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HOMOGENEOUS MODELS OF BIANCHI TYPE-II WITH ANISOTROPIC DARK ENERGY IN MODIFIED SCALE-COVARIANT THEORY OF GRAVITATION

S. Chandel* and **Shri Ram****
*Department of Engineering, Science and Humanities,
Thakur College of Engineering and Technology, Mumbai, India
**Department of Mathematical Sciences, Indian Institute of Technology,
(BHU), Varanasi-2210 05, India
Email: chandel.savita7@gmail.com, srmathitbhu@rediffmail.com

Abstract: In this paper, we investigate anisotropic Bianchi type-II space-time with variable equation of state parameter and constant deceleration parameter within the framework of modified scale-covariant theory of gravitation formulated by Canuto et al. [11]. Exact solutions of the field equations are obtained by applying a special law of variation of Hubble parameter which yields a constant value of deceleration parameter. Two different physically viable models of the universe are presented in two types of cosmologies, one with power-law expansion and the other one with exponential expansion. The dark energy models are new and quite general than the models obtained earlier by Singh and Sharma [31]. Some physical and kinematical properties of dark energy models are discussed. Cosmological model with power-law expansion has an initial big-bang singularity at the time t=0, whereas the model with exponential expansion has a singularity in the infinite past. We observe that the models of the universe in two types of cosmologies are compatible with the results of the recent observations on the present-day universe.

Keywords: Bianchi Type-II cosmological model, dark energy, scale covariant theory.

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1. Introduction

In recent years experimental studies of CMBR and many other effects have stimulated much theoretical interests in the study of anisotropic cosmological models. The interests have been due to the realization that the standard cosmological models, which are in very good agreement with the present-day universe, do not provide a clear description of the early phase of evolution of the universe. The physically realistic description of the early evolution of the universe is best given by anisotropic models. The spatially homogeneous and anisotropic Bianchi models I-IX [28] present a middle way between FRW models and completely inhomogeneous and anisotropic models, and thus play an important role in modern cosmology. This is due to the fact that close to big-bang singularity, neither the assumption of spherical symmetry nor of the isotropy can be strictly valid. This stimulates cosmologists to obtain exact anisotropic solutions of Einstein's field equations which yield cosmologically acceptable physical models of the universe. Bianchi type-II space-times play a fundamental role in constructing models with richer structure both geometrically and physically for describing the early stages of evolution of the universe. Asseso and Sol [3], Bali and Singh [5] emphasized the cosmological importance of Bianchi type-II space-times.

The limitations of general theory of relativity in providing satisfactory explanation of different phases of the evolution of the universe have led researchers to adopt various hypotheses and study their implications in this context. These hypotheses include the formulation of alternative or modified theories of gravitation by many cosmologists viz. Brans and Dicke [10], Nordtvedt [15], Sen [30], Wagoner [35], Saez and Saez-Ballester [29] etc. Such theories are expected to bring out a number of aspects of mathematical and physical interests. Canuto et al. [11] formulated a self-covariant theory of gravitation by associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space-time distances. They derived the generalized Einstein's field equations and studied their several astrophysical tests. This theory provides the necessary theoretical framework to discuss sensibly the possible variation of the gravitational constant G without compromising the validity of general relativity (Wesson[36] and Will [37]). In this theory, Einstein's field equations are valid in gravitational units whereas the physical quantities are measured in atomic units. The components of metric tensors in two systems of units are related by a conformal transformation.

$$\overline{g_{ij}} = \phi^2(x^k) g_{ij}, \qquad \text{i,j,k,=1,2,3,4} \tag{1}$$

where the barred quantities are refer to gravitational units and unbarred quantities denote atomic units. The gauge function ϕ is a function of all space-time coordinates. Using the conformal transformation (1), Canuto et al. [11] transformed Einstein's field equations to the form

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\phi) g_{ij} \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} \text{-} 4\phi_i\phi_j - g_{ij}\left(\phi\phi^{;k}_{;k} - \phi^k\phi_k\right) \tag{3}$$

where a semicolon denotes covariant derivative and ϕ_i denotes ordinary derivative with respect to x^i . Other symbols have their usual meanings.

