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A TWO LAYER MATHEMATICAL MODEL FOR MUCUS TRANSPORT IN CONSTRICTED AIRWAYS: EFFECT OF SLIP PARAMETER

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Abstract: This paper presents a two-layer cylindrical quasi-steady co-axial flow model of air and mucus in the constricted airways subjected to a time varying pressure gradient. In this model, air and mucus both are assumed as an incompressible Newtonian fluid which follow quasi-steady flow. Mucus is taken as highly viscous fluid, whereas air is less viscous. The effect of slip parameter due to immotile cilia bed which forms porous matrix, is also incorporated into the model. The analysis reveals that the flow rates of air and mucus both decrease as the mucus viscosity increases. It is also shown that both air and mucus flow rates increase as the magnitude of pressure gradient and slip parameter increases. It is also observed that the mucus flow rate increases as the mucus thickness increases, whereas the air flow rate decreases as the mucus thickness increases. This study also states that the flow rates of air and mucus both decrease as the thickness of constriction increases.

Keywords: Quasi-steady state, Time varying pressure gradient, Immotile cilia, Constricted airways.

1. Introduction

Under pathological conditions, human lungs are adversely affected by various diseases such as cystic fibrosis, chronic bronchitis, bronchial asthma, lung cancer, ciliary dyskinesia, etc. In most of these diseases, cilia become immotile which may induce the loss of cilia. In case of diseases such as cystic fibrosis, the periciliary fluid may be absent and cilia may be immotile. The mammalian bronchi (respiratory tracts) are very sensitive so that foreign particles (such as dust particles, straw particles, carcinogens, etc.) or other means of irritation (such as sneezing) in airways can initiate forced expiration or the cough reflex causing the flow of fluids (air and mucus) in the airways. The coughing or cough reflex is associated with asthma, chronic bronchitis, etc. The human cough has mainly two functions: it helps to protect the lungs against aspiration and also helps to propel fluid secretions and other materials upward through the airways [1, 2].

Respiratory diseases such as chronic bronchitis and cystic fibrosis leads to the excessive production of mucus, which is expelled through forced expiration or coughing. In cases, where the airways are affected by immotile cilia syndrome, coughing serves as the primary defense mechanism for mucus clearance. Over the past few decades, numerous experiments have explored two-phase flow in respiratory tubes under externally applied pressure to simulate mucus transport in the airways during coughing. In particular, Clarke et al. [3] have demonstrated a significant increase in airflow resistance in liquid-lined tubes across all flow rates compared to dry tubes. They have observed that the nonlinear pressure-flow relationships under laminar flow conditions in liquid-lined tubes, attributed to increased flow resistance due to the presence of high-viscosity fluid occupying air space. Scherer and Burtz [17] have conducted fluid mechanical experiments relevant to coughing, utilizing air and liquid blown out of a straight tube by turbulent air jets. Zahm et al. [24] have highlighted an increase in mucus transport due to the presence of a sol phase at the bottom plate in their cough machine experiments. Furthermore, they studied the effect of repetitive coughing on mucus transport in a simulated cough machine, noting a considerable decrease in mucus viscosity due to high shear rates during coughing. Many more investigators have contributed or contributing in this field by their experimental findings [3, 4, 18].

In this regard, the theoretical models are also playing their important roles. Many mathematical approaches and models are also being used in this field [6, 7, 8, 10, 19, 20]. Verma and Rana [21] have presented a mathematical model for mucus transport in human lung airways by taking the effects of air motion, cilia beating and porosity parameter under steady-state conditions by considering mucus as a viscoelastic fluid. They have shown that the mucus transport rate increases with increase in air motion, cilia beating, porosity parameter and decreases with increase in viscosities of the serous and mucus layers. Rana et al. [15] have proposed a two-layer circular steady-state mathematical model to investigate mucus transport in human lung airways by taking the effects of mucus viscoelasticity, cilia beating, and porosity parameter. Kumar et al. [14] have investigated mucus transport in a diseased airway by considering the effect of constriction on the airways. They have shown that the mucus transport rate decreases as the diameter of the airways increases. Chitra and Shabana [12] have provided a two-layer model for the air-mucus interface in the constricted human airways under a time-varying pressure gradient by considering the effect of slip parameter. They have shown that the mucus transport rate increases as the slip parameter increases. Raut et al. [16] have presented a two-layer planar unsteady state model by considering the porosity parameter, pressure gradient and shear stress caused by air-motion. They have shown that the mucus transport rate increases as the porosity parameter, pressure gradient, and shear stress caused by air motion increase.

