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ON THE CHARACTERIZATION OF TRIPLY DIFFUSIVE CONVECTION IN A SPARSELY DISTRIBUTED POROUS MEDIUM

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Abstract: In the present paper, it is analytically proved that the triply diffusive convection in a sparsely distributed porous medium cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two salinity Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations respectively, the Viscosity ratio Λ, the Darcy number D_a, the Heat capacity ratio E, a constant E_1 analogous to E corresponding to first salinity component and a constant E_2 analogous to E corresponding to second salinity component satisfy the inequality $R_1 + R_2 \le \left[\frac{\left(\frac{27\pi^4}{4}\Lambda + 4\pi^2 D_a^{-1}\right)}{\left(1+E\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\right)}\right]$. This result is uniformly valid for any combination of rigid and free boundaries.

Mathematics Subject Classification:

Keywords: Triply diffusive convection, Oscillatory motions, Rayleigh number, Concentration Rayleigh number, Darcy-Brinkman Model.

1. Introduction

Research on double diffusive convective motion in porous media has attracted several researchers over the last few decades due to its relevance and applications in many fields such as oceanography, geophysics, engineering geology, ground water movement in aquifers, chemical process industry, food processing industry, petroleum industry, nuclear engineering, centrifugal casting of metals and rotating machinery, solidification, electrochemistry and biomechanics (Brandt and Fernando [2] and Nield and Bezan [9]). Double diffusive convection in porous medium has been extensively studied. For a broad view of the subject one may be referred to Murray and Chen [7], Nield [8], Taunton et al. [21], Kuznetsov and Nield [5], Vafai [25], Malashetty and Begum [6], Prakash and Kumar [13] and Radko[18].

Above mentioned researchers have studied two component hydrodynamic systems only. However, there are many situations where more than two diffusing components are involved e.g. solidification of molten alloys, Earth core, geothermally heated lakes, magmas and their laboratory models and sea water (Griffiths [3, 4]). The subject with more than two components (in porous and non-porous medium) has attracted the attention of many researchers e.g. Poulikakos [11], Pearlstein et al. [10], Terrones and Pearlstein [22], Rudraiah and Vortmeyer [19], Tracey [23, 24], Prakash et al. [16], Prakash et al. [14] and Prakash and Manan [15]. The essence of the works of these researchers is that the small mass concentration of a third component with a smaller mass diffusivity can have a remarkable impact upon the nature of instability; and the manifestation of 'oscillatory' and direct 'salt finger' modes are simultaneous possible under a wide range of conditions (Griffiths [3, 4]).

The Banerjee et al.'s [1] characterization theorem, which states that oscillatory motions of neutral or growing amplitude cannot manifest in an initially bottom heavy thermohaline convection of Veronis type whenever the concentration Rayleigh number is less than a critical value, provides a classification of the neutral and unstable double diffusive convection configuration into two classes namely the bottom heavy class and the top heavy class and differentiate between them by means of characterization theorem which disallow the existence of oscillatory motions in the bottom heavy class. Prakash et al. [12, 17] further extended Banerjee et al.'s [1] work to triply diffusive convection configurations.

In the present attempt such a criterion has been established for characterizing non oscillatory motions which may be neutral or unstable for triply diffusive convection configuration in a sparsely distributed porous medium using the Darcy-Brinkman model. The result derived herein is uniformly valid for all the combinations of rigid and free boundaries.

2. Mathematical Formulation

We consider an initially quiescent viscous finitely heat conducting Boussinesq fluid layer, saturating a porous medium, of infinite horizontal extension statically confined between two horizontal planes $z = 0$ and $z = d$. The lower surface is kept at uniform temperature T_0 and concentrations S_{10} ,S_{20} while the upper surface is kept at uniform temperature $T_1 (< T_0)$ and uniform concentrations $S_{11} (< S_{10}), S_{21} (< S_{20})$ (see in Fig.1). The porous medium is assumed to be a constant porosity medium. The cross-diffusion effects of the salinity components are ignored. The Darcy-Brinkman model is used to investigate the problem.

