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HEAT TRANSFER ANALYSIS OF HYDROMAGNETIC VISCOUS RADIATING FLOW OVER A STRETCHING SHEET BY HOMOTOPY PERTURBATION METHOD

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Abstract: In this paper we examine the heat transfer of hydromagnetic viscous radiating flow over a stretching sheet. The governing equations are converted into linear ordinary differential equations by using similarity transformations and have been solved by Homotopy Perturbation Method (HPM). Later, the effects of different physical parameters like Prandtl Number, Radiation parameter, Magnetic field parameter and stretching sheet parameter on the flow and heat transfer characteristics of viscous fluid is studied. The influence of various physical parameters over a velocity and temperature are presented with the help of graph and discussed.

Keywords: Homotopy Perturbation Method, hydromagnetic viscous radiating flow, Stretching sheet.

1. Introduction

The study of a magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of significant interest in modern world of fast technological applications such as mineral, metal-working processes, MHD power generation, MHD pumps etc. In mineral processes, the rate of cooling and stretching of the strips can be controlled by drawing the strip in an electrically conducting fluid subject to a magnetic field, so that a final product of desired characteristics can be achieved. Most of the boundary-layer problems are non-linear in nature and we know that only a limited number of these problems are solved numerically, most of them do not have any analytical solutions. Therefore, with the passage of time many new methods were developed to solve these nonlinear equations such as, Adomian decomposition method given by Hayat et al. [8] and Ganji and Ganji [6]. Laplace decomposition method by Khan [12], Khan and Faraz [13]. Homotopy analysis method by Sajid and Hayat [19] and Sajid et al. [20] etc. Most of these techniques faces the inherent deficiencies and involve vast computational work. He [9,10] developed and work out on homotopy perturbation method (HPM) by merging the standard homotopy and perturbation technique. The He's homotopy perturbation method (HPM) proved to be amiable with the flexible nature of the physical problems and has been used in wide classes of dynamic functional equations. The variable separation method requires initial and boundary conditions but homotopy perturbation (HPM) provides an analytical solution by using initial conditions only.

The study of a boundary layer flow over a stretching sheet has gained much attention in the advanced world of fast changing technological applications in engineering and industrial processes for example designing cooling nuclear reactor system, fiber coating, rubber production, colloidal suspension, production of glass socks, metal spinning and drawing of plastic film, textile and paper production, use of geothermal energy, food processing, plasma studies and aerodynamics, bulges processes, metal spinning, extrusion of polymer sheet, rubber sheet manufacturing and design of various heat exchangers are some physical applications of such flows. The steady two-dimensional laminar flow of an incompressible, viscous fluid past a stretching sheet has become a classical problem in fluid dynamics. It was Crane [5] who first investigated the flow past a stretching plate. Later, Gupta and Gupta [7] added surface suction and blowing on such stretching sheet. Andersson [1] has analyzed the flow of an electrically conducting viscoelastic fluid under the influence of uniform transverse magnetic field. The flow past a stretching sheet need not be necessarily two-dimensional because the stretching of the sheet can take place in a variety of ways. It can be three-dimensional, axisymmetric when the sheet is stretching radially, this problem has no closed form exact analytical solution. Accordingly, many researchers focussed their attention on the study of boundary layer flow of Newtonian and Non-Newtonian fluids over different stretching sheets i.e., quadratic, power law and non-isothermal stretching sheets etc. The three-dimensional flow due to a stretching flat surface was analysed by Wang [23]. Ariel [2] had investigated the MHD flow of a viscoelastic fluid past a stretching sheet with suction. Further Ariel [3] applied HPM and extended HPM to derive the analytical solution of the axisymmetric flow past a stretching sheet. Chamkha et al. [4] has studied the natural convection from an inclined plate embedded in a variable porosity porous medium due to solar radiation. Raftari and Yildirim [18] has used the homotopy perturbation method to solve MHD flows of upper convected Maxwell (UCM) fluids above porous stretching sheets. Magyari and Chamkha [17] analyzed the combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. Sandeep and Sulochana [18] has studied the dual solutions for unsteady mixed convection flow of MHD micropolar fluid over a stretching/shrinking sheet with non-uniform heat source/sink. Madhu et al. [16] has investigated the unsteady flow of a Maxwell nanofluid over a stretching surface in the presence of magnetohydrodynamic and thermal radiation effects. Jahan et al. [11] has also investigated the heat transfer in nanofluid past a convectively heated permeable stretching/shrinking sheet. Sreedevi et al. [22] has analyzed the heat and mass transfer of unsteady hybrid nanofluid flow over a stretching sheet with thermal radiation. Kumar et al. [15] has used modified homotopy perturbation approach to solve the system of fractional partial differential equations.

