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LRS BIANCHI TYPE II HOMOGENEOUS COSMOLOGICAL MODEL FOR PERFECT FLUID WITH ELECTROMAGNETIC FIELD AND VARIABLE Λ

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Abstract: We have investigated LRS Bianchi type II cosmological model for perfect fluid distribution with electromagnetic field and cosmological term Λ. The magnetic field is due to electric current produced along x-axis, thus F_{23} is the only non-vanishing component of F_{ij}. To get the deterministic model of the universe, we have also assumed the condition $A = B^n$ where A and B are metric potentials. The physical and geometrical aspects of the model are also discussed.

Keywords: LRS Bianchi Type II model, Perfect fluid, Electromagnetic field, cosmological term Λ.

1. Introduction

Homogeneous and anisotropic cosmological models have been studied in the framework of general relativity in the search of realistic picture of the universe in its early stage. Bianchi type II models play an important role in modern cosmology, for simplification and description of the large scale behaviour of the actual universe. Bianchi type II space time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of universe. Asseo and Sol [2] considered the importance of Bianchi type II space time for the study of universe.

The large scale distribution of galaxies in our universe shows that the matter distribution can satisfactorily be described by perfect fluid. Also, the occurrence of magnetic field on galatic scale is well established fact today and their importance for a variety of astrophysical phenomenon is generally acknowledged, as pointed out by Zeldovich et al. [24]. So, we should include magnetic field in energy momentum tensor of early universe. Lorentz [11] has investigated Bianchi type cosmologies in presence of magnetic field.

One of the fundamental and challenging problem in cosmology is the cosmological constant (Λ) problem. Einstein considered the introduction of Λ term into the field equations to study static, homogeneous & isotropic models as per status of universe at that time but researchers Bertolami [7, 8], Ozer and Taha [12], Weinberg [23] investigated more significant cosmological models with cosmological constant Λ. Ratra and Peebles [14] discussed in detail the cosmological constant problem and cosmology with time varying cosmological constant. A number of cosmological models with vacuum energy density have been investigated by Bali et al.[3, 4 , 5]. Singh and Agarwal [17] studied Bianchi type II, VIII, IX models in scalar tensor theory under the assumption of a relationship between the cosmological constant (Λ) and scalar field (ψ). Singh [16] also studied string cosmology with electromagnetic fields in Bianchi type II, VIII, IX space times.

Bianchi cosmologies with variable cosmological term in presence of perfect fluid have been studied by number of author viz. Chakravarty and Biswas [9], Adhav et. al.[1]. Bali et al. [6] also studied Bianchi type I massive string magnetized barotropic perfect cosmological model in general relativity. Pradhan et. al. [13] discussed some homogeneous cosmological models with electromagnetic field in presence of perfect fluid with variable Λ. Tiwari [18] has discussed Bianchi type III cosmological model filled with perfect fluid in presence of cosmological constant. Dunn and Tupper [10] discussed properties of Bianchi type VI_0 models with perfect fluid and magnetic field. Roy et. al. [15] explored the effects of cosmological constant in Bianchi type I and VI_o models with perfect fluid and homogeneous magnetic field in axial direction.

Recently, Tyagi and Sharma [21] have studied LRS Bianchi type II magnetized string cosmological model with bulk viscous fluid in general relativity. Tyagi and Sharma [20] also investigated Bianchi type IX string cosmological model for perfect fluid distribution in general relativity. Some anisotropic Bianchi type II cosmological model with variable Λ are investigated by Tiwari et. al.[19]. Also Tyagi and Chhajed [22] investigated homogeneous anisotropic Bianchi type IX cosmological model for perfect fluid distribution with electromagnetic field.

In this paper, we have investigated LRS Bianchi type II homogeneous cosmological model for perfect fluid with electromagnetic field and variable Λ. To get determinate solution, we have assumed the condition $A = B^n$ between metric potentials, where n is a constant. Various physical and geometrical features of the model are also discussed.

2. The Metric and Field Equations

We consider LRS Bianchi type II metric of the form

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2 \tag{1}$$

where the metric potentials A and B are functions of t alone and $\sqrt{-g} = A^2 B$

We have considered distribution of matter to consist of perfect fluid with an infinite electrical conductivity and magnetic field. So, the energy momentum tensor is taken in the form

$$T_i^j = (p + \rho)\, \mathrm{v}_i v^j + p g_i^j + E_i^j \tag{2}$$

where ρ is energy density, p is pressure.