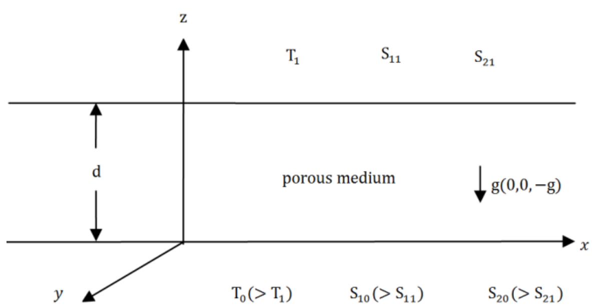

Fig.1 Physical configuration.

Following Prakash et al. [13], the hydrodynamic equations that govern the problem can easily be obtained as (for detailed derivation of the equations one may refer to Prakash et al. [13])

$$\Lambda(D^2 - a^2)^2 w - (p + D_a^{-1})(D^2 - a^2)w = R a^2\theta - R_1 a^2 \phi_1 - R_2 a^2 \phi_2 , \qquad (1)$$

$$(D^2 - a^2 - E\,\sigma p)\theta = -w , \qquad (2)$$

$$\left(D^2 - a^2 - \frac{E_1 \sigma p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1} , \qquad (3)$$

$$\left(D^2 - a^2 - \frac{E_2 \sigma p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2} . \qquad (4)$$

The equations (1) - (4) are to be solved by using the following boundary conditions:

$w = \theta = \phi_1 = \phi_2 = Dw = 0$ at $z = 0$ and at $z = 1$, (both the boundaries are rigid) (5)

or $w = \theta = \phi_1 = \phi_2 = D^2 w = 0$ at $z = 0$ and at $z = 1$, (both the boundaries are free) (6)

or $w = \theta = \phi_1 = \phi_2 = Dw = 0$ at $z = 0$ (lower boundary is rigid)

and $w = \theta = \phi_1 = \phi_2 = D^2 w = 0$ at $z = 1$, (upper boundary is free) (7)

or $w = \theta = \phi_1 = \phi_2 = D^2 w = 0$ at $z = 0$ (lower boundary is free)

and $w = \theta = \phi_1 = \phi_2 = Dw = 0$ at $z = 1$ (upper boundary is rigid) (8)

where z is the real independent variable such that $0 \le z \le 1$, D is the differentiation w.r.t. z, a^2 is square of the wave number, $\sigma > 0$ is the Prandtl number, $\tau_1 > 0$ and $\tau_2 > 0$ are the Lewis numbers for two concentration components respectively, $R = \frac{g\,\alpha\,\beta\,d^4}{\kappa\,\nu} > 0$ is the thermal Rayleigh number, $R_1 = \frac{g\,\alpha_1 \beta_1\, d^4}{\kappa\ \nu} > 0$ and $R_2 = \frac{g\,\alpha_2 \beta_2\, d^4}{\kappa\ \nu} > 0$ are the two salinity Rayleigh numbers, $D_a = \frac{k_1}{d^2} > 0$ is the Darcy number, $p = p_r + ip_i$ is the complex growth rate where p_r and p_i are the real constants, w is the vertical velocity, θ is the temperature, ϕ_1 and ϕ_2 are the respective concentrations of the two components, $E > 0$, $E_1 > 0$ and $E_2 > 0$ are constants. It may further be noted that Eqs. (1) - (8) describe an eigen value problem for p and govern triply diffusive convection in a porous medium for any combination of dynamically free and rigid boundaries.

3. Mathematical Analysis

Now we prove the following theorem:

Theorem:

If $R > 0, R_1 > 0, R_2 > 0, p_r \ge 0, p_i \ne 0$, and $R_s = R_1 + R_2 \le \left[\frac{\left(27\frac{\pi^4}{4}\Lambda + 4\pi^2 D_a^{-1}\right)}{1 + E\left(\frac{\tau_1}{E_1} + \frac{\tau_2}{E_2}\right)}\right]$, then a necessary condition for the existence of nontrivial solution $(w, \theta, \phi_1, \phi_2, p)$ of Eqs. (1) - (4) together with boundary conditions (5) or (6) or (7) or (8) is that $R_s = R_1 + R_2 < R$.

