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SOME PROPERTIES OF THE BIQUADRATIC SEQUENCE SPACE Γ^4

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Abstract: In this paper, we introduce the biquadratic sequence spaces Γ^4 and study some basic properties of it. We obtain certain results for separability, reflexivity, rotund and inner product on Γ^4. Comparison of weak and strong convergence is also the part of our study.

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1. Introduction

Throughout ω, Γ and Λ denote the classes of all, entire and analytic scalar valued single sequences respectively. We write ω^4 for the set of all complex sequences (x_{mnkl}), where $m, n, k, l \in N$, the set of positive integers. Then, ω^4 is a linear space under the coordinate-wise addition and scalar multiplication.

We can represent triple and biquadratic sequences by matrix. In the case of double sequence, we write in the form of a square. In the case of a biquadratic sequence, it will be in the form of a box in four dimensional case.

Some initial work on sequence space and series is found in Kamthan and Gupta [5], double series is found in Apostol [1] and infinite series is found in Bromwhich [3]. Later on, it was investigated by some initial work on double sequence spaces is found in Hardy [4], Basarir and Solancan [2], Moricz [6], Moricz and Rhoades [7], Subramanian [13-14], Tripathy [15]. Some initial work on triple sequence space is found in Sahiner [8], Subramanian [9-12] and many others.

Let (x_{mnkl})be a biquadratic sequence of real or complex numbers. Then the series $\sum_{m,n,k,l=1}^{\infty} x_{mnkl}$ is called a biquadratic series.

The biquadratic series $\sum_{m,n,k,l=1}^{\infty} x_{mnkl}$ is said to be convergent if and only if the biquadratic sequence (S_{mnkl}) is convergent, where

$$S_{mnkl} = \sum_{i,j,q,r=1}^{m,n,k,l} x_{ijqr} \;\; (m,n,k,l = 1,2,3,\ldots)$$

A sequence $x = (x_{mnkl})$ is said to be biquadratic analytic sequence if

$$sup_{mnkl}|x_{mnkl}|^{1/m+n+k+l} < \infty$$

The vector space of all biquadratic analytic sequences is usually denoted by Λ^4.

A sequence $x = (x_{mnkl})$ sequence is called biquadratic entire sequence if

$$|x_{mnkl}|^{1/m+n+k+l} \to 0 \; as \; m,n,k,l \to \infty$$

The vector space of all biquadratic entire sequences is usually denoted by Γ^4.

The spaces Λ^4 and Γ^4 are metric space with the metric

$$(\mathbf{1.1}) \; d(x,y) = sup_{m,n,k,l} \left\{ |x_{mnkl} - y_{mnkl}|^{1/m+n+k+l} \; : m,n,k,l = 1,2,3,\ldots \right\}$$

for all $x = (x_{mnkl})$ and $y = (y_{mnkl})$ in Γ^4.

Consider a biquadratic sequence $x = (x_{mnkl})$. The $(m,n,k,l)^{th}$ section $x^{[m,n,k,l]}$ of the sequence is defined by

$$x^{[m,n,k,l]} = \sum_{i,j,q,r=0}^{m,n,k,l} x_{ijqr} \;\; \delta_{ijqr} \; \forall \; m,n,k,l \in N$$

$$\delta_{mnkl} = \begin{bmatrix} 0 & 0 & ..0 & 0 & \ldots \\ 0 & 0 & ..0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & ..1 & 0 & \ldots \\ 0 & 0 & ..0 & 0 & \ldots \end{bmatrix}$$

with 1 in the $(m,n,k,l)^{th}$ position and zero otherwise.

An FK-space (or a metric space) x is said to have AK-property if (δ_{mnkl}) is a Schauder basis for X. Or equivalently $x^{[m,n,k,l]} \to x$. An FDK-space is a biquadratic sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings are continuous.

If X is a sequence space, we give the following definitions:

(i) $X' =$ the continuous dual of X;

(ii) $X^\alpha = \{a = (a_{mnkl}) : \sum_{m,n,k,l=1}^{\infty} |a_{mnkl} x_{mnkl}| < \infty, \text{ for each } x \in X\}$;

(iii) $X^\beta = \{a = (a_{mnkl}) : \sum_{m,n,k,l=1}^{\infty} a_{mnkl} x_{mnkl} \text{ is convergent, for each } x \in X\}$;

(iv) $X^{\gamma} = \left\{a = (a_{mnkl}): sup_{m,n,k,l\geq 1}\left|\sum_{m,n,k,l=1}^{\infty} a_{mnkl}x_{mnkl}\right| < \infty, \text{for each } x \in X\right\}$;

(v) $X^{\Lambda} = \left\{a = (a_{mnkl}): sup_{m,n,k,l} \left|a_{mnkl}x_{mnkl}\right|^{1/m+n+k+l} < \infty, \text{for each } x \in X\right\}$;

$X^{\alpha}, X^{\beta}, X^{\gamma}$ *and* X^{Λ} are called $\alpha-$(or Kothe- Toeplitz) dual of X, $\beta-$(generalized-Kothe Toeplitz) dual of X, $\gamma-$ dual of X and $\Lambda-$dual of X, respectively.