Here, v^i describes the four velocity vector of the matter satisfying the following conditions,

$$v_i v^i = -1 \tag{3}$$

The electromagnetic field E_i^j is defined by

$$E_i^j = \bar{\mu}\left[|h|^2 \left(v_i v^j + \frac{1}{2} g^j{}_i \right) - h_i h^j \right] \tag{4}$$

where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \varepsilon_{ijkl} F^{kl} v^j \tag{5}$$

where F^{kl} is the electromagnetic field tensor and ε_{ijkl} is the Levi-Cevita tensor density.

The coordinates are considered to be co-moving so that

$$v^i = (0,0,0,1) \tag{6}$$

The incident magnetic field is taken along x-axis so that

$$h_1 \neq 0; h_2 = h_3 = h_4 = 0 \tag{7}$$

The first set of Maxwell's equations is

$$F_{[ij;k]} = 0 \tag{8}$$

which leads to

$$F_{23} = H = constant \tag{9}$$

which is the only non-vanishing component.

The components of electromagnetic field with help of equations (4), (5) and (9) are obtained as

$$E_2^2 = E_3^3 = \frac{H^2}{2\bar{\mu}A^2B^2} = E_1^1 = -E_4^4 \tag{10}$$

The Einstein's field equation in the geometrized unit (c = 1, 8πG = 1) with time dependent cosmological term Λ is given by

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \tag{11}$$

The Einstein's field equation (11) for the metric (1) together with (10) leads to

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{A} + \frac{B^2}{4A^4} + \Lambda = -\left(p - \frac{H^2}{2\bar{\mu} A^2 B^2} \right) \tag{12}$$

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{3B^2}{4A^4} + \Lambda = -\left(p + \frac{H^2}{2\bar{\mu} A^2 B^2} \right) \tag{13}$$

$$\frac{A_4^2}{A^2} + \frac{2A_4 B_4}{AB} - \frac{\mathrm{B}^2}{4A^4} + \Lambda = -\left(-\rho - \frac{H^2}{2\bar{\mu} A^2 B^2} \right) \tag{14}$$

3. Solution of Field Equations

The field equations (12) - (14) are system of three equations in five unknown A, B, p, ρ and Λ. To get deterministic solution of the model, we use condition between metric potentials i.e.

$$A = B^n \tag{15}$$

From equations (12) and (13) we get

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{A_4^2}{A^2} - \frac{A_4 B_4}{AB} - \frac{B^2}{A^4} = -\frac{K}{A^2 B^2} \tag{16}$$

where $K = \dfrac{H^2}{\bar{\mu}}$

Equation (15) and (16) leads to

$$BB_{44} + 2nB_4^2 = \frac{1}{(n-1)} \left(-\frac{\mathrm{K}}{B^{2n}} + \frac{1}{B^{4n-4}} \right) \tag{17}$$

Now on putting $B_4 = f(B)$ in equation (17), we get

$$\frac{df^2}{dB} + 4n \frac{f^2}{B} = \frac{1}{(n-1)} \left(-\frac{2K}{B^{2n+1}} + \frac{2}{B^{4n-3}} \right) \tag{18}$$

Equation (18) leads to

$$f^2 = \frac{1}{(n-1)} \left(-\frac{K}{nB^{2n}} + \frac{1}{2B^{4n-4}} \right) + \frac{L}{B^{4n}} \tag{19}$$

where L is constant of integration.

Equation (19) leads to

$$\int \frac{dB}{\sqrt{\frac{1}{(n-1)}\left[-\frac{K}{nB^{2n}}+\frac{1}{2B^{4n-4}}\right]+\frac{L}{B^{4n}}}}=t+M \tag{20}$$

where M is constant of integration. The value of B can be determine by equation (20). After a suitable transformation of co-ordinates, metric (1) reduces to

$$ds^2=\frac{-dT^2}{\frac{1}{(n-1)}\left[-\frac{K}{nT^{2n}}+\frac{1}{2T^{4n-4}}\right]+\frac{L}{T^{4n}}}+T^{2n}\left(dX^2+dZ^2\right)+T^2(dY-XdZ)^2 \tag{21}$$

where B = T, x = X, y = Y and z = Z.

