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LOVE WAVES IN A FIBRE-REINFORCED MEDIUM OVER A HETEROGENEOUS ORTHOTROPIC HALF-SPACE

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Abstract: The objective of this paper is to investigate Love waves in a fibre-reinforced medium over a heterogeneous orthotropic half-space under initial stress. The lower heterogeneous half-space is caused by consideration of exponential variation in initial stress, rigidity and shear moduli. The method of separation of variables is applied to find the dispersion equation. As a special case when the layer and the half-space both are homogeneous our computed equation coincides with the standard equation of Love wave. Numerical results analyzing the dispersion equation are discussed and presented by number of graphs. The dispersion equation shows that the reinforcement as well as heterogeneity, initial stress parameters have remarkable effect on the propagation of Love waves.

Keywords: Love waves, initial stress, heterogeneity, fibre-reinforced medium, orthotropic, phase velocity.

1. Introduction

When an earthquake occurs, the shockwaves of released energy which shake the Earth and temporarily turn soft deposits are called seismic waves. These are waves of energy that travel at different speeds when they pass through different types of material and move similarly to other types of waves, like sound waves, light waves and water waves. These waves contain vital information about the internal structure of the Earth. Because the speed of seismic waves depends on the material properties, one can use the travel-time of seismic waves to map change in density with depth, and show that the Earth is composed of several layers. There are two basic types of seismic waves – body waves and surface waves. Surface waves are similar in nature to water waves and travel just under the Earth's surface.

Since our planet is a spherical body having finite dimension and the elastic waves generated must receive the effect of the boundaries. Naturally, this concept leads us to study surface waves. The study of surface waves in elastic media is very important due to their devastating damage capabilities during earthquake and for homogeneous, non-homogeneous and layered media it has been central interest to theoretical seismologists in recent time. One type of surface waves is Love wave that may be available in non-homogeneous Earth. Love waves are transverse waves that vibrate the ground in the horizontal direction perpendicular to the direction that the waves are travelling. Quite a good amount of information about the propagation of seismic waves is contained in the well-known book by Ewing et al. [14]. A large number of papers has been published in different journals after publishing this book. Chattopadhyay et al. [9] investigated the propagation of torsional surface waves in an inhomogeneous layer over an inhomogeneous half-space. Georgiadis et al. [15] showed the existence of torsional surface waves in a gradient-elastic half space. Dey et al. [12] presented a study of Love wave propagation in an elastic layer with void pores. Love-type surface wave in homogeneous micropolar elastic media was studied by Midya [31]. Davini et al. [11] studied the propagation of torsional waves in a thin rectangular domain using asymptotic approach. Gupta et al. [18] pointed that in a homogeneous layer over a heterogeneous half-space torsional waves do exist. Ghorai and Tiwary [16] found that in-homogeneity of rigidity and density of the medium influences the velocity of torsional surface wave. Sethi et al. [35] made an attempt to investigate the effect of viscoelastic material on the phase velocity of torsional wave. Manna et al. [30] discussed Love wave propagation in a piezoelectric layer lying over an inhomogeneous elastic half-space. Kundu et al. [28] showed that the appearance of heterogeneities, initial stress, and viscoelasticity in the velocity equation significantly affects the attenuation and dispersion characteristics of Love-type waves. Kumhar et al. [25] discussed the response of all mechanical parameters such as heterogeneities, sandiness, hydrostatic stress, thickness ratio, attenuation and viscoelasticity on both the phase and damped velocity of Love waves. Singh and Alam [36] studied the dispersion and attenuation characteristics of corrugation, reinforcement, heterogeneity and initial stress on propagation of Love waves. The commendable works by Islam et al. [21], Abd-Alla et al. [2], Chattopadhyay et al. [10] in the study of seismic waves may be cited.

The study of wave propagation in fibre-reinforced medium plays an important role in geomechanics and civil engineering. The characteristic property of a fibre-reinforced material is that its components, i.e., concrete and steel act as a single anisotropic unit as long as they remain in elastic condition, i.e., the components are bound together so that there are no relative displacement between them. There are some hard and soft rocks inside the Earth that show reinforced property. There are also artificial fibre-reinforced composites used to minimize the damage due to earthquake. Many papers have been published on the propagation of seismic waves in fibre-reinforced medium. Pradhan et al. [33] considered the influence of anisotropy on the Love waves in self-reinforced layer lying over an elastic non-homogeneous half-space. The effects of reinforcement, gravity and porosity on the propagation of Love waves were discussed by Chattaraj and Samal [7]. Dhua et al. [13] observed that the presence of reinforcement in the layer increases the phase velocity of tosrional wave significantly. Kundu et al. [27] investigated Love waves in a fibre-reinforced layered medium lying over an initially stressed orthotropic half-space. Vishwakarma [38] showed the effect of reinforced and viscoelastic parameters on torsional wave propagation. Alam et al. [4] investigated the dispersion and attenuation characteristics of Love-type wave propagation in a fiber-reinforced layer laid on an inhomogeneous viscoelastic half-space. Alam and Singh [6] showed the effects of sandiness, irregular boundary interfaces, heterogeneity and viscoelasticity on the phase velocity of Love waves. Kumhar et al. [24] investigated the effects of all affecting parameters such as reinforcement, porosity, initial stress, inhomogeneity, undulated and position parameters on shear waves graphically for different cases.