In this paper, we wish to study a two-layer cylindrical co-axial flow model to investigate air and mucus flows in the constricted airways. In the central lumen of the annular region, air is assumed to flow under quasi-steady condition. However, mucus surrounding the central lumen is also assumed to flow under quasi-steady state condition. We consider the effect of immotile cilia bed which is approximated by a porous matrix within which periciliary fluid flows following the Darcy's law. This model considers the simultaneous flow of air and mucus in a cylindrical tube during coughing. Here, we make following assumptions which have also been used in previous researches of various investigators:

a) The flow of air and mucus is symmetrical about the central axis of the cylindrical tube.

b) The pressure gradient generated due to coughing in the fluid layers is time-varying.

c) Mucus behaves as an incompressible Newtonian fluid due to high shear rates experienced during coughing [Zahm et al. [25]].

d) Since air is saturated with watery fluid during coughing, it is also assumed to behave as an incompressible Newtonian fluid within the lung airways.

e) Air and mucus flows are assumed to be laminar and quasi-steady during coughing.

f) It is also assumed that there is no existence of a periciliary layer and therefore, mucus is assumed to be directly attached to the epithelial walls of the airways.

g) In larger airways, there may exist a slip parameter at the interface of the mucus layer and immotile cilia bed which is saturated with periciliary fluid and forms a porous matrix following the Darcy's law.

2. Mathematical Model

In real situation, the airways in the human lungs are cylindrical in nature. Therefore, the physical situation of movement in the lung airways is idealized by circular tube geometry. The inner surface wall being ciliated (Fig.1) and it is assumed that the central lumen is filled with air surrounded by a highly viscous mucus fluid, which is covered by a watery periciliary layer with much lower viscosity than that of the mucus. It is assumed that the certain cilia are immotile and form a porous matrix bed in periciliary layer in contact with the epithelium. No net flow is assumed in the periciliary layer in contact with the epithelium.

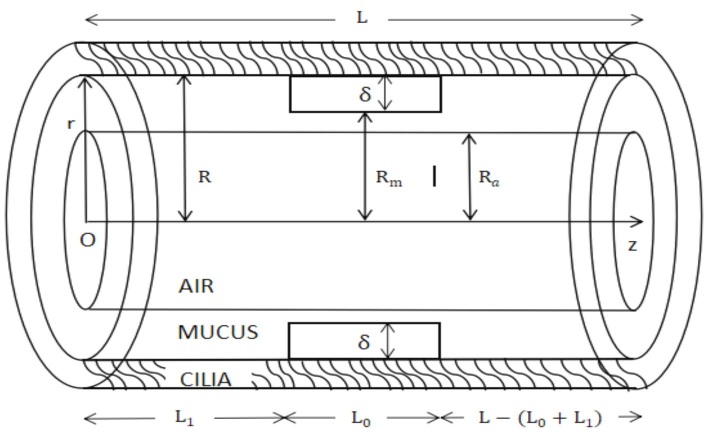

Figure 1: Circular tube geometry for mucus transport in constant constricted airways.

Based on the constriction geometry, the radius of the cylindrical tube can be expressed as follows:

$$\mathrm{R_m} = \mathrm{R} - \delta \tag{1}$$

where R is the radius of circular tube, $\mathrm{R_m}$ is the radius of circular tube in constricted area, $\delta(<< \mathrm{R_m})$ is the thickness of constriction which is assumed as constant.