Beesham [7] has discussed power asymptotic singularities in the scale-covariant theory of gravitation. Bali and Kumawat [4] investigated cosmological model with variable G in

C-field cosmology, Reddy and Venkateswarlu [26], Venkateswarlu and Kumar [34] Reddy [25], Adhav et al. [1], Shri Ram et al. [22-24] and Belinchon [8] are some of the authors who have studied several aspects of this theory.

The recent observational data of high red-shift from I_a supernovae (Riess et al. [27], Perlmutter et al. [17]), cosmic microwave background (CMB) anisotropy (Netterfield et al. [14]), large scale structure (LSS) (Spergel et al. [32]) all have suggested convincing indication that the present-day universe is undergoing a late-time accelerated expansion. The late-time cosmic acceleration is assumed to be driven by an exotic fluid, known as dark energy (DE), whose origin is even a mystery in modern cosmology. It is held that the accelerating expansion is driven by a negative pressure of DE, which tends to increase the rate of expansion of the universe (Peebles and Ratra [16]). At present much interests are focused on the study of cosmological models with a variable equation of state (EoS) parameter $\omega(t)=\frac{p}{\rho}$, where p is the fluid pressure and ρ is the energy density of matter. The cosmological constant Λ (vacuum energy) is the most efficient and simplest candidate for explaining the observed accelerated background of universe with $\omega=-1$. A lucid introduction and nice review of work done on DE models in general relativity is given by Farook et al. [12] and Pradhan et al. [19], Pradhan and Amirhaschi [18], Amirhaschi et al. [2], Pradhan et al. [19] have discussed DE models in anisotropic Bianchi type space-times with variable EoS parameter. It deserves to mention that Bamba et al. [6] have presented different isotropic DE cosmologies with early deceleration and late-time acceleration.

Singh and Sharma [31] investigated spatially homogeneous and anisotropic Bianchi type-II dark energy cosmological models in the scale-covariant theory of gravitation formulated by Canuto et al. [1]. In this paper, we consider the field equations and obtain new exact solutions for general Bianchi type-II dark energy models. The plan of the paper is as follows: In sect.2, we present the metric and field equations. In Sect.3, we obtain exact solution of the field equations by applying a special law of variation for Hubble parameter together with an additional assumption that the component σ_1^1 of shear tensor σ_i^j is proportional to the mean Hubble parameter. Two different physically viable models in two types of cosmologies, one with power-law expansion and the other one with exponential expansion are presented. We also discuss the physical and kinematical properties of the dark energy models. Some concluding remarks are given in Sect.4.

2. Metric and Field Equations

We consider the metric of spatially homogeneous and anisotropic Bianchi type-II space-time given by

$$ds^2=dt^2-A^2\left(dx-zdy\right)^2-B^2dy^2-C^2dz^2 \tag{4}$$

Where *A, B* and *C* are functions of cosmic time *t*.

The energy-momentum tensor T_i^j of the anisotropic fluid is given by

$$T_i^j = \text{diag}\left[\rho, -p_x, -p_y, -p_z\right) = \text{diag}\left[1, -\omega_x, -\omega_y, -\omega_z\right)\rho \tag{5}$$

Where ρ is the energy density of the fluid, p_x, p_y and p_z are the pressure and ω_x, ω_y and ω_z are the directional EoS parameters of the fluid in the direction of x, y and z respectively.

Now, parameterizing the deviation from isotropy by setting $\omega_x = \omega_y = \omega_z = \omega$ and then introducing the skewness parameter δ which is the deviation from ω on x-axis. The Eqn.(5) can be written as:

$$T_i^j = \text{diag}\left[1, -(\omega+\delta), -\omega, -\omega\right)\rho. \tag{6}$$

Here ω and δ are not necessarily constant and can be functions of the cosmic time *t*.

In a comoving coordinate system, the field equations (2) and (3) for the metric (4) with the help of Eqn.(6) can be written as :

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3}{4}\frac{A^2}{B^2C^2} - \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) + \frac{\dot{\phi}^2}{\phi^2} = -8\pi G(\omega+\delta), \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{1}{4}\frac{A^2}{B^2C^2} - \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right) + \frac{\dot{\phi}^2}{\phi^2} = -8\pi G\omega\rho, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{A^2}{B^2C^2} - \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \frac{\dot{\phi}^2}{\phi^2} = -8\pi G\omega\rho, \tag{9}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{4}\frac{A^2}{B^2C^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - 3\frac{\dot{\phi}^2}{\phi^2} = 8\pi G\rho \tag{10}$$

where an overdot denotes ordinary differentiation with respect to the cosmic time *t*.