Krishna et al. [14] has studied radiative MHD flow of Casson hybrid nanofluid over an infinite exponentially accelerated vertical porous surface.

Motivated by above researches in the present paper we intent to analyze the heat transfer in the steady hydromagnetic viscous radiating flow over a horizontally oriented stretching sheet in the presence of magnetic field by using the homotopy perturbation method (HPM). The expression for velocity and temperature have been evaluated by using MATLAB and MAPLE software. The impact of various relevant parameters occurring in the governing equation has been expressed with the help of graph and tables.

2. Mathematical Formulation

We consider the two-dimensional steady flow of a viscous radiating incompressible electrically conducting fluid over a stretching surface. We assumed the constant free stream velocity U_∞ over a flat plate along the X-axis direction. A cartesian co-ordinate system with X-axis oriented horizontally and Y-axis oriented vertically upward through the origin is considered. The sheet is parallel to the X-axis and subjected to a perpendicular magnetic field $B(x) = B_0 x^{\frac{n-1}{2}}$ where B_0 is constant magnetic field. The sheet velocity, $U_w(x) = cx^n$ is assumed to vary as a nonlinear function of the distance from the origin, where $c > 0$ is the stretching rate and n is a nonlinear stretching parameter. The temperature of the fluid $T_w(x) = T_\infty + T_0 x^{\frac{n-1}{2}}$ is also consider as a nonlinear function of the distance, where T_0 is a positive constant temperature of fluid at the origin and T_∞ is the free stream temperature of the fluid. Under the boundary layer approximation and constant fluid property assumption, the continuity, momentum and energy equations for the flow is given as below

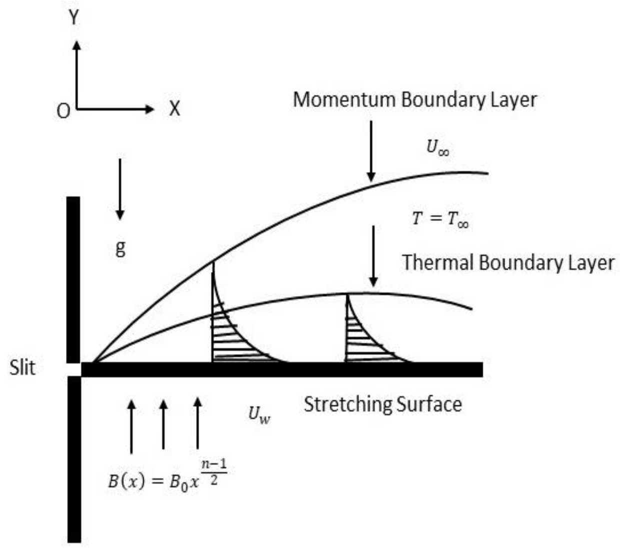

Fig. 1: Geometrical configuration of the physical problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial x^2}-\sigma\frac{B^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where u and v are the velocity components in the x and y directions respectively. Also ν, c_p, σ, k, ρ and q_r are respectively, the kinematic viscosity, heat capacity at constant pressure, electrical conductivity, thermal conductivity, fluid density and radiative heat flux.

The associated boundary conditions are

$$u=U_w(x)=cx^n, v=0, T=T_w(x)=T_\infty+T_0x^{\frac{n-1}{2}}, at\ y=0,$$

$$u\to 0, v\to 0, T\to T_\infty\ as\ y\to\infty \tag{4}$$

where the subscripts w and ∞ refer to stretching at the wall and free stream, respectively. The magnetic field $B(x)$ is considered to have the form in order to simplify the similarity solution.

$$B(x)=B_0x^{\frac{n-1}{2}}, \tag{5}$$

where B_0 is constant.