The equations governing the flow of air and mucus in lung airways under quasi-steady state and low Reynolds number flow approximations, are written as follows:

Region I Air Flow Region $(0 \le \mathrm{r} \le \mathrm{R}_a)$:

$$\frac{\partial^2 u_a}{\partial r^2} + \frac{1}{r}\frac{\partial u_a}{\partial r} = \frac{1}{\mu_a}\frac{\partial p}{\partial z} \tag{2}$$

Region II Mucus Flow Region $(\mathrm{R}_a \le \mathrm{r} \le \mathrm{R_m})$:

$$\frac{\partial^2 u_m}{\partial r^2} + \frac{1}{r}\frac{\partial u_m}{\partial r} = \frac{1}{\mu_m}\frac{\partial p}{\partial z} \tag{3}$$

where z is the axial coordinate along the tube axis which is in the direction of flow and r is the radial coordinate in the radial direction which is perpendicular to the fluid flow, R_a is the thickness up to air-mucus interface, p is the mean pressure that is constant across the two layers, u_a and u_m are the mean velocity components of air and mucus in the direction of z and μ_a , μ_m are the viscosities of air and mucus respectively [11, 12, 18].

The following boundary and matching conditions are taken into account to solve the problem:

Boundary Conditions:

$$\frac{\partial u_a}{\partial r} = 0, \qquad r = 0 \tag{4}$$

$$u_m = -\beta\tau_m, \quad r = R_m \tag{5}$$

Matching Conditions:

$$u_a = u_m, \qquad r = R_a \tag{6}$$

$$\tau_a = \tau_m, \qquad r = R_a \tag{7}$$

where τ_a and τ_m are the mean shear stress in the air and mucus layers respectively given by

$$\tau_a = \mu_a \left(\frac{\partial u_a}{\partial r}\right) \tag{8}$$

$$\tau_m = \mu_m \left(\frac{\partial u_m}{\partial r}\right) \tag{9}$$

The negative sign in (5) is taken into account because of negative value of τ_m taken in the mucus layer. Here, it is kept in mind that β is slip parameter arising at the interface between mucus and the immotile cilia saturated with the periciliary layer forming a porous matrix in the airways. The conditions (6) and (7) represent the continuity of velocity and stress components at the interface between air and mucus layers.

During coughing, the pressure gradient generated in lung airways changes with time. Therefore, we may assume that

$$-\frac{\partial p}{\partial z} = P = P_0 f(t) \tag{10}$$

where t is time, P_0 is constant and influenced by intensity of cough. The higher intensity of cough leads to proportional increase in flow rates as the flow duration progresses. The function $f(t)$ in equation (10) is taken from the paper of Satpathi et al. [5] which is defined as follows:

$$f(t) = \begin{cases} \frac{27\, t(T-t)^2}{4T^3}, & 0 \le t \le T \\ 0, & t > T \end{cases} \tag{11}$$

For the sake of simplicity, we may also assume that the cough duration T ranges from 0.001 sec to 0.035 sec. The graphical representation of equation (11) is shown in Figure 2.

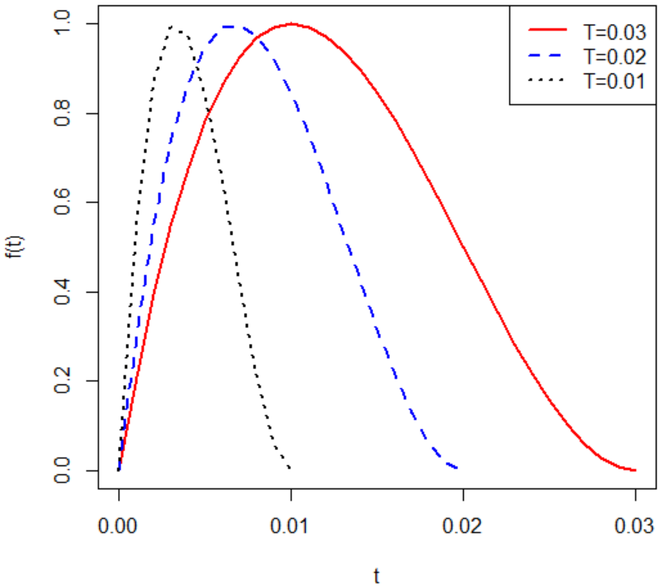

Figure 2: Graphical Representation of $f(t)$ with t for various values of T

3. Analytical Solution

Solving equations (2) and (3) by using boundary and matching conditions (4)-(7), the stress and velocity components in air and mucus layers are computed which are given below:

$$\tau_a = \tau_m = -\frac{Pr}{2} \tag{12}$$

$$u_a = \frac{P}{2}\left[\frac{1}{2\mu_a}(R_a^2 - r^2) + \frac{1}{2\mu_m}(R_m^2 - R_a^2) + \beta R_m\right] \tag{13}$$

$$u_m = \frac{P}{2}\left[\frac{1}{2\mu_m}(R_m^2 - r^2) + \beta R_m\right] \tag{14}$$