Proof: Multiplying Eq. (1) by w* (the superscript * henceforth denotes complex conjugation) on both sides and integrating over vertical range of z, we obtain

$$\Lambda\int_0^1 w^*(D^2-a^2)^2 w\,dz-(p+D_a^{-1})\int_0^1 w^*(D^2-a^2)w\,dz=$$

$$R\,a^2\int_0^1 w^*\,\theta\,dz-R_1a^2\int_0^1 w^*\,\phi_1\,dz-R_2a^2\int_0^1 w^*\,\phi_2\,dz. \tag{9}$$

Using Eqs. (2) - (4) and the boundary conditions (5) - (8), we can write

$$R\,a^2\int_0^1 w^*\,\theta\,dz=-Ra^2\int_0^1\theta\,(D^2-a^2-E\,\sigma\,p^*)\theta^*dz, \tag{10}$$

$$R_1a^2\int_0^1 w^*\,\phi_1 dz=-R_1a^2\tau_1\int_0^1\phi_1\left(D^2-a^2-\frac{E_1\sigma\,p^*}{\tau_1}\right)\phi_1^*dz, \tag{11}$$

$$R_2a^2\int_0^1 w^*\,\phi_2 dz=-R_2a^2\tau_2\int_0^1\phi_2\left(D^2-a^2-\frac{E_2\sigma\,p^*}{\tau_2}\right)\phi_2^*dz. \tag{12}$$

Combining Eqs. (9) – (12), we obtain

$$\Lambda\int_0^1 w^*(D^2-a^2)^2 w\,dz-(p+D_a^{-1})\int_0^1 w^*(D^2-a^2)w\,dz=$$

$$-Ra^2\int_0^1\theta\,(D^2-a^2-E\,\sigma\,p^*)\theta^*dz+R_1a^2\tau_1\int_0^1\phi_1\left(D^2-a^2-\frac{E_1\sigma\,p^*}{\tau_1}\right)\phi_1^*dz$$

$$+R_2a^2\tau_2\int_0^1\phi_2\left(D^2-a^2-\frac{E_2\sigma\,p^*}{\tau_2}\right)\phi_2^*dz. \tag{13}$$

Integrating both sides of equation (13), by parts, for an appropriate number of times and using the boundary conditions (5) - (8), we get

$$\Lambda\int_0^1(|D^2w|^2+2a^2|Dw|^2+a^4|w|^2)\,dz+(p+D_a^{-1})\int_0^1(|Dw|^2+a^2|w|^2)\,dz$$

$$=Ra^2\int_0^1(|D\theta|^2+a^2|\theta|^2+E\sigma p^*|\theta|^2)\,dz$$

$$-R_1a^2\tau_1\int_0^1\left(\left|D\phi_1\right|^2+a^2\left|\phi_1\right|^2+\frac{E_1\sigma\,p^*}{\tau_1}\left|\phi_1\right|^2\right)dz$$

$$-R_2a^2\tau_2\int_0^1\left(\left|D\phi_2\right|^2+a^2\left|\phi_2\right|^2+\frac{E_2\sigma\,p^*}{\tau_2}\left|\phi_2\right|^2\right)dz. \tag{14}$$

Equating the real and imaginary parts of both sides of Eq. (14) and cancelling p_i ($\neq 0$) throughout from the imaginary part, we get

$$\Lambda\int_0^1 (|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)\ dz + (p_r + D_a^{-1})\int_0^1 (|Dw|^2 + a^2|w|^2)\,dz$$

$$= Ra^2\int_0^1 (|D\theta|^2 + a^2|\theta|^2 + E\sigma p_r|\theta|^2)\,dz$$

$$- R_1a^2\tau_1\int_0^1 \left(|D\phi_1|^2 + a^2|\phi_1|^2 + \frac{E_1\sigma p_r}{\tau_1}|\phi_1|^2\right)dz$$

$$-R_2a^2\tau_2\int_0^1 \left(|D\phi_2|^2 + a^2|\phi_2|^2 + \frac{E_2\sigma p_r}{\tau_2}|\phi_2|^2\right)dz, \tag{15}$$

and

$$\int_0^1(|Dw|^2 + a^2|w|^2)dz = -Ra^2E\sigma\int_0^1|\theta|^2dz + R_1a^2E_1\sigma\int_0^1|\phi_1|^2\,dz + R_2a^2E_2\sigma\int_0^1|\phi_2|^2\,dz. \tag{16}$$