The initial stresses have important role on the propagation of waves. These initial stresses exist due to different factors such as external pressure, slow process of creep, manufacturing process, differences in temperature etc. Gupta et al. [19] assumed linear variations of rigidity, density and initial stress in the layer and observed significant role of initial stress on phase velocity of torsional waves.. Kepceler [22] discussed the effects of the imperfectness of the boundary condition on the influence of the initial stresses on the wave propagation velocity. Ahmed and Abo-Dahab [3] used Fourier transform method to find the dispersion equation of Love waves in an orthotropic Granular layer under initial stress overlying a semi-infinite Granular medium. Abd-Alla and Ahmed [1] also used Fourier transform method to find the dispersion equation of Love waves and showed that the velocity of Love waves lies between two quantities which are dependent on the non-homogeneities of two media. Sethi et al. [34] studied the effect of non-homogeneity of the orthotropic media as well as the changeable initial stress on the dispersion equation of Love waves. References can be made to Kundu et al. [26], Ozturk and Akbarov [32], Chattaraj et al. [8] for their excellent contributions in investigating seismic waves in various mediums under various circumstances. Alam and Kundu [5] concluded that that the heterogeneities, initial stresses and isotropy of the proposed Earth model have remarkable effect on the Love-type surface wave propagation. Kumhar et al. [23] examined the dispersion behavior on the Love waves propagate through pre-stressed anisotropic fluid saturated porous viscoelastic layer laid over an inhomogeneous half-space affected by gravity.

So far it has been found that the propagation of Love waves in a fibre-reinforced layer over a heterogeneous orthotropic half-space has remained un-attempted. So in the present paper, the effects of reinforcement, initial stress and heterogeneity parameter are shown on the propagation of Love waves. The crust region of our planet is composed of various heterogeneous layers with different geological parameters. For the present study the heterogeneity in the lower half-space is caused by exponential variation in initial stress, density and shear moduli. The dispersion equation of Love waves under these conditions has been derived. The study reveals that the reinforcement as well as heterogeneity, initial stress have remarkable effect on the propagation of Love waves. The graphical representation has shown the relation between dimensionless phase velocity and wave number.

2. Formulation of the Problem

We consider the positive z-axis vertically downwards and the x-axis along the direction of wave propagation. Let H be the thickness of the fibre-reinforced layer lying over a heterogeneous orthotropic half-space. The free surface of the reinforced layer is assumed to be traction free. The heterogeneity in the half-space is considered in initial stress, density and shear moduli. The following variations for the half-space are taken into account:

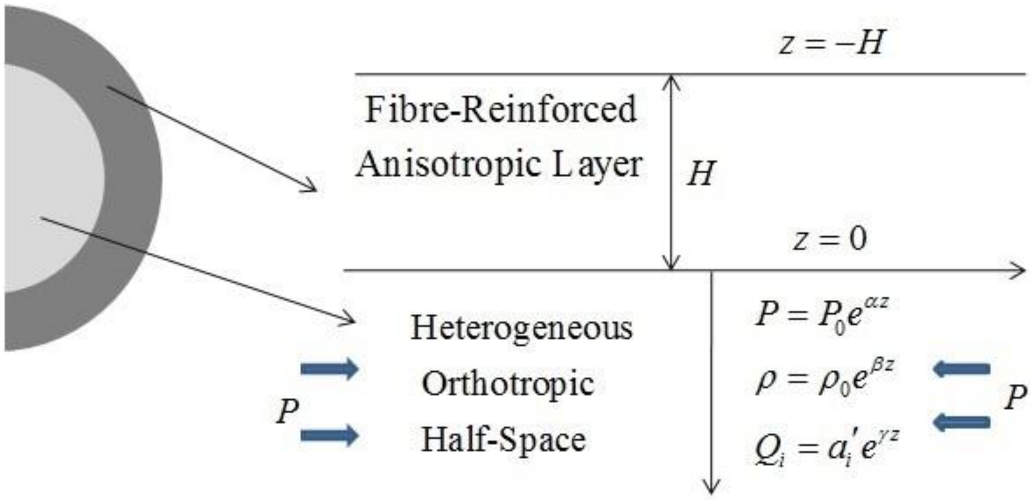

Figure 1: Geometry of the problem

$$\left.\begin{aligned} P &= P_0 e^{\alpha z}, \\ \rho &= \rho_0 e^{\beta z}, \\ Q_i &= a_i' e^{\gamma z}, \end{aligned}\right\} \tag{1}$$

where P is initial stress, ρ is density, Q_i are shear moduli, α, β, γ are constants having dimension that is inverse of length.