The radiative heat flux q_r can be approximated by the Rosseland approximation as:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

Where σ^*signifies Stefan-Boltzmann constant and k^*symbolizes Rosseland mean absorption coefficient. Assuming that T^4 can be represented as a linear function of temperature $T^4=4T_\infty^3T-3T_\infty^4$ and that the temperature variations within the fluid flow are suitably low. With this, Equation (6) can be written as

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^*T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

Substituting equation (7) into equation (3) gives

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^*T_\infty^3}{\rho c_p 3k^*}\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

To solve the problem, momentum and energy equations are non-dimensionalised by introducing the following dimensionless variables:

$$\eta=\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}y,\quad u=cx^nf'(\eta),\quad v=-\sqrt{\frac{c\nu(n+1)}{2}}\,x^{\frac{n-1}{2}}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right],$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \Psi(x,y) = \sqrt{\frac{2cvU_w}{(n+1)}} f(\eta). \tag{9}$$

where η represents the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ represent the non-dimensional temperature and the stream function $\Psi(x,y)$ satisfies the continuity equation (1) such that the components of the velocity u and v are defined as

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x}. \tag{10}$$

The momentum and energy equations can be transformed into the corresponding ordinary non-linear differential equation by using non-dimensional variable as given in equation (9), where equation (2) and equation (8) transforms to the following equations

$$f''' + ff'' - \beta f'^2 - Mf' = 0 \tag{11}$$

$$\gamma\theta'' + f\theta' = 0 \tag{12}$$

the corresponding boundary conditions are:

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0,\ \theta(0) = 1,\ \theta(\infty) = 0 \tag{13}$$

where $\gamma = \frac{1}{Pr}\left(1 + \frac{4}{3}R\right)$ and f is related to the u velocity by $f' = \frac{u}{U_\infty}$, $M = \frac{2\sigma B_0^2}{\rho c(n+1)}$, is the dimensionless magnetic field parameter, $R = \frac{4\sigma^* T_\infty^3}{kk^*}$, is the radiation parameter, $Pr = \frac{\rho c_p v}{k}$, is the Prandtl number and $\beta = \frac{2n}{n+1}$, is the stretching sheet parameter.

3. Solution by using Homotopy Perturbation Method (HPM)

According to the HPM, the homotopy form of equation (11) and equation (12) are constructed as follows

$$(1-p)(f''' - f_0''') + p(f''' + ff'' - \beta f'^2 - Mf') = 0 \tag{14}$$

$$(1-p)(\gamma\theta'' - \gamma\theta_0'') + p(\gamma\theta'' + f\theta') = 0 \tag{15}$$

the associated boundary conditions are

$$f(0) = 0,\ f'(0) = 1,\ f''(0) = \alpha,\ \theta(0) = 1,\ \theta(\infty) = 0. \tag{16}$$

where $\alpha > 0$, is an arbitrary constant.

We consider f and θ as the following

$$f = f_0 + pf_1 + p^2 f_2 + \cdots, \qquad \theta = \theta_0 + p\theta_1 + p^2\theta_2 + \cdots. \tag{17}$$

Assuming $f_0''' = \theta_0'' = 0$ for initial guess and substituting f and θ from equation (17) into equation (14) and (15) and on simplification and rearranging the term based on powers of p, we have

$$p^0: \quad f_0''' = 0,\ \gamma\theta_0'' = 0, \tag{18}$$

$$f_0(0) = 0 \quad f_0'(0) = 1,\ f_0''(0) = \alpha,\ \theta_0(0) = 1,\ \theta_0(\infty) = 0, \tag{19}$$

$$p^1: \quad f_1''' - Mf_0' + f_0 f_0'' - \beta f_0'^2 = 0,\ \gamma\theta_1'' + f_0\theta_0' = 0, \tag{20}$$

$$f_1(0) = 0, \quad f_1'(0) = 0,\ f_1''(0) = 0,\ \theta_1(0) = 0,\ \theta_1(\infty) = 0, \tag{21}$$