2. Preliminaries

Let ω^4 denote the set of all complex sequence (x_{mnkl}) where $m, n, k, l \in N$. A sequence $x = (x_{mnkl})$ is said to be **biquadratic analytic sequence** if

$$sup_{m,n,k,l}|x_{mnkl}|^{1/m+n+k+l} < \infty$$

The set of all biquadratic analytic sequences will be denoted by Λ^4.

A sequence $x = (x_{mnkl})$ sequence is called **biquadratic entire sequence** if

$$|x_{mnkl}|^{1/m+n+k+l} \to 0 \text{ as } m, n, k, l \to \infty$$

The set of all biquadratic entire sequences will be denoted by Γ^4.

The spaces Λ^4 and Γ^4 are metric space with the metric

$$d(x, y) = sup_{m,n,k,l} \left\{|x_{mnkl} - y_{mnkl}|^{1/m+n+k+l} : m, n, k, l = 1,2,3, \ldots\right\}$$

for all $x = (x_{mnkl})$ and $y = (y_{mnkl})$ in Γ^4.

3. Main Results

Proposition 3.1*: Γ^4 has monotonic metric.*

Proof : We know that

$$d(x, y) = sup_{m,n,k,l} \left\{|x_{mnkl} - y_{mnkl}|^{1/m+n+k+l} : m, n, k, l = 1,2,3, \ldots\right\}$$

$$d(x^n, y^n) = sup_{n,n,n,n} \left\{|x_{nnnn} - y_{nnnn}|^{1/n+n+n+n}\right\}$$

$$d(x^n, y^n) = sup_{n,n,n,n} \left\{|x_{nnnn} - y_{nnnn}|^{1/4n}\right\}$$

and

$$d(x^m, y^m) = sup_{m,m,m,m} \left\{|x_{mmmm} - y_{mmmm}|^{1/4m}\right\}$$

Let $m > n$. Then

$$sup_{m,m,m,m} \left\{|x_{mmmm} - y_{mmmm}|^{1/4m}\right\} \geq sup_{n,n,n,n} \left\{|x_{nnnn} - y_{nnnn}|^{1/4n}\right\}$$

(3.1) $d(x^m, y^m) \geq d(x^n, y^n), m > n$

Also $\{d(x^n, y^n): n = 1,2,3, \ldots\}$ is monotonically increasing bounded by $d(x, y)$.

For such a sequence

$$(\mathbf{3.2})\ sup_{n,n,n,n}\left\{|x_{nnnn}-y_{nnnn}|^{1/n+n+n+n}\right\}=\lim_{n\to\infty}d(x^n,y^n)=d(x,y)$$

From (3.1) and (3.2) it follows that

$$d(x,y)=sup_{m,n,k,l}\left\{|x_{mnkl}-y_{mnkl}|^{1/m+n+k+l}:m,n,k,l=1,2,3,\ldots\right\}$$

is a monotonic metric for Γ^4.

Proposition 3.2: *The dual space of* Γ^4 *is* Λ^4.

Proof : We recall that

$$\delta_{mnkl}=\begin{bmatrix}0&0&..0&0&\ldots\\0&0&..0&0&\ldots\\.\\.\\.\\0&0&..1&0&\ldots\\0&0&..0&0&\ldots\end{bmatrix}$$

With 1 in the position $(m,n,k,l)^{th}$and zero's else where. With

$$x=\delta_{mnkl},|x_{mnkl}|^{1/m+n+k+l}$$

$$=\begin{bmatrix}0^{1/4}&\ldots&0^{1/m+n+k+l}&\ldots0^{1/m+n+k+l}\\.\\.\\.\\0^{1/m+1+1+1}&\ldots&1^{1/m+n+k+l}&\ldots0^{1/m+n+k+l}\\.\\0^{1/(m+2)+1+1+1}&\ldots&0^{1/(m+2)+n+k+l}&\ldots0^{1/(m+2)+n+k+l}\end{bmatrix}$$

$$=\begin{bmatrix}0&0&\ldots0&\ldots0\\.\\.\\.\\0&0&\ldots(1)^{1/m+n+k+l}&\ldots0\\.\\0&0&\ldots0&\ldots0\end{bmatrix}$$

This is a biquadratic entire sequence.

Hence $\delta_{mnkl}\in\Gamma^4$. We have $f(x)=\sum_{m,n,k,l=1}^{\infty}x_{mnkl}y_{mnkl}$ with $x\in\Gamma^4$ and $f\in(\Gamma^4)^*$ the dual of Γ^4.