3. Solution for Fibre-reinforced Layer

The constitutive equation for a transversely isotropic linear elastic material with preferred direction $\vec{a}$ (Spencer [37]) is

$$\begin{aligned} \sigma_{ij} = {} & \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(\mathrm{a}_k \mathrm{a}_m \mathrm{e}_{km}\,\delta_{ij} + \mathrm{a}_i \mathrm{a}_j \mathrm{e}_{kk}) + \\ & 2(\mu_L - \mu_T)(\mathrm{a}_i \mathrm{a}_k \mathrm{e}_{kj} + \mathrm{a}_j \mathrm{a}_k \mathrm{e}_{ki}) + \beta(\mathrm{a}_k \mathrm{a}_m \mathrm{e}_{km} \mathrm{a}_i \mathrm{a}_j) \end{aligned} \tag{2}$$

where σ_{ij} are components of stress; e_{ij} are the infinitesimal strain components; δ_{ij} is Kronecker delta; $\mathrm{a}_i = (\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3)$ are the direction cosines of $\vec{a}$ with respect to Cartesian coordinate system such that $a_1^2 + a_2^2 + a_3^2 = 1$; μ_T and μ_L are transverse and longitudinal elastic shear modulus respectively; λ is elastic parameter; α, β, $\mu_L - \mu_T$

are reinforced anisotropic elastic parameters; u_i are the components of displacement vector. We assume the direction of fibre along x and z -axis, i.e., $\vec{a} = \vec{a}(\mathrm{a}_1, 0, \mathrm{a}_3)$.

Using the conventional Love wave conditions $u_1 = u_3 = 0, u_2 = v(\mathrm{x,z,t})$ in equation (2), we get the non-zero stress components as

$$\sigma_{12} = \mu_T \left[P\frac{\partial v}{\partial x} + R\frac{\partial v}{\partial z} \right], \ \sigma_{23} = \mu_T \left[Q\frac{\partial v}{\partial z} + R\frac{\partial v}{\partial x} \right] \tag{3}$$

Where

$$\left.\begin{aligned} P &= 1 + \left(\frac{\mu_L}{\mu_T} - 1 \right) a_1^2, \\ Q &= 1 + \left(\frac{\mu_L}{\mu_T} - 1 \right) a_3^2, \\ R &= \left(\frac{\mu_L}{\mu_T} - 1 \right) a_1 \,\mathrm{a}_3 . \end{aligned}\right\} \tag{4}$$

The equation of motion for Love waves in a fibre reinforced layer is

$$\frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2}. \tag{5}$$

With the help of (3) and (4), the equation of motion (5), takes the form

$$P\frac{\partial^2 v}{\partial x^2} + Q\frac{\partial^2 v}{\partial z^2} + 2R\frac{\partial^2 v}{\partial x \partial z} = \frac{\rho}{\mu_T}\frac{\partial^2 v}{\partial t^2}. \tag{6}$$

For the wave changing harmonically, we assume

$$v = V(\mathrm{z})\,\mathrm{e}^{ik(\mathrm{x}-\mathrm{ct})}, \tag{7}$$

where k is wave number, $\omega(=\mathrm{kc})$ =circular frequency, c is the speed of simple harmonic waves.

On substituting (7) into (6), one gets

$$Q\frac{d^2V}{dz^2} + 2Rik\frac{dV}{dz} + k^2\left(\frac{c^2}{c_0^2} - P \right)V = 0, \tag{8}$$

where $c_0 = \sqrt{\dfrac{\mu_T}{\rho}}$ is shear wave velocity in the reinforced layer.