The volumetric flow rates in the two layers (air and mucus) can be defined as follows:

$$Q_a = \int_0^{R_a} 2\pi u_a dr, \quad Q_m = \int_{R_a}^{R_m} 2\pi u_m dr \tag{15}$$

Substituting the value of u_a from equation (13) and the value of u_m from (14) into equation (15), we obtain

$$Q_a = \frac{\pi P R_a^2}{2}\left[\frac{R_a^2}{4\mu_a} + \frac{(R_m^2 - R_a^2)}{2\mu_m} + \beta R_m\right] \tag{16}$$

$$Q_m = \frac{\pi P (R_m^2 - R_a^2)}{2}\left[\frac{(R_m^2 - R_a^2)}{4\mu_m} + \beta R_m\right] \tag{17}$$

Now, to find the pressure drop in each layer, we know from equation of continuity that Q_a and Q_m are constants. Therefore, from equations (16) and (17), we get

$$-\frac{\partial p}{\partial z} = \frac{Q_a}{\pi K_1 (R_m^2 + K_2 R_m - K_3)} \tag{18}$$

$$-\frac{\partial p}{\partial z} = \frac{Q_m}{\pi K_4 [(R_m^2 - R_a^2)^2 + 2K_2 R_m (R_m^2 - R_a^2)]} \tag{19}$$

where $K_1 = \frac{R_a^2}{4\mu_m}$, $K_2 = 2\beta\mu_m$, $K_3 = R_a^2\left(1 - \frac{\mu_m}{2\mu_a}\right)$, $K_4 = \frac{1}{8\mu_m}$.

Replacing R_m by R for non-constricted regions, the pressure gradient for non-constricted regions $0 \le z \le L_1$ and $L_1 + L_0 \le z \le L$ becomes

$$-\frac{\partial p}{\partial z} = \frac{Q_a}{\pi K_1 (R^2 + K_2 R - K_3)} \tag{20}$$

$$-\frac{\partial p}{\partial z} = \frac{Q_m}{\pi K_4 [(R^2 - R_a^2)^2 + 2K_2 R (R^2 - R_a^2)]} \tag{21}$$

Since the pressure is present only at two ends of the tube i.e. $p = p_0$ at $z = 0$, $p = p_L$ at $z = L$. Then, we define the pressure drop as $\Delta P = p_0 - p_L$.

Now, integrating equations (18) and (20), we get

$$\Delta P = -\int_0^L dp = \int_0^{L_1} \frac{Q_a dz}{\pi K_1 (R^2 + K_2 R - K_3)} + \int_{L_1}^{L_1 + L_0} \frac{Q_a dz}{\pi K_1 (R_m^2 + K_2 R_m - K_3)} + \int_{L_1 + L_0}^{L} \frac{Q_a dz}{\pi K_1 (R^2 + K_2 R - K_3)}$$

Putting the value of R_m from (1) in above equation, we get

$$\Delta P = \frac{Q_a}{\pi K_1}\left[\frac{(L - L_0)}{(R^2 + K_2 R - K_3)} + \frac{L_0}{\{(R-\delta)^2 + K_2 (R-\delta) - K_3\}}\right] \tag{22}$$

The volumetric flow rate of air i.e.; Q_a can be found as follows:

$$Q_a = \frac{\pi K_1 \Delta P}{\left[\frac{(L - L_0)}{(R^2 + K_2 R - K_3)} + \frac{L_0}{\{(R-\delta)^2 + K_2 (R-\delta) - K_3\}}\right]} \tag{23}$$