We write Eq. (15) in an alternative form as

$$\Lambda\int_0^1(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)dz + p_r\int_0^1(|Dw|^2 + a^2|w|^2)\,dz + D_a{}^{-1}\int_0^1(|Dw|^2 + a^2|w|^2)\,dz = Ra^2\int_0^1(|D\theta|^2 + a^2|\theta|^2)dz - R_1a^2\tau_1\int_0^1\left(|D\phi_1|^2 + a^2|\phi_1|^2\right)dz - R_2a^2\tau_2\int_0^1\left(|D\phi_2|^2 + a^2|\phi_2|^2\right)dz + p_r\left[Ra^2E\sigma\int_0^1|\theta|^2dz - R_1a^2E_1\sigma\int_0^1|\phi_1|^2dz - R_2a^2E_2\sigma\int_0^1|\phi_2|^2\,dz\right], \tag{17}$$

and derive the validity of the theorem from the resulting inequality obtained by replacing each terms of this equation by its appropriate estimate.

We first note that since w, θ, ϕ_1 and ϕ_2 satisfy $w(0) = 0 = w(1), \theta(0) = 0 = \theta(1), \phi_1(0) = 0 = \phi_1(1)$ and $\phi_2(0) = 0 = \phi_2(1)$, we have by Rayleigh-Ritz inequality (Schultz [20])

$$\int_0^1|Dw|^2dz \geq \pi^2\int_0^1|w|^2dz, \tag{18}$$

$$\int_0^1|D\theta|^2dz \geq \pi^2\int_0^1|\theta|^2dz, \tag{19}$$

$$\int_0^1|D\phi_1|^2dz \geq \pi^2\int_0^1|\phi_1|^2dz, \tag{20}$$

$$\int_0^1|D\phi_2|^2dz \geq \pi^2\int_0^1|\phi_2|^2dz. \tag{21}$$

Further, since $w(0) = 0 = w(1)$, it follows that (Banerjee et al. [1])

$$\int_0^1|Dw|^2dz = -\int_0^1 w^*D^2w\,dz,$$

$$\leq \left|-\int_0^1 w^*D^2w\,dz\right|,$$

$\leq \int_0^1 |w^*||D^2 w| dz,$

$\leq \int_0^1 |w||D^2 w| dz,$

$\leq \left(\int_0^1 |w|^2 dz\right)^{1/2} \left(\int_0^1 |D^2 w|^2 dz\right)^{1/2}$, (using Schwartz inequality)

$\leq \frac{1}{\pi} \left(\int_0^1 |Dw|^2 dz\right)^{1/2} \left(\int_0^1 |D^2 w|^2 dz\right)^{1/2}$, (using inequality (18))

which implies that

$$\int_0^1 |D^2 w|^2 dz \geq \pi^2 \int_0^1 |Dw|^2 dz. \tag{22}$$

Combining inequality (18) and (22), we get

$$\int_0^1 |D^2 w|^2 dz \geq \pi^4 \int_0^1 |w|^2 dz, \tag{23}$$

and thus by using in equality (18) and (23), we have

$$\int_0^1 (|D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2) dz \geq (\pi^2 + a^2)^2 \int_0^1 |w|^2 dz. \tag{24}$$

Secondly, since $p_r \geq 0$, we note that

$$p_r \int_0^1 (|Dw|^2 + a^2 |w|^2) dz \geq 0. \tag{25}$$

Now, multiplying Eq. (2) by θ^* and integrating the various terms on the left hand side of the resulting equation by parts for an appropriate number of times by making use of the boundary conditions on θ, namely $\theta(0) = 0 = \theta(1)$, then equating the real parts of both sides of the final equation, we obtain