The solution of (8) may be taken as

$$V(\mathrm{z}) = \mathrm{D}_1\, e^{-ik\zeta_1 z} + \mathrm{D}_2\, e^{-ik\zeta_2 z},$$

where

$$\zeta_1 = \frac{R + \sqrt{R^2 + Q\left(\dfrac{c^2}{c_0^2} - P\right)}}{Q}, \ \zeta_2 = \frac{R - \sqrt{R^2 + Q\left(\dfrac{c^2}{c_0^2} - P\right)}}{Q} \tag{9}$$

Thus the solution for the upper reinforced layer is

$$v = v_0(\mathrm{say}) = (\mathrm{D}_1\, e^{-ik\zeta_1 z} + \mathrm{D}_2\, e^{-ik\zeta_2 z})e^{ik(\mathrm{x}-\mathrm{ct})}. \tag{10}$$

4. Solution for Orthotropic Half-Space

The equations of motion without body force under initial stress $\left(\tau_{11} = -P\right)$ are given by

$$\left.\begin{aligned} &\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z} - P\left(\frac{\partial w_z}{\partial y} - \frac{\partial w_y}{\partial z}\right) = \rho\frac{\partial^2 u}{\partial t^2}, \\ &\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} - P\left(\frac{\partial w_z}{\partial x}\right) = \rho\frac{\partial^2 v}{\partial t^2}, \\ &\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{23}}{\partial y} + \frac{\partial \tau_{33}}{\partial z} - P\left(\frac{\partial w_y}{\partial x}\right) = \rho\frac{\partial^2 w}{\partial t^2}, \end{aligned}\right\} \tag{11}$$

where τ_{ij} are the incremental stress components; u, v, w are the components of displacement vector ; w_x, w_y, w_z are the components of rotational vector given by

$$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right),\ w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right),\ w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{12}$$

The stress-strain relations are

$$\left.\begin{aligned}\tau_{11} &= \beta_{11}e_{xx} + \beta_{12}e_{yy} + \beta_{13}e_{zz},\\ \tau_{22} &= \beta_{21}e_{xx} + \beta_{22}e_{yy} + \beta_{23}e_{zz},\\ \tau_{33} &= \beta_{31}e_{xx} + \beta_{32}e_{yy} + \beta_{33}e_{zz},\\ \tau_{23} &= 2Q_1e_{yz}, \tau_{31} = 2Q_2e_{zx}, \tau_{12} = 2Q_3e_{xy}.\end{aligned}\right\} \tag{13}$$

In above β_{ij} are the incremental normal elastic coefficients and Q_i are the shear moduli.

Now the strain-displacement relations are given by

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),\ \mathrm{e}_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),\ \mathrm{e}_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right). \tag{14}$$

For the propagation of Love waves along x-direction having the displacement of particles along y-direction, we have

$$u = 0, \mathrm{w} = 0, \mathrm{v} = v(\mathrm{x}, \mathrm{z}, \mathrm{t}). \tag{15}$$

Using the equations (12)-(15), the dynamical equation of motion which is not automatically satisfied is

$$Q_1\frac{\partial^2 v}{\partial z^2} + \left(Q_3 - \frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2} + \frac{dQ_1}{dz}\frac{\partial v}{\partial z} = \rho\frac{\partial^2 v}{\partial t^2} \tag{16}$$

We assume the harmonic solution of equation (16) as

$$v = V(\mathrm{z})\,\mathrm{e}^{ik(\mathrm{x}-\mathrm{ct})} \tag{17}$$

where $V(\mathrm{z})$ satisfies the following equation

$$Q_1\frac{d^2V}{dz^2} + \frac{dQ_1}{dz}\frac{dV}{dz} + k^2\left(\rho c^2 + \frac{P}{2} - Q_3\right)V = 0. \tag{18}$$

In above equation, k is the angular wave number and $w(= \mathrm{kc})$ is angular frequency.

Substituting $V = \dfrac{V_1}{\sqrt{Q_1}}$, above equation reduces to

$$\frac{d^2V_1}{dz^2} + \left[\frac{1}{4Q_1^2}\left(\frac{dQ_1}{dz}\right)^2 - \frac{1}{2Q_1}\frac{d^2Q_1}{dz^2} + k^2\left(\frac{\rho c^2}{Q_1} + \frac{P}{2Q_1} - \frac{Q_3}{Q_1}\right)\right]V_1 = 0. \tag{19}$$

Using equation (2.1), equation (4.9) gives

$$\frac{d^2V_1}{dz^2}+k^2\left[\frac{d_1+d_2z}{1+\gamma z}-\frac{\gamma^2}{4k^2}-\frac{a_3'}{a_1'}\right]V_1=0, \tag{20}$$

where

$d_1=\frac{c^2}{c_1^2}+\frac{P_0}{2a_1'}$,

$d_2=\beta\frac{c^2}{c_1^2}+\alpha\frac{P_0}{2a_1'}$,

$c_1=\sqrt{\frac{a_1'}{\rho_0}}$ = Characteristic velocity of transverse waves for lower heterogeneousorthotropichalf-space.

Using dimensionless parameters $\eta=\frac{2\gamma_1 k(1+\gamma z)}{\gamma}$ and $\gamma_1=\sqrt{\frac{a_3'}{a_1'}+\frac{\gamma^2}{4k^2}-\frac{d_2}{\gamma}}$ in equation (20), one gets

$$\frac{d^2V_