4. Some Physical and Geometrical Features

The pressure and density for the model (21) are given by

$$p=\frac{K(n-3)}{2(n-1)T^{2n+2}}+\frac{(2n^2-n-3)}{4(n-1)T^{4n-2}}+\frac{n(n+2)L}{T^{4n+2}}-\Lambda \tag{22}$$

$$\rho=-\frac{3K(n+1)}{2(n-1)T^{2n+2}}+\frac{(2n^2+3n+1)}{4(n-1)T^{4n-2}}+\frac{n(n+2)L}{T^{4n+2}}+\Lambda \tag{23}$$

For the specification of Λ, we assume that the fluid obeys an equation of state of the form

$$p=\gamma\rho \tag{24}$$

Using equations (22) and (23) in (24), we get

$$\Lambda(1+\gamma)=\frac{K\left((n-3)+3\gamma(n+1)\right)}{2(n-1)T^{2n+2}}+\frac{(2n^2-n-3)-\gamma(2n^2+3n+1)}{4(n-1)T^{4n-2}}+\frac{n(n+2)L(1-\gamma)}{T^{4n+2}}$$

The scalar expansion θ calculated for flow vector v^i is given by

$$\theta=(2n+1)\sqrt{\frac{K}{n(n-1)T^{2n+2}}+\frac{1}{2(n-1)T^{4n-2}}+\frac{L}{T^{4n+2}}} \tag{26}$$

The shear σ for the model (21) is given by

$$\sigma^2=\frac{1}{3}(n-1)^2\left[\frac{K}{n(n-1)T^{2n+2}}+\frac{1}{2(n-1)T^{4n-2}}+\frac{L}{T^{4n+2}}\right] \tag{27}$$

From equations (26) and (27) we have

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3}\frac{(n-1)^2}{(2n+1)^2} = \text{constant} \tag{28}$$

5. Solution in Absence of Magnetic Field

In the absence of magnetic field, the metric (21) reduces to

$$ds^2 = \frac{-dT^2}{\left[\frac{1}{2(n-1)T^{4n-4}} + \frac{L}{T^{4n}}\right]} + T^{2n}(dX^2 + dZ^2) + T^2(dY - XdZ)^2 \tag{29}$$

The pressure and density for the model (29) are given by

$$p = \frac{(2n^2 - n - 3)}{4(n-1)T^{4n-2}} + \frac{n(n+2)L}{T^{4n+2}} - \Lambda \tag{30}$$

$$\rho = \frac{(2n^2 + 3n + 1)}{4(n-1)T^{4n-2}} + \frac{n(n+2)L}{T^{4n+2}} + \Lambda \tag{31}$$

Also, the cosmological term Λ for the model (29) is given by

$$\Lambda(1+\gamma) = \frac{(2n^2 - n - 3) - \gamma(2n^2 + 3n + 1)}{4(n-1)T^{4n-2}} + \frac{n(n+2)L(1-\gamma)}{T^{4n+2}} \tag{32}$$

6. Conclusion

We have obtained a new class of anisotropic cosmological model for perfect fluid in presence of electromagnetic field. In general the model represents expanding, shearing and non-rotating universe.

The model starts with big bang at $T = 0$ and expansion in the model decreases as time increases. The expansion in the model stops when $n = -\frac{1}{2}$ and stops at $T \to \infty$. Also as $T \to \infty$, $p \to 0$ and $\rho \to 0$. Since $T \to \infty, \frac{\sigma}{\theta} \neq 0$ so the model does not approach isotropy for large values of T. However, when $n = 1$, the model is isotropized.

The cosmological constant for both the models is decreasing function of time and approaches a small value in the present epoch. The value of cosmological constant for these models are small and positive, which are supported by the result from recent supernova observations recently obtained by the High-Z Supernova team and Supernova cosmological project.

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References

[1] Adhav, K.S., Vgale, M.R., Kale, C.B. and Bhinde, M.P. (2007). *Bulg. Journal of Physics* 34, 260.

[2] Asseo E. and Sol. H. (1987). Extragalactic magnetic fields, *Physics Reports*, 148(6), 307-436.