$\int_0^1 (|D\theta|^2 + a^2 |\theta|^2 + E\sigma p_r |\theta|^2) dz = \text{Real part of} \int_0^1 \theta^* w dz,$

$\leq \left|\int_0^1 \theta^* w dz\right|,$

$\leq \int_0^1 |\theta^* w| dz,$

$$\leq \left(\int_0^1 |\theta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2} \text{(using Schwartz inequality)} \tag{26}$$

Using the inequality (19) and the fact that $p_r \geq 0$, in equation (26), we have

$$(\pi^2 + a^2) \int_0^1 |\theta|^2 dz \leq \left(\int_0^1 |\theta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2},$$

which gives that

$$\left(\int_0^1 |\theta|^2 dz\right)^{1/2} \leq \frac{1}{(\pi^2 + a^2)} \left(\int_0^1 |w|^2 dz\right)^{1/2},$$

and thus inequality (26) gives

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)dz \le \frac{1}{(\pi^2+a^2)}\int_0^1 |w|^2 dz. \quad (27)$$

Further, combining the above inequality with inequality (19), we get

$$\int_0^1 |\theta|^2 dz \le \frac{1}{(\pi^2+a^2)^2}\int_0^1 |w|^2 dz\,. \quad (28)$$

Now using inequalities (20) and (21), it follows that

$$R_1 a^2 \tau_1 \int_0^1 \left(|D\phi_1|^2 + a^2|\phi_1|^2\right) dz + R_2 a^2 \tau_2 \int_0^1 \left(|D\phi_2|^2 + a^2|\phi_2|^2\right) dz$$

$$\ge (\pi^2 + a^2)\left(R_1 a^2 \tau_1 \int_0^1 |\phi_1|^2 dz + R_2 a^2 \tau_2 \int_0^1 |\phi_2|^2 dz\right)$$

$$\ge (\pi^2 + a^2)\left\{\frac{\tau_1}{E_1\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz - \frac{R_2 a^2 E_2 \tau_1}{E_1}\int_0^1 |\phi_2|^2 dz + \frac{\tau_2}{E_2\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz - \frac{R_1 a^2 E_1 \tau_2}{E_2}\int_0^1 |\phi_1|^2 dz\right\},$$ (utilizing Eq. (16))

and thus

$$-R_1 a^2 \tau_1 \int_0^1 \left(|D\phi_1|^2 + a^2|\phi_1|^2\right) dz - R_2 a^2 \tau_2 \int_0^1 \left(|D\phi_2|^2 + a^2|\phi_2|^2\right) dz$$

$$\le (\pi^2 + a^2)\left[\frac{R_1 a^2 E_1 \tau_2}{E_2}\int_0^1 |\phi_1|^2 dz + \frac{R_2 a^2 E_2 \tau_1}{E_1}\int_0^1 |\phi_2|^2 dz\right] - (\pi^2 + a^2)\left(\frac{\tau_1}{E_1\sigma} + \frac{\tau_2}{E_2\sigma}\right)\int_0^1 (|Dw|^2 + a^2|w|^2)dz. \quad (29)$$

Again, using Eq. (16), we obtain

$$\frac{R_1 a^2 E_1 \tau_2}{E_2}\int_0^1 |\phi_1|^2 dz \le \frac{\tau_2}{E_2\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz + \frac{Ra^2 E\tau_2}{E_2}\int_0^1 |\theta|^2 dz, \quad (30)$$

and

$$\frac{R_2 a^2 E_2 \tau_1}{E_1}\int_0^1 |\phi_2|^2 dz \le \frac{\tau_1}{E_1\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz + \frac{Ra^2 E\tau_1}{E_1}\int_0^1 |\theta|^2 dz. \quad (31)$$