Take $x=(x_{mnkl})=\delta_{mnkl}\in\Gamma^4$.

Then $|y_{mnkl}| \leq fd(\delta_{mnkl}, 0) < \infty \, for \, all \, m, n, k, l.$

Thus (y_{mnkl}) is bounded sequence and hence a biquadratic analytic sequence.

Proposition 3.3: Γ^4 is separable but its dual Λ^4 is not separable.

Proof: Part (i)- It is routine verification. So omitted.

Part (ii) - Since

$$|x_{mnkl}|^{1/m+n+k+l} \to 0 \, as \, m, n, k, l \to \infty,$$

It may be happen that first row or first column may not be convergent, and may not be bounded.

Let A be a set of biquadratic sequences such that first row is construct by sequences of zero's or one's.

Then A is uncountable.

Consider an open ball of radius 3^{-1} units. Then these open balls will not cover Λ^4.

Hence Λ^4 is not separable.

Proposition 3.4: Γ^4 is not reflexive.

Proof: Use embedding to prove it.

Proposition 3.5: Γ^4 is not an inner product space and hence not a Hilbert space.

Proof : Let

$$x = x_{mnkl} = \begin{bmatrix} 1 & 1/4 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix}, y = y_{mnkl} = \begin{bmatrix} 1 & -1/4 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix}$$

$$d(x, 0) = sup \begin{bmatrix} |x_{1111} - 0|^{1/4} & |x_{1222} - 0|^{1/7} & \dots \\ |x_{2111} - 0|^{1/5} & |x_{2222} - 0|^{1/8} & \dots \\ . & & \\ . & & \\ . & & \end{bmatrix}$$

$$= sup \begin{bmatrix} |1 - 0|^{1/4} & \left|\frac{1}{4} - 0\right|^{1/7} & \dots \\ |0 - 0|^{1/5} & |0 - 0|^{1/8} & \dots \\ . & & \\ . & & \\ . & & \end{bmatrix} = sup \begin{bmatrix} (1)^{1/4} & \left(\frac{1}{4}\right)^{1/7} & 0 & \dots \\ 0 & 0 & 0 & \dots \\ . & & & \\ . & & & \\ . & & & \end{bmatrix} = 1$$

$d(x, 0) = 1.$

Similarly, $d(0, y) = 1$.

Hence $d(x, 0) = d(0, y) = 1$.

$$x + y = \begin{bmatrix} 1 & 1/4 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix} + \begin{bmatrix} 1 & -1/4 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

$$d(x + y, x - y) = \sup\{ (|x_{mnkl} + y_{mnkl}| - |x_{mnkl} - y_{mnkl}|)^{1/m+n+k+l} : m, n, k, l = 1,2,3, \dots \}$$

$$d(x_{mnkl} + y_{mnkl}, 0) = sup \begin{bmatrix} |x_{1111} + y_{1111}|^{1/4} & |x_{1222} + y_{2222}|^{1/7} & \dots \\ . & & \\ . & & \\ . & & \end{bmatrix}$$

$$= sup \begin{bmatrix} |1 + 1|^{1/4} & \left|\frac{1}{4} - \frac{1}{4}\right|^{1/7} & \dots \\ . & & \\ . & & \\ . & & \end{bmatrix}$$

$$= sup \begin{bmatrix} |2|^{1/4} & |0|^{1/7} & \dots \\ . & & \\ . & & \\ . & & \end{bmatrix} = sup \begin{bmatrix} 1.189 & 0 & \dots \\ . & & \\ . & & \\ . & & \end{bmatrix} = 1.189$$

Therefore $d(x + y, 0) = 1.189$

Similarly, $d(x - y, 0) = 0.908$.

By parallelogram law,

$\Rightarrow [d(x + y, 0)]^2 + [d(x - y, 0)]^2 = 2[(d(x, 0))^2 + (d(0, y))^2]$

$\Rightarrow (1.189)^2 + (0.908)^2 = 2[1 + 1]$

$\Rightarrow 1.414 + 0.824 = 4$

$\Rightarrow 2.238 = 4$

Hence it is not satisfied by the law. Therefore Γ^4 is not an inner product space. Assume that Γ^4 is a Hilbert space. But then Γ^4 would satisfy reflexivity condition. Γ^4 is not reflexive. Thus Γ^4 is not a Hilbert space.

Proposition 3.6: Γ^4 is not rotund.

Proof : Let

$$x = x_{mnkl} = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix}, y = y_{mnkl} = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix}$$

Then $x = (x_{mnkl})$ and $y = (y_{mnkl})$ are in Γ^4.