[3] Bali, R. and Singh, J.P. (2008). Bulk viscous Bianchi Type I cosmological models with time dependent cosmological term Λ, *Int. J. Theor. Phys.*47, 3288-3297.

[4] Bali, R., Singh, P. and Singh, J.P. (2012). Bianchi Type V viscous fluid cosmological models in presence of decaying vacuum energy, *Astrophys. Space-Sci.* 341, 701-706.

[5] Bali, R. and Singh, S. (2014). LRS Bianchi Type II massive string cosmological model for stiff fluid distribution with decaying vacuum energy, *Int. J. Theor. Phys*. 53, 2082-2090.

[6] Bali, R., Pareek, U.K. and Pradhan, A. (2007). Bianchi type I massive string magnetized barotropic perfect cosmological model in general relativity, *Chinese Physics Letter* 24, 2455-2458.

[7] Bertolami, O. (1986). Time Dependent Cosmological Term. *Nuovo Cimento*, B93, 36-42.

[8] Bertolami, O. (1986). Brans-dicke Cosmology with a Scalar Field Dependent Cosmological Term, *Fortschr. Physics* 34, 829-833.

[9] Chakravarty, N. and Biswas, L. (2007). Bianchi type I cosmological model with variable Λ-term, *Astrophysics and Space Science,* 312(1), 13-15.

[10] Dunn, K.A. and Tupper, B.O.J. (1976). A Class of Bianchi Type VI_0 Cosmological Models with Electromagnetic Field, *The Astrophysical Journal* 204(1), 322-329.

[11] Lorentz, D. (1982). Tilted electromagnetic Bianchi type III cosmological solution, *Astrophysics and Space Science*, 85(1), 59-61.

[12] Ozer, M. and Taha, M.O. (1987). A model of the universe free of cosmological problems, *Nuclear Physics B*, 287, 776-796.

[13] Pradhan, A., Pandey, P. and Rai, K.K. (2006). Magnetized anisotropic cosmological models with varying Λ, *Czechoslovak Journal of Physics* 56(3), 303-313.

[14] Ratra, B. and Peebles, P.I.E. (1988). Cosmological consequence of a rolling homogeneous scalar field. *Physics Review D,* 37(12), 3406.

[15] Roy, S.R., Singh, J.P. and Narain, S. (1985). Bianchi type-VI_0 perfect fluid cosmological models with magnetic field, *Astrophysics and Space Science*, 111(2), 389-397.

[16] Singh, G.P. (1995). String cosmology with electromagnetic fields in Bianchi type II, VIII, IX space times, *Nuovo Cimento B*, 110(12).

[17] Singh, T. and Agarwal, A.K. (1997). Bianchi type II, VIII, IX in certain new theories of gravitation, *Astrophysics and Space Science*, 191, 61-88.

[18] Tiwari, R.K. (2009). Bianchi type-III universe with perfect fluid, *Astrophysics and Space Science*, 319(1), 85-87.

[19] Tiwari, R.K., Tiwari, D.K. and Tiwari, A.K.S. (2014). Anisotropic Bianchi Type-II Cosmological Model with Varying Λ, *International Journal of (IJETCAS)*, 14-852.

[20] Tyagi, A. and Sharma, K. (2010). Bianchi Type-IX String Cosmological Models for Perfect Fluid Distribution in General Relativity, *Chinese Physics Letter*, 27, 7,079801.

[21] Tyagi, A. and Sharma, K. (2011). Locally Rotationally Symmetric Bianchi Type-II Magnetized String Cosmological Model with Bulk Viscous Fluid in General Relativity, *Chinese Physics Letter*, 28(8), 089802.

[22] Tyagi, A. and Chhajed, D. (2012). Homogeneous anisotropic Bianchi type IX cosmological model for perfect fluid distribution with electromagnetic field, *American Journal of Mathematics and Statistics*, 2(3), 19-21.

[23] Weinberg, S. (1989). The cosmological constant problem, *Reviews of Modern Physics*, 61, 1.

[24] Zeldovich, Ya, B., Ruzmainkin, A.A. and Sokoloff, D.D. (1983). Magnetic Field in Astrophysics. (Gordon and Breach, New York).