Now using inequalities (30) and (31), we can write

$$\frac{R_1 a^2 E_1 \tau_2}{E_2}\int_0^1 |\phi_1|^2 dz + \frac{R_2 a^2 E_2 \tau_1}{E_1}\int_0^1 |\phi_2|^2 dz \le \left(\frac{\tau_1}{E_1\sigma} + \frac{\tau_2}{E_2\sigma}\right)\int_0^1 (|Dw|^2 + a^2|w|^2)dz + Ra^2 E\left(\frac{\tau_1}{E_1} + \frac{\tau_2}{E_2}\right)\int_0^1 |\theta|^2 dz. \quad (32)$$

Using this inequality (32) in inequality (29), we get

$$-R_1a^2\tau_1\int_0^1\left(|D\phi_1|^2+\ a^2|\phi_1|^2\right)dz-R_2a^2\tau_2\int_0^1\left(|D\phi_2|^2+\ a^2|\phi_2|^2\right)dz$$

$\leq(\pi^2+a^2)Ra^2E\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\int_0^1|\theta|^2dz,$

$\leq\frac{Ra^2E}{(\pi^2+a^2)}\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\int_0^1|w|^2dz$ (using inequality (28)). (33)

Also from Eq. (16) and the fact that $p_r\geq 0$, we obtain

$p_r\left[Ra^2E\sigma\int_0^1|\theta|^2dz-R_1a^2E_1\sigma\int_0^1|\phi_1|^2dz-R_2a^2E_2\sigma\int_0^1|\phi_2|^2dz\right]<0.$ (34)

Now, if permissible, assume that $R_s=R_1+R_2\geq R$. Then, we obtain from Eq. (17) and the inequalities (18), (24), (25), (27), (33) and (34) that

$\left[\Lambda(\pi^2+a^2)^2+D_a^{-1}(\pi^2+a^2)-\frac{R_sa^2}{(\pi^2+a^2)}\left(1+E\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\right)\right]\int_0^1|w|^2dz<0$, (35)

which clearly implies that

$$R_s>\frac{\left(\Lambda\frac{(\pi^2+a^2)^3}{a^2}+D_a^{-1}\frac{(\pi^2+a^2)^2}{a^2}\right)}{\left(1+E\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\right)}.$$

Since the minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ (for $a^2=\frac{\pi^2}{2}$) is $\frac{27\pi^4}{4}$ and the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ (for $a^2=\pi^2$) is $4\pi^2$, thus we necessarily have

$R_s>\left[\frac{\left(\frac{27\pi^4}{4}\Lambda+4\pi^2D_a^{-1}\right)}{\left(1+E\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\right)}\right],$ (36)

Hence if $R_s=R_1+R_2\leq\left[\frac{\left(\frac{27\pi^4}{4}\Lambda+4\pi^2D_a^{-1}\right)}{\left(1+E\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\right)}\right]$, then we must have

$R_s=R_1+R_2<R.$ (37)

This proves the theorem.

The essential content of the theorem from the physical point of view are that triply diffusive convection in a porous medium cannot manifest as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two salinity Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two mass concentrations respectively, the Viscosity ratio Λ, the Darcy number D_a, the Heat capacity ratio E, a constant E_1 analogous to E corresponding to first salinity component and a constant E_2 analogous to E corresponding to second salinity component satisfy the inequality

$R_1 + R_2 \leq \left[\frac{\left(\frac{27\pi^4}{4}\Lambda + 4\pi^2 D_a^{-1}\right)}{\left(1+E\left(\frac{\tau_1}{E_1}+\frac{\tau_2}{E_2}\right)\right)}\right]$. This result is uniformly valid for the quite general nature of the bounding surfaces.

4. Conclusions

A linear stability analysis is performed for the onset of triply diffusive convection in a sparsely distributed isotropic and homogeneous porous medium using the Darcy-Brinkman model. The layer is considered to be heated and soluted from below. The neutral or unstable triply diffusive configuration has been classified into two classes namely the bottom-heavy class and top-heavy class. Then the two classes are differentiated by means of characterization theorem which does not allow the existence of oscillatory perturbations of growing amplitude in an initially bottom-heavy triply diffusive convection. The result derived herein will certainly pave the way for further theoretical and experimental investigations in this domain of enquiry.

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