Similarly, integrating equations (19) and (21), we get

$$\Delta P = -\int_0^L dp = \int_0^{L_1} \frac{Q_m dz}{\pi K_4 [(R^2 - R_a^2)^2 + 2K_2 R (R^2 - R_a^2)]} + \int_{L_1}^{L_1 + L_0} \frac{Q_m dz}{\pi K_4 [(R_m^2 - R_a^2)^2 + 2K_2 R_m (R_m^2 - R_a^2)]}$$

$$+\int_{L_1+L_0}^{L}\frac{Q_m dz}{\pi K_4[(R_m^2-R_a^2)^2+2K_2R(R^2-R_a^2)]}$$

Putting the value of R_m from (1) in above equation, we get

$$\Delta P=\frac{Q_m}{\pi K_4}\left[\frac{(L-L_0)}{[(R^2-R_a^2)^2+2K_2R(R^2-R_a^2)]}+\frac{L_0}{[((R-\delta)^2-R_a^2)^2+2K_2(R-\delta)((R-\delta)^2-R_a^2)]}\right] \quad (24)$$

The volumetric flow rate of mucus i.e.; Q_m can be found as follows:

$$Q_m=\frac{\pi K_4\Delta P}{\left[\frac{(L-L_0)}{[(R^2-R_a^2)^2+2K_2R(R^2-R_a^2)]}+\frac{L_0}{[((R-\delta)^2-R_a^2)^2+2K_2(R-\delta)((R-\delta)^2-R_a^2)]}\right]} \quad (25)$$

4. Results and Discussion

To explore the effects of various model parameters on mucus and air flow rates, the values of Q_a and Q_m are given by equations (23) and (25) have been computed by using the following datasets [Weibal [22], Shukla [18]]:

$R=41.45\times10^{-2}$cm, $R_m=38.45\times10^{-2}$ cm, $R_a=31.45\times10^{-2}$cm

t= 0 – 0.035 sec, T = 0.035 sec, L = 1.0 cm

L_0= 0.5 cm, $P_0=1-2\times10^5$ gm cm^{-2} sec^{-2}, β= 0 – 0.10 gm cm^{2}sec

μ_m = 1.00 – 10.00 poise, μ_a= 0.0002 poise, δ = 0 – 0.02 cm

The variations in volumetric flow rates Q_a and Q_m with respect to time t are shown in following figures:

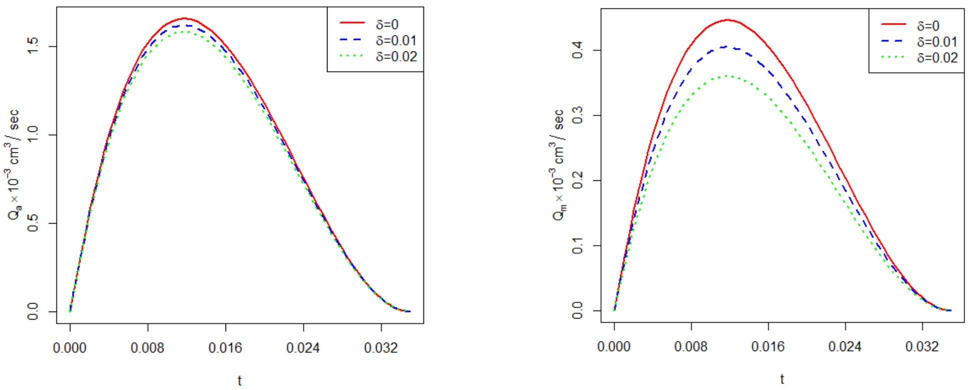

Figure 3: Variations of Q_a and Q_m with t for different values of δ

Figure 3 illustrate the impact of time on the flow rates of air and mucus, for fixed values of T = 0.035 sec, $R=41.45\times10^{-2}$cm, $R_m=38.45\times10^{-2}$ cm, $R_a=31.45\times10^{-2}$cm, L = 1.0 cm, L_0= 0.5 cm, $P_0=1\times10^5$ gm cm^{-2} sec^{-2}, β= 0.05 gm cm^{2}sec, μ_m = 1 poise and μ_a= 0.0002 poise for different values of δ. It is observed that the volumetric flow rates of air and mucus decreases with the increase in thickness of constriction. These results match with the results of Kumar et al. [11] and Chitra et al. [12].