For the metric (4), the average scale factor *a* and the spatial volume *V* are given by

$$V = a^3 = ABC. \tag{11}$$

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and mean Hubble parameter. These are defined as

$$H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C} \tag{12}$$

and

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\dot{a}}{a}. \tag{13}$$

The dynamical scalars such as expansion scalar θ, the shear scalar σ and anisotropy parameter A_m are defined to be

$$\theta = 3H = H_1 + H_2 + H_3, \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right), \tag{15}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{16}$$

The dimensionless deceleration parameter q, which tells whether the universe exhibits accelerated expansion or not, is an important kinematical quantity defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{17}$$

The universe exhibits accelerating volumetric expansion if $q < 0$, decelerating volumetric expansion if $q > 0$ and exhibits constant rate volumetric expansion if $q = 0$.

3. Solutions of the Field Equations

We now obtain the solutions of the field equations (7)-(10) which constitute a system of four highly non-linear differential equations in seven unknowns A, B, C, ϕ, ω, δ and ρ.

Subtracting Eqn.(8) from Eqn.(9), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{A}}{A} - 2\frac{\dot{\phi}}{\phi}\right)\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0. \tag{18}$$

The first integral of Eqn.(18) can be written in the form

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{X\phi^2}{a^3} \tag{19}$$

where X is an integration constant. Eqn.(19) can be further integrated for $\frac{B}{C}$ if the scalar function ϕ and the average scale factor a are explicitly known as functions of cosmic time t. Therefore, we consider a power-law relation between the scale factor a and the scalar function ϕ of the form (Johri and Desikan [33])

$$\phi = la^{\alpha} \tag{20}$$

where l is a proportionality constant and a is the power index. We further determine the average scale factor a by using the variation law of Hubble parameter proposed by Berman [34] of the form

$$H = ka^{-n} \tag{21}$$

where $k > 0$ and $n \geq 0$. Combining Eqn.(13) and Eqn.(21), we get

$$\dot{a} = ka^{-n+1}. \tag{22}$$

Integrating Eqn.(22), we obtain

$$a = \left(nkt + c_1\right)^{\frac{1}{n}}, n \neq 0 \tag{23}$$

and

$$a = c_2 e^{kt}, \qquad n = 0 \tag{24}$$

Where c_1 and c_2 are arbitrary constants. From Eqn.(17) and Eqn.(22), we find the deceleration parameter

$$q = n - 1,\ n \neq 0,\ and\ q = -1,\ n = 0 \tag{25}$$

Now, we use the values of ϕ and a given in Eqn.(20), Eqn.(23) and Eqn.(24) and integrate Eqn.(19) to obtain further the corresponding cosmological models for $n \neq 0$ and $n = 0$ separately.

3.1 Dark Energy Model of the Universe when $n \neq 0$

In this section, we derive the model of the universe with power-law expansion when $n \neq 0$. In this case ϕ takes the form

$$\phi = l\left(nkt + c_1\right)^{\frac{\alpha}{n}}. \tag{26}$$

Substituting Eqn.(26) in Eqn.(19), we get

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=Xl^{2}\left(nkt+c_{1}\right)^{\frac{2\alpha-3}{n}}. \tag{27}$$

Integrating Eqn.(27), we obtain

$$B=k_{1}C\exp\left[\frac{Xl^{2}}{k\left(2\alpha+n-3\right)}\left(nkt+c_{1}\right)^{\frac{2\alpha+n-3}{n}}\right) \tag{28}$$

where k_1 is an integration constant which can be taken unity without loss of any generality.

