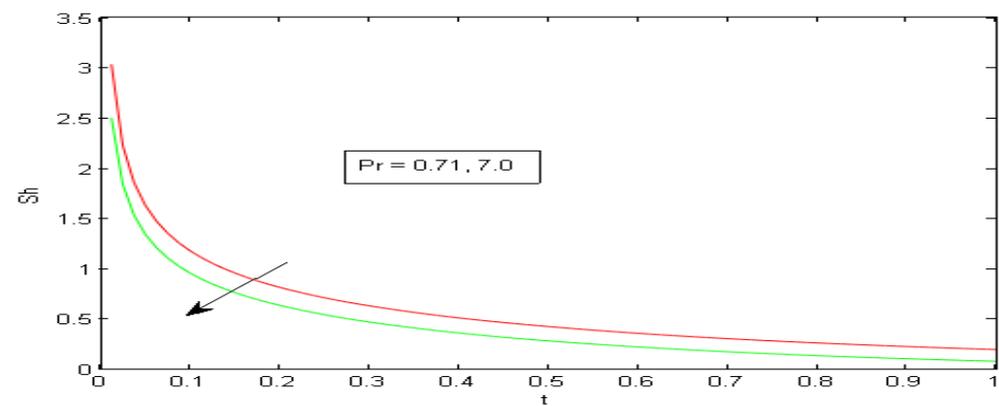


Figure 21 : Sherwood number Sh versus t for variations in Pr when $\alpha = 1; Sr = 0.2; Q = 1; Re = 1; K = 1$

9. Conclusions

The present investigation leads to arrive at the following conclusions:

- The fluid velocity decreases with the increase in magnetic field parameter and heat sink parameter whereas it is accelerated under the effect of thermal diffusion.
- The temperature decreases with the increase in Prandtl number and heat sink parameter.
- The concentration level of the fluid falls with the increase in Schmidt number and chemical reaction parameter.
- The viscous drag on the plate increases under the influence of heat sink and Soret effect.
- The rate of heat transfer at the plate increases with the increase in Prandtl number.
- The rate of mass transfer from plate to the fluid is increased due to heat sink and decreased under the effect of thermal diffusion.

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