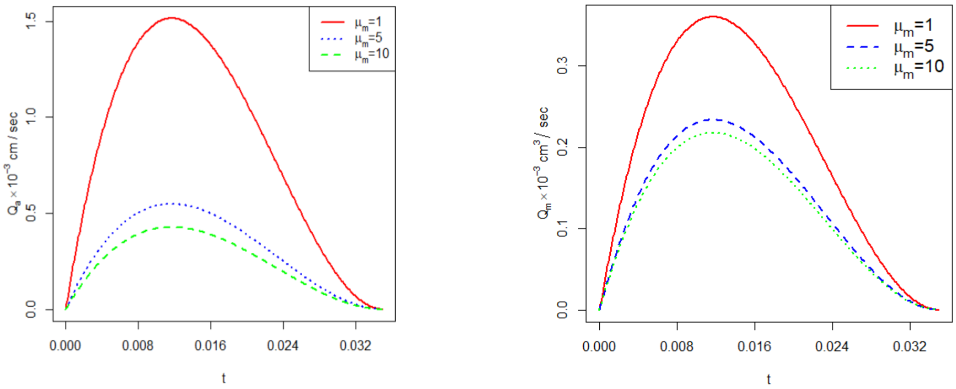

Figure 4: Variations of Q_a and Q_m with t for different values of μ_m

Figure 4 illustrate the impact of time on air and mucus flow rates for fixed values of $R = 41.45 \times 10^{-2}$cm, $R_m = 38.45 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$cm, T = 0.035 sec, L = 1.0 cm, L_0= 0.5 cm, $P_0 = 1 \times 10^5$ gm cm^{-2} sec^{-2} , β= 0.05 gm cm^{2}sec, μ_a= 0.0002 poise and δ = 0.02 cm for various values of μ_m. The observation reveals that air and mucus flow rates decrease with increase in mucus viscosity. These results match with the results of Kumar et al. [11] and Verma and Rana et al. [15, 21].

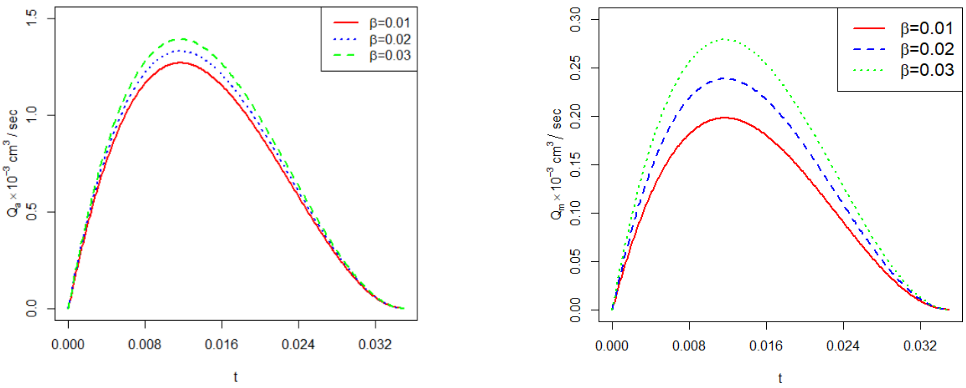

Figure 5: Variations of Q_a and Q_m with t for different values of β

Figure 5 depict the impact of time on air and mucus flow rates for fixed values of $R = 41.45 \times 10^{-2}$cm, $R_m = 38.45 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$cm, T = 0.035 sec, L = 1.0 cm, L_0= 0.5 cm, $P_0 = 1 \times 10^5$ gm cm^{-2} sec^{-2}, μ_m = 1 poise, μ_a= 0.0002 poise and δ = 0.02 cm poise for different values of β. It is observed that air and mucus flow rates increase as the slip parameter β increases at a constant constriction size. These results match with the results of Satpathi et al. [5], Chitra et al. [12] and Raut et al. [16].