We next assume that the component σ_1^1 of the shear tensor σ_i^j is proportional to the mean Hubble parameter. This assumption leads to the following relation between metric functions of the form

$$A=\left(BC\right)^{m} \tag{29}$$

where m is a positive constant. The motive behind this condition is explained by Thorne [33]. Combining Eqn.(11), Eqn.(23) and Eqn.(29), we obtain

$$A=\left(nkt+c_{1}\right)^{\frac{3m}{n(m+1)}}. \tag{30}$$

From Eqn. (28), Eqn.(29) and Eqn.(30), the solutions of the metric functions B and C are obtained as :

$$B=\left(nkt+c_{1}\right)^{\frac{3m}{2n(m+1)}}\exp\left[\frac{Xl^{2}}{2k\left(2\alpha+n-3\right)}\left(nkt+c_{1}\right)^{\frac{2\alpha+n-3}{n}}\right), \tag{31}$$

$$C=\left(nkt+c_{1}\right)^{\frac{3m}{2n(m+1)}}\exp\left[-\frac{Xl^{2}}{2k\left(2\alpha+n-3\right)}\left(nkt+c_{1}\right)^{\frac{2\alpha+n-3}{n}}\right). \tag{32}$$

Hence, using an appropriate scale transformation, the metric (4) can be written in the form

$$ds^{2}=dt^{2}-\left(nkt\right)^{\frac{6m}{n(m+1)}}\left(dx^{2}-zdy\right)^{2}-\left(nkt\right)^{\frac{3}{n(m+1)}}\exp\left[\frac{Xl^{2}}{2k\left(2\alpha+n-3\right)}\left(nkt\right)^{\frac{2\alpha+n-3}{n}}\right)dy^{2} \tag{33}$$
$$-\left(nkt\right)^{\frac{3}{n(m+1)}}\exp\left[-\frac{Xl^{2}}{2k\left(2\alpha+n-3\right)}\left(nkt\right)^{\frac{2\alpha+n-3}{n}}\right)dz^{2}.$$

For this cosmological model, the physical and kinematical parameters have values given below:

$$V=\left(nkt\right)^{\frac{3}{n}}, \tag{34}$$

$$H_1 = \frac{3m}{n(m+1)t}, \tag{35}$$

$$H_2 = \frac{3}{2n(m+1)t} + \frac{Xl^2}{2}(nkt)^{\frac{2\alpha-3}{n}}, \tag{36}$$

$$H_3 = \frac{3}{2n(m+1)t} - \frac{Xl^2}{2}(nkt)^{\frac{2\alpha-3}{n}}, \tag{37}$$

$$H = \frac{1}{nt}, \tag{38}$$

$$\theta = \frac{3}{nt}, \tag{39}$$

$$\sigma^2 = \frac{3}{4n^2}\left(\frac{2m-1}{m+1}\right)^2 \frac{1}{t^2} + \frac{X^2 l^4}{4}(nkt)^{4\alpha-6}, \tag{40}$$

$$A_m = \frac{1}{2}\left(\frac{2m-1}{m+1}\right)^2 + \frac{n^2 t^2 X^2 l^4}{6}(nkt)^{4\alpha-6}, \tag{41}$$

$$\phi = (nkt)^{\frac{\alpha}{n}}. \tag{42}$$

We observe that for this power-law expanding model, the spatial volume V is zero at $t=0$ which increases as time increases and ultimately tends to infinite at late-time. At the epoch $t=0$, the physical parameters H_1 , H_2 , H_3 and H are all infinite. The expansion scalar θ, shear scalar σ are infinite and anisotropy parameter A_m is constant at this epoch. Therefore, the model has a big-bang type singularity at $t=0$. The metric functions *A*, *B* and *C* vanish at this point. The expansion scalar tends to zero at late time. The shear scalar and anisotropy parameter are decreasing functions of time and tend to zero as $t \to \infty$ if $\alpha < \frac{3}{2}$. Hence the present DE model is highly anisotropic for $0 < t < \infty$.