Also,

$$d(x,y) = sup \begin{bmatrix} |x_{1111} - y_{1111}|^{1/4} & \ldots & |x_{1nkl} - y_{1nkl}|^{1/1+n+k+l} & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \\ |x_{m1kl} - y_{m1kl}|^{1/m+1+k+l} & \ldots & |x_{mnkl} - y_{mnkl}|^{1/m+n+k+l} & 0 & \ldots \\ 0 & \ldots & & 0 & \ldots \end{bmatrix}$$

Therefore

$$d(x,0) = sup \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \ldots \end{bmatrix}$$

Hence $d(x,0) = 1$

Similarly, $d(0,y) = 1$

$$x_{mnkl} + y_{mnkl} = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \ldots \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \ldots \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

$$d\left(\frac{x_{mnkl}+y_{mnkl}}{2},0\right)$$

$$= sup \begin{bmatrix} \left(\frac{x_{1111}+y_{1111}}{2}\right)^{1/4} & \dots & \left(\frac{x_{1nkl}+y_{1nkl}}{2}\right)^{1/1+n+k+l} & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \\ \left(\frac{x_{m1kl}+y_{m1kl}}{2}\right)^{1/m+1+k+l} & \dots & \left(\frac{x_{mnkl}+y_{mnkl}}{2}\right)^{1/m+n+k+l} & 0 & \dots \\ 0 & \dots & & 0 & \dots \end{bmatrix}$$

$$d\left(\frac{x_{mnkl}+y_{mnkl}}{2},0\right) = sup \begin{bmatrix} \left(\frac{1+1}{2}\right)^{1/4} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

$$d\left(\frac{x_{mnkl}+y_{mnkl}}{2},0\right) = sup \begin{bmatrix} \left(\frac{2}{2}\right)^{1/4} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

$$d\left(\frac{x_{mnkl}+y_{mnkl}}{2},0\right) = sup \begin{bmatrix} (1)^{1/4} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & 0 & \dots \end{bmatrix} = 1$$

Therefore $\Gamma^{\mathbf{4}}$ is not rotund.

Proposition 3.7: Weak convergence and strong convergence are equivalent in Γ^4.

Proof : Step 1. Always strong convergence implies weak convergence in $\Gamma^{\mathbf{4}}$**.**

Step 2. So it is enough to show that weakly convergence implies strongly convergence in $\Gamma^{\mathbf{4}}$.

y^{η} tends to weakly in Γ^{4}, where $\left(y^{\eta}_{mnkl}\right) = y^{\eta}$, and $y = y_{mnkl}$. Take any $x = (x_{mnkl}) \in \Gamma^{4}$ and

$$(3.3)\ f(z) = \sum_{m,n,k,l=1}^{\infty} |z_{mnkl}x_{mnkl}|^{1/m+n+k+l} \text{ for each } z \in (z_{mnkl}) \in \Gamma^{4}$$

Then $f \in (\Gamma^{4})^{*}$ (by proposition 3.2). By hypothesis $(y^{\eta}) \to f(y)$ as $\eta \to \infty$.

(3.4) $f(y^{\eta} - y) \to 0$ as $\eta \to \infty$

An appeal to (3.3) and (3.4),

$$\Rightarrow \sum_{m,n,k,l=1}^{\infty} \left(\left|y^{\eta}_{mnkl} - y_{mnkl}\right|^{1/m+n+k+l} |x_{mnkl}|^{1/m+n+k+l} \right) \to 0, \text{as } \eta \to \infty.$$

(Since $x = (x_{mnkl}) \in \Lambda^{4}$ we have $\sum_{m,n,k,l=1}^{\infty} |x_{mnkl}|^{1/m+n+k+l} < \infty$, $\forall\, x \in \Lambda^{4}$)

$$\Rightarrow \sum_{m,n,k,l=1}^{\infty} \left(\left|y^{\eta}_{mnkl} - y_{mnkl}\right|^{1/m+n+k+l} \right) \to 0, \text{as } \eta \to \infty.$$

$$\Rightarrow sup_{m,n,k,l} \left(\left|\left(y^{\eta}_{mnkl} - y_{mnkl}\right) - 0\right|^{1/m+n+k+l} \right) \to 0, \text{as } \eta \to \infty.$$

$\Rightarrow d\big((y^{\eta} - y), 0\big) \to 0, \text{as } \eta \to \infty.$

$\Rightarrow y^{\eta} - y \to 0, \text{as } \eta \to \infty.$

$\Rightarrow y^{\eta} \to y, \text{as } \eta \to \infty.$

4. Conclusion

Biquadratic Sequence spaces has great importance in study of functional analysis, specially their monotonicity, separability and convergence. We have proved that Γ^{4} is not reflexive and hilbert space but weak convergence and strong convergence are equivalent. We also conclude that the dual space of Biquadratic Sequence is Λ^{4} but not separable although Γ^{4} is separable.

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