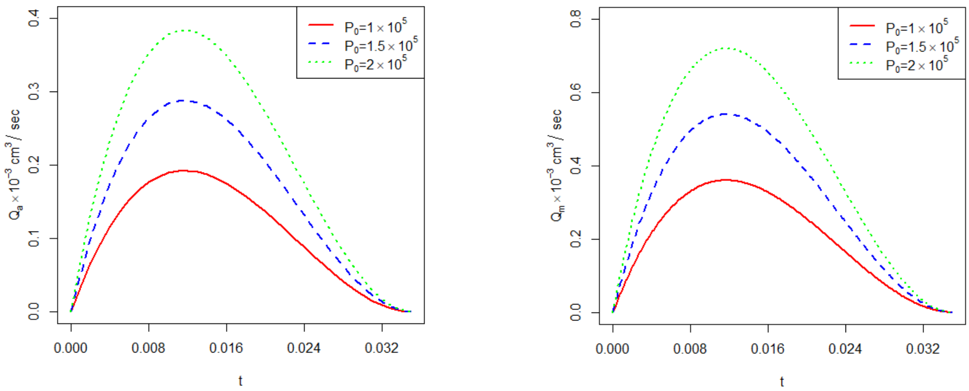

Figure 6: Variations of Q_a and Q_m with t for different values of P_0

Figure 6 depict the impact of time on air and mucus flow rates for fixed values of $R = 41.45 \times 10^{-2}$cm, $R_m = 38.45 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$cm, T = 0.035 sec, L = 1.0 cm, L_0= 0.5 cm, β= 0.05 gm cm^{2}sec, μ_m = 1 poise, μ_a= 0.0002 poise and δ = 0.02 cm for different values of P_0 . It is observed that air and mucus flow rates increase as the pressure drop in the two layers increases. These results match with the results of almost all investigators [1, 2, 3, 19, 20, 21].

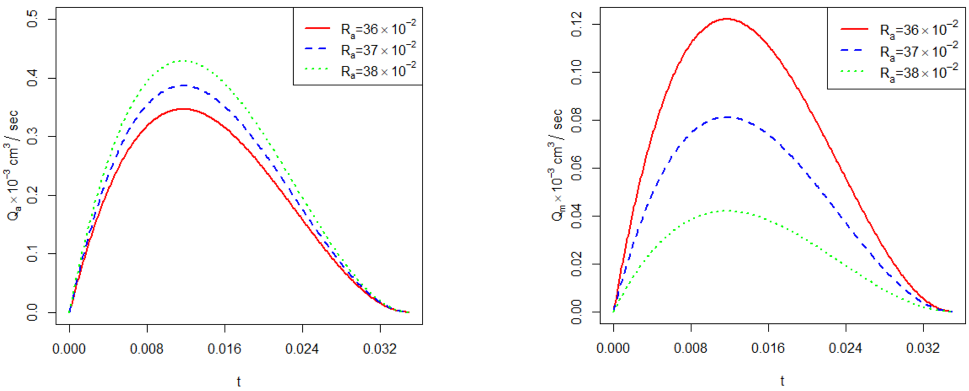

Figure 7: Variations of Q_a and Q_m with t for different values of R_a

Figure 7 depict the impact of time on air and mucus flow rates for fixed values of $R = 41.45 \times 10^{-2}$cm, $R_m = 38.45 \times 10^{-2}$ cm, T = 0.035 sec, L = 1.0 cm, L_0= 0.5 cm, β= 0.05 gm cm^2 sec, μ_m = 1 poise, μ_a= 0.0002 poise and δ = 0.02 cm for different values of R_a. It is observed that mucus flow rate increases as the mucus thickness increases, where as the air flow rate decreases as the mucus thickness increases. These results match with the results of Agarwal et al. [1, 2], Satpathi et al. [5], King et al. [9], Shukla et al. [18], Verma and Tripathee [20].

5. Conclusion

This paper describes a two-layer cylindrical quasi-steady co-axial flow model of air and mucus flow in constricted lung airways by considering the effects of coughing, airflow and the stresses in two layers. The effect of slip parameter is also incorporated into the model by using boundary conditions. By analytical and graphical analysis, the following conclusions are drawn:

a) Both air and mucus flow rates decrease as the thickness of constriction increases.

b) Both air and mucus flow rates increase as the viscosity of mucus decreases.

c) Both air and mucus flow rates increase as the slip parameter increases.

d) Both air and mucus flow rates increase as the pressure gradient (influenced by intensity of cough) increases.

e) The mucus flow rate increases as the mucus thickness increases, whereas the air flow rate decreases as the mucus thickness increases.

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