For the anisotropic DE model (33) with power-law expansion the energy density ρ, EoS parameter ω and Skewness parameter δ are calculated from Eqns.(7)-(10) as

$$\rho = \frac{1}{8\pi G}\left[\frac{9(4m+1)-4(2\alpha^2+3\alpha+n\alpha)(m+1)^2}{4(m+1)^2(nt)^2} - \frac{X^2 l^4}{4}(nkt)^{\frac{4\alpha-6}{n}} + \frac{1}{4}(nkt)^{\frac{6(m-1)}{n(m+1)}}\right), \quad (43)$$

$$\omega = -\frac{1}{8\pi G\rho}\left[\frac{4\alpha n(m+1)^2+9m^2+2(m+1)(9m-6m\alpha-3n-6mn)+9}{4(m+1)^2 n^2 t^2}\right) \quad (44)$$

$$-\frac{1}{8\pi G\rho}\left[\frac{nk^2(2\alpha-3)}{8nk(m+1)}X^2 l^4(nkt)^{\frac{2\alpha-n-3}{n}} + \frac{X^2 l^4}{4}(nkt)^{\frac{4\alpha-6}{n}} + \frac{1}{4}(nkt)^{\frac{6(m-1)}{n(m+1)}}\right),$$

$$\delta = \frac{1}{8\pi G\rho}\left[\frac{4\alpha(6m-n)+36m+6n-12mn-9}{4(m+1)n^2t^2} - \frac{9(4m^2+1)}{(m+1)^2 n^2 t^2}\right) \quad (45)$$

$$+\frac{1}{8\pi G\rho}\left[\frac{3Xl^2k+3Xl^2m-2Xnkl^2(m+1)}{2nk(m+1)}(nkt)^{\frac{2\alpha-n-3}{n}} - (nkt)^{\frac{6(m-1)}{n(m+1)}}\right),$$

From Eqn.(43), we observe that the energy density ρ is infinite at $t=0$ and is a decreasing function of time and ultimately tends to zero as $t \to \infty$ if $\alpha < \frac{3}{2}$ and $m<1$. Thus, this model essentially gives an empty universe for large time. For physical reality of the model $\alpha < \frac{3}{2}$ and $m<1$.

From Eqn.(44), we see that the EoS parameter ω is infinite at $t=0$ which decreases as time increases and tends to a constant negative value at late time.

3.2 Dark Energy Model of the Universe when n=0

In this section, we obtain an exponentially expanding DE cosmological model having singularity in the infinite past. We consider the average scale factor of the form

$$a = e^{kt} \quad (46)$$

taking $c_2 = 1$ in Eqn.(24) without the loss of any generality.

From Eqn.(20) and Eqn.(46), we get

$$\phi = l e^{k\alpha t}. \quad (47)$$

Using Eqn.(46) and Eqn.(47) in Eqn.(19) and integrating, we obtain

$$\frac{B}{C} = k_2 \exp\left[\frac{Xl^2}{k(2\alpha-3)}e^{k(2\alpha-3)t}\right) \quad (48)$$

where k_2 is an integration constant which can be taken as unity.

Again from Eqn.(11), Eqn.(24) and Eqn.(29), we get

$$A = e^{\frac{3mkt}{(m+1)}}. \tag{49}$$

From Eqn.(29) and Eqn.(49), we find that

$$BC = e^{\frac{3kt}{(m+1)}}. \tag{50}$$

Combining Eqn.(48) and Eqn.(50), we obtain

$$B = \exp\left[\frac{3kt}{2(m+1)} + \frac{Xl^2}{2k(2\alpha-3)} e^{k(2\alpha-3)t}\right), \tag{51}$$

$$C = \exp\left[\frac{3kt}{2(m+1)} - \frac{Xl^2}{2k(2\alpha-3)} e^{k(2\alpha-3)t}\right). \tag{52}$$

Hence, the metric of our solution can be written in the form

$$ds^2 = dt^2 - e^{\frac{6mkt}{(m+1)}}(dx - zdy)^2 - \exp\left[\frac{3kt}{(m+1)} + \frac{Xl^2}{k(2\alpha-3)} e^{k(2\alpha-3)t}\right)dy^2 \tag{53}$$

$$-\exp\left[\frac{3kt}{(m+1)} - \frac{Xl^2}{k(2\alpha-3)} e^{k(2\alpha-3)t}\right)dz^2.$$