$$p^2: \quad f_2''' - Mf_1' - 2\beta\, f_0' f_1' + f_1\, f_0'' + \ f_0 f_1'' = 0,\ \gamma\theta_2'' + f_0\theta_1' + f_1\theta_0' = 0, \tag{22}$$

$$f_2(0) = 0, \quad f_2'(0) = 0,\ f_2''(0) = 0,\ \theta_2(0) = 0,\ \theta_2(\infty) = 0, \tag{23}$$

$$p^3: \quad f_3''' - Mf_2' - \beta(f_1'^2 + 2f_0' f_2') + \ f_1 f_1'' + f_0 f_2'' + f_2 f_0'' = 0,$$

$$\gamma\theta_3'' + f_0\theta_2' + f_1\theta_1' + f_0\theta_0' = 0, \tag{24}$$

$$f_3(0) = 0, \quad f_3'(0) = 0,\ f_3''(0) = 0,\ \theta_3(0) = 0,\ \theta_3(\infty) = 0. \tag{25}$$

Solving equations (18,20,22 & 24) by using the boundary conditions giving in equations (19,21,23 & 25), we obtain the following solutions

$$f_0 = \frac{1}{2}\alpha\eta^2 + \eta, \tag{26}$$

$$f_1 = \frac{1}{60}\alpha^2\beta\eta^5 - \frac{1}{120}\alpha^2\eta^5 + \frac{1}{24}\alpha M\eta^4 + \frac{1}{12}\alpha\beta\eta^4 - \frac{1}{12}\alpha\eta^4 + \frac{1}{6}M\eta^3 + \frac{1}{6}\beta\eta^3, \tag{27}$$

$$f_2 = \frac{1}{2016}\alpha^3\beta^2\eta^8 - \frac{1}{1260}\alpha^3\beta\eta^8 + \frac{11}{40320}\alpha^3\eta^8 + \frac{1}{504}\alpha^2\beta M\eta^7 + \frac{1}{252}\alpha^2\beta^2\eta^7 - \frac{1}{630}\alpha^2 M\eta^7 - \frac{2}{315}\alpha^2\beta\eta^7 + \frac{11}{5040}\alpha^2\eta^7 + \frac{1}{720}\alpha M^2\eta^6 + \frac{1}{72}\alpha\beta M\eta^6 + \frac{1}{72}\alpha\beta^2\eta^6 - \frac{1}{90}\alpha M\eta^6 - \frac{1}{60}\alpha\beta\eta^6 + \frac{1}{240}\alpha\eta^6 + \frac{1}{120}M^2\eta^5 + \frac{1}{40}\beta M\eta^5 + \frac{1}{60}\beta^2\eta^5 - \frac{1}{60}M\eta^5 - \frac{1}{60}\beta\eta^5, \tag{28}$$

$$f_3 = \frac{1}{630}\beta\eta^7 + \frac{1}{252}\beta^2 M\eta^7 - \frac{13}{2520}\beta M\eta^7 + \frac{1}{12096}\alpha^3\beta^2 M\eta^{10} - \frac{13}{86400}\alpha^3\beta M\eta^{10} + \frac{1}{8640}\alpha^2\beta M^2\eta^9 + \frac{5}{6048}\alpha^2\beta^2 M\eta^9 - \frac{13}{8640}\alpha^2\beta M\eta^9 + \frac{1}{960}\alpha\beta M^2\eta^8 + \frac{1}{336}\alpha\beta^2 M\eta^8 - \frac{7}{1440}\alpha\beta M\eta^8 - \frac{5}{532224}\alpha^4\eta^{11} - \frac{5}{48384}\alpha^3\eta^{10} - \frac{43}{120960}\alpha^2\eta^9 - \frac{1}{2688}\alpha\eta^8 + \frac{1}{5040}M^3\eta^7 + \frac{1}{504}\beta^3\eta^7 - \frac{1}{504}M^2\eta^7 - \frac{1}{315}\beta^2\eta^7 + \frac{1}{630}M\eta^7 + \frac{1}{66528}\alpha^4\beta^3\eta^{11} - \frac{19}{475200}\alpha^4\beta^2\eta^{11} + \frac{233}{6652800}\alpha^4\beta\eta^{11} + \frac{1}{6048}\alpha^3\beta^3\eta^{10} - \frac{19}{43200}\alpha^3\beta^2\eta^{10} + \frac{3}{44800}\alpha^3 M\eta^{10} + \frac{233}{604800}\alpha^3\beta\eta^{10} + \frac{5}{6048}\alpha^2\beta^3\eta^9 - \frac{13}{120960}\alpha^2 M^2\eta^9 - \frac{71}{36288}\alpha^2\beta^2\eta^9 + \frac{3}{4480}\alpha^2 M\eta^9 + \frac{137}{90720}\alpha^2\beta\eta^9 + \frac{1}{40320}\alpha M^3\eta^8 + \frac{1}{504}\alpha\beta^3\eta^8 - \frac{13}{13440}\alpha M^2\eta^8 - \frac{83}{20160}\alpha\beta^2\eta^8 + \frac{1}{504}\alpha M\eta^8 + \frac{103}{40320}\alpha\beta\eta^8 + \frac{11}{5040}\beta M^2\eta^7, \tag{29}$$

$$\theta_0 = -\frac{1}{5}\eta + 1, \tag{30}$$

$$\theta_1 = \frac{1}{40(3+4R)}Pr\alpha\eta^4 + \frac{1}{10(3+4R)}Pr\eta^3, \tag{31}$$

$\theta_2 = \frac{1}{200(3+4R)^2}\Pr(4RM + 4R\beta + 3M - 9Pr + 3\beta)\,\eta^5 - \frac{1}{8400(3+4R)^2}\Pr\alpha^2(-8R\beta + 30Pr + 4R - 6\beta + 3)\eta^7 + \frac{1}{1200(3+4R)^2}\Pr\alpha(4RM + 8R\beta + 3M - 30Pr - 4R + 6\beta - 3)\eta^6,$ (32)

$\theta_3 = \frac{1}{6048000(3+4R)^3}\Pr\alpha^3(320R^2\beta^2 - 2016PrR\beta - 512R^2\beta + 480R\beta^2 + 2520Pr^2 + 1008PrR - 1512Pr\beta + 176R^2 - 768R\beta + 180\beta^2 + 756Pr + 264R - 288\beta + 99)\,\eta^{10} + \frac{1}{8400(3+4R)^3}\Pr(16R^2M^2 + 48R^2\beta M + 32R^2\beta^2 + 24RM^2 - 180PrRM - 32R^2M + 72R\beta M - 180PrR\beta - 32R^2\beta + 48R\beta^2 + 9M^2 - 135PrM - 48RM + 27\beta M + 135Pr^2 - 135Pr\beta - 48Pr\beta + 18\beta^2 - 18M - 18\beta)\,\eta^7 - \frac{1}{604800(3+4R)^3}\Pr\alpha^2(-160R^2\beta M - 320R^2\beta^2 + 672PrRM + 128R^2M - 240R\beta M + 2016PrR\beta + 512R^2\beta - 480R\beta^2 + 504PrM + 192RM - 90\beta M - 2520Pr^2 - 1008PrR + 1512Pr\beta - 176R^2 + 768R\beta - 180\beta^2 + 72M - 756Pr - 264R + 288\beta - 99)\eta^9 + \frac{1}{67200(3+4R)^3}\Pr\alpha(160R^2M^2 + 160R^2\beta M + 160R^2\beta^2 + 24RM^2 - 672PrRM - 128R^2M + 240R\beta M - 924PrR\beta - 192R^2\beta + 240R\beta^2 + 9M^2 - 504PrM - 192RM + 90\beta M + 945Pr^2 + 252PrR - 693Pr\beta + 48R^2 - 288R\beta + 90\beta^2 - 72M + 189Pr + 72R - 108\beta + 27)\eta^8$ (33)

From equation (17) by assuming $p = 1$, we get

$$f = f_0 + f_1 + f_2 + f_3 + \cdots, \theta = \theta_0 + \theta_1 + \theta_2 + \theta_3 + \cdots. \quad (34)$$

where $f_0, f_1, f_2, f_3, \ldots$ and $\theta_0, \theta_1, \theta_2, \theta_3, \ldots$ are given above.

4. Some Important Characteristics of Flow Field

4.1 Skin friction coefficient

The dimensionless expression for skin friction coefficient is given by

$$C_f = \frac{\tau_w}{\rho U_w^2}, \text{ where } \tau_w = \left[\mu \frac{\partial u}{\partial y}\right]_{y=0}. \quad (35)$$

The skin friction coefficient in terms of transformation variables equation (9) and equation (10) can be obtained as

$$C_f = \frac{1}{(Re)^{\frac{1}{2}}}\left(\frac{n+1}{2}\right)^{\frac{1}{2}} f''(0) \quad (36)$$

where $f''(0)$ is obtained using equation (34).

4.2 The dimensionless coefficient of heat transfer (Nusselt number)

The dimensionless expression for Nusselt number is given by

$$Nu = \frac{xq_w}{k(T_w - T_\infty)}\text{, where } q_w = \left[-\left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial T}{\partial y}\right]_{y=0} \tag{37}$$

is heat flux at the stretched surface.

The dimensionless expression for Nusselt number in terms of transformation variables equations (9) & (10) and using equation (34) turned out to be

$$Nu = (Re)^{\frac{1}{2}}\left(\frac{n+1}{2}\right)^{\frac{1}{2}}\left(1 + \frac{4}{3}R\right)\theta'(0) \tag{38}$$

where $Re = \frac{U_w x}{\nu}$ is the local Reynolds number.

5. Results and Discussion

In this paper we have studied the heat transfer of a hydromagnetic viscous flow over a stretching sheet by using homotopy perturbation method. The numerical calculations of velocity and temperature profile have been computed for different values of governing parameters such as magnetic field parameter (M), Prandtl number (Pr), radiation parameter (R) and stretching sheet parameter (β) by using MATLAB and MAPLE software and graph have been lotted by using bvp4c of MATLAB software.

The influence of the magnetic field parameter (M) over the dimensionless velocity f is shown in Figure 2, It is observe that increasing values of the magnetic field parameter retard the velocity at all points of the flow field. It is because that the application of the transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus results in reducing its velocity. Figure 3, shows the influence of nonlinear stretching sheet parameter (β) on velocity. It is observed that with increase in stretching sheet parameter the velocity increases. It is due to the fact that large value of β tends to produce more deformation in the liquid, which implies an increase in straining motion near the stagnation point. It results in thining of the boundary layer and leads to an increased acceleration in the external stream. Figure 4, shows that with increase of Prandtl number (Pr) the temperture θ decreases. An increase in the Prandtl number reduces the thermal boundary layer thickness. When Pr is small, heat diffuses quickly compared to the velocity. Figure 5, shows the characteristics of radiation parameter (R) with the temperature profiles. It is observed from this graph that the thermal field increased with the increase in R. More heat is transferred to the working liquid via the radiation phenomena because the radiation parameter is inversely related to the mean absorption coefficient. A higher radiation parameter value causes k^* to decline, which raises the temperature. As a result, the rate of thermal convection into the fluid increases. From Figure 4 and Figure 5 it is evident that for $\eta \geq 25$ the Prandtl number (Pr) and radiation parameter (R) has negligible effect on heat transfer and it tends to stables.

From Table 1, it is observed that the value of the skin friction coefficient is inversely proportional to Reynolds number (Re) and directly proportional to the stretching sheet parameter (n). From Table 2, it is observed that the Nusselt number enhances with increasing value of any of the three parameters i.e. Reynold number (Re), stretching sheet parameter (n) and radiation heat parameter (R).

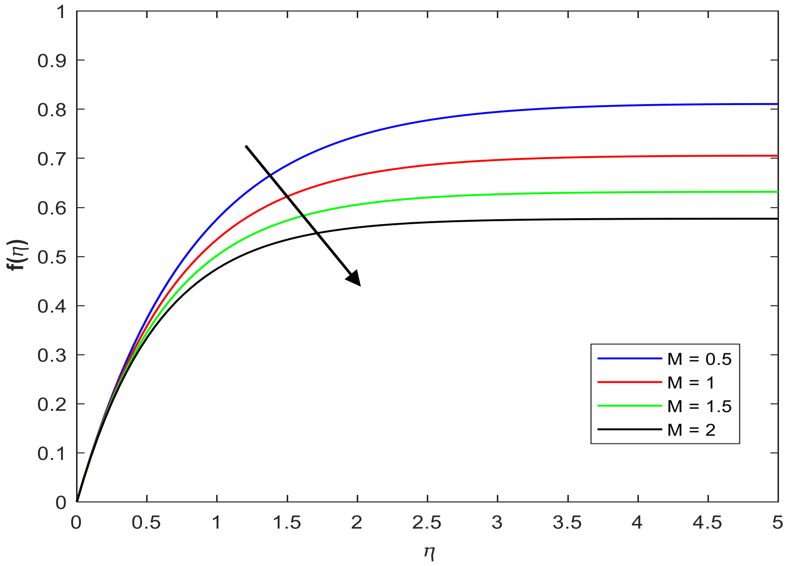

Fig. 2. Velocity profiles f for various values of magnetic field parameter M when $\beta = 1$.

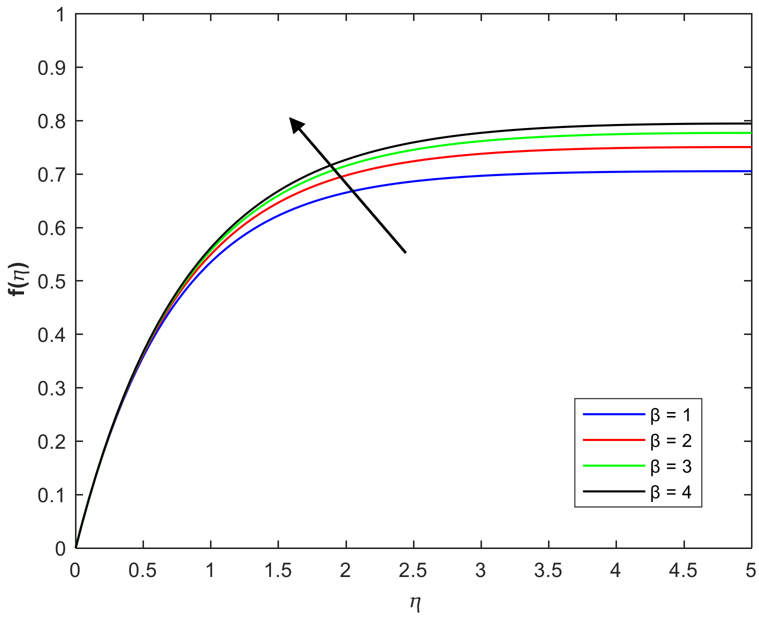

Fig. 3. Velocity profiles f for various values of stretching sheet parameter β when M = 1

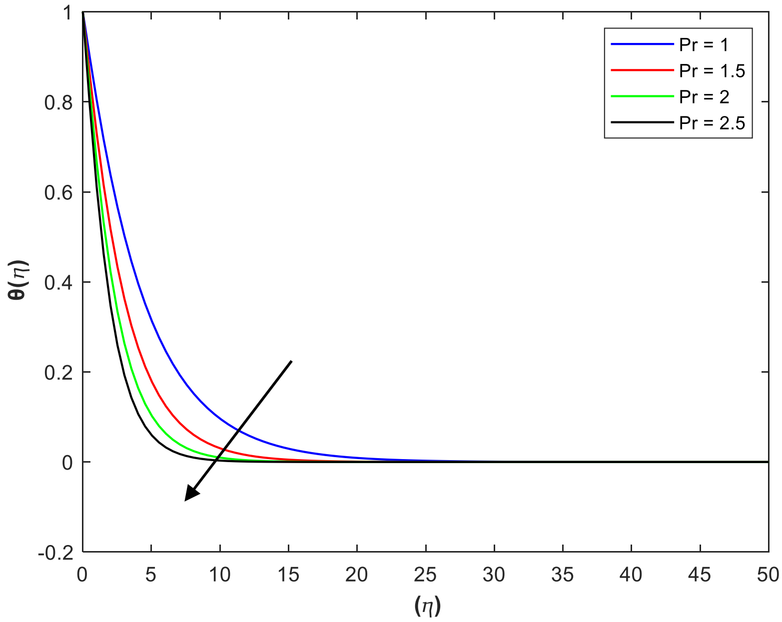

Fig. 4. Temperature profiles θ for various values of Prandtl number Pr when β=1.5, M = 0.5 and R = 1.5.

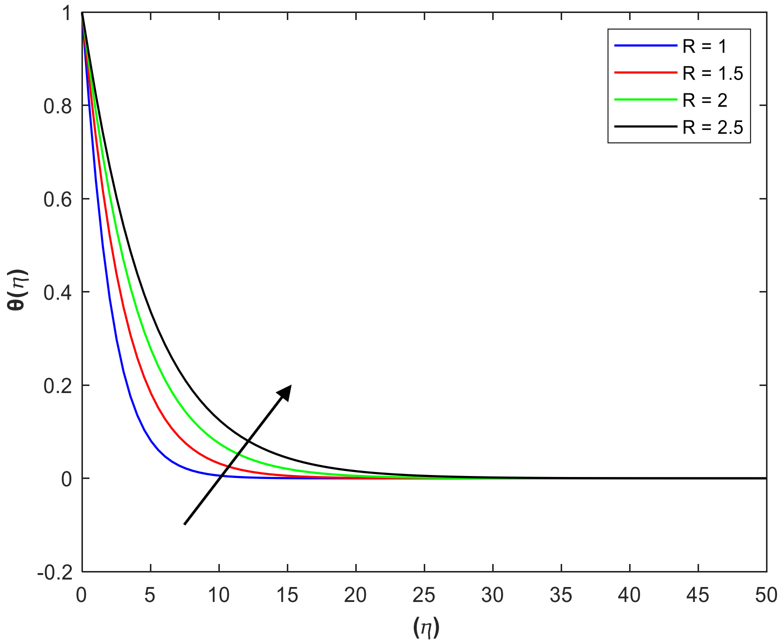

Fig. 5. Temperature profiles θ for various values of radiation parameter R when β=1, M = 0.5 and Pr = 1.5.

Table 1: Numerical value for Skin friction coefficient with various values of Re and β.

SL No	Re	β	C_f
1	1	1.5	0.1000
2	2	1.5	0.0500
3	1	2.0	0.1250
4	1	2.5	0.1500

Table 2: Numerical value for Nusselt number with various values of Re, β and R.

SL No	Re	β	R	Nu
1	1	1.5	0.5	0.3333
2	2	1.5	0.5	0.4714
3	1	2.0	0.5	0.4167
4	1	2.5	0.5	0.5000
5	1	1.5	1.0	0.4667
6	1	1.5	1.5	0.6000

6. Conclusion

In this paper, the analytic solution for the hydromagnetic boundary layer viscous radiating flow with heat transfer over a stretching sheet has been obtained. The governing equations are transformed to a set of ordinary differential equations and then solved analytically by using homotopy perturbation method (HPM). The effects of various non-dimensional parameters on velocity and temperature distribution are discussed and presented through graphs. Also, the effect of physical parameters on the skin friction and Nusselt number are investigated and listed in tables. The main findings of this investigation can be summarized as follows

i. The effects of the transvarse magnetic field (M) on a viscous fluid flow reduces the velocity.

ii. The incresasing stretching sheet parameter (β) helps in elavating the velocity.

iii. The increasing value of radiation parameter (R) enhance the temperature field.

iv. The Prandtl number (Pr) inversely effects the temperature field.

v. The increasing Reynolds number (Re) reduce the skin friction coefficient (C_f), while the increasing value of stretching sheet parameter (β) enhances it.

vi. The Reynolds number (Re), stretching sheet parameter (β) and the radiation parameter (R) directly effects the heat transfer.

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