For the model (53) the spatial volume is given by

$$V = e^{3kt}. \tag{54}$$

The directional Hubble parameters H_1, H_2 and H_3 are obtained as

$$H_1 = \frac{3mk}{m+1}, \tag{55}$$

$$H_2 = \frac{3k}{2(m+1)} + \frac{Xl^2}{2} e^{k(2\alpha-3)t}, \tag{56}$$

$$H_3 = \frac{3k}{2(m+1)} - \frac{Xl^2}{2} e^{k(2\alpha-3)t} \tag{57}$$

whereas the mean Hubble parameter H is given by

$$H = k = \text{constant}. \tag{58}$$

The scalar expansion and shear scalar are obtained as

$$\theta = 3k, \tag{59}$$

$$\sigma^2 = \frac{3}{4}\left[k^2 \left(\frac{2m-1}{m+1} \right)^2 + X^2 l^4 e^{2k(2\alpha-3)t} \right). \tag{60}$$

The anisotropy parameter A_m is calculated as

$$A_m = \frac{1}{2}\left(\frac{2m-1}{m+1} \right)^2 + \frac{X^2 l^4}{6k^2} e^{2k(2\alpha-3)t}. \tag{61}$$

The deceleration parameter q is given as

$$q = -1. \tag{62}$$

For this model the deceleration parameter $q = -1$ indicates that the universe represented by this set of solutions is inflationary. As an exponential function e^{kt} is never zero for finite values of t, spatial volume is never zero for finite t therefore, the universe has a physical singularity in the infinite past. The spatial volume tends to zero as $t \to -\infty$, which shows that the universe is infinitely old and has exponential inflationary phase. The directional Hubble parameters are time-dependent while the mean Hubble parameter is constant. The expansion scalar is constant throughout the time of evolution right from beginning. The shear scalar and anisotropy parameter associated with expansion are decreasing functions which tend to constant value as $t \to \infty$ value if $\alpha < \frac{3}{2}$. Thus, the model is anisotropic for all time.

For the anisotropic dark energy model (53) with exponential expansion, the energy density, EoS parameter and skewness parameter have the values

$$\rho = \frac{1}{8\pi G}\left[\frac{45mk^2}{4(m+1)} - \frac{X^2 l^4}{4} e^{2k(2\alpha-3)} - 3k^2\alpha(1+\alpha) - \frac{1}{4} e^{\frac{6(m-1)kt}{m+1}} \right), \tag{63}$$

$$\omega = -\frac{1}{8\pi G\rho}\left[\frac{36m^2k^2 + 9k^2 + 18mk^2 - 12k^2\alpha m - 4k^2\alpha^2(m+1)^2}{4(m+1)^2} \right) \tag{64}$$

$$-\frac{1}{8\pi G\rho}\left[\frac{(2\alpha-3)Xl^2k(m+1) + 3Xl^2 + 3Xmkl^2 - 2k(m+1)}{2(m+1)} e^{(2\alpha-3)kt} \right),$$

$$\delta=\frac{1}{8\pi G\rho}\left[\frac{36m^2k^2-13k^2+18mk^2-12k^2\alpha(m+1)+12k^2\alpha}{4(m+1)^2}-\frac{1}{4}e^{\frac{6(m-1)kt}{(m+1)}}\right) \tag{65}$$

$$\frac{+1}{8\pi G\rho}\left[\frac{Xl^2k(2\alpha-3)(m+1)+6kXl^2+3Xmkl^2(m+1)-2X\alpha kl^2(m+1)}{2(m+1)}\right)e^{k(2\alpha-3)t}.$$

Clearly the energy density ρ of dark energy and skewness parameter δ are non-zero for sufficiently large value of t if $\alpha<\frac{3}{2}$ and $m<1$.

4. Conclusion

We have presented exact solutions to the field equations in the scale-covariant theory of gravitation formulated by Canuto et al. [1] by applying a special law of variation of Hubble parameter for a spatially homogeneous and anisotropic Bianchi type-II space time in the presence of an anisotropic dark energy. We have obtained two different physically viable models of the universe, one with power-law expansion and other one with exponential expansion. Dark energy model with power-law expansion has an initial big-bang singularity, whereas the model with exponential expansion has a singularity in the infinite past. The evolution of the universe in such a scenario is consistent with present observations predicted by accelerated expansion. We have also discussed the physical and kinematical behaviors of the models of universe in two types of cosmologies. These models could give appropriate description of the universe at its early and late stages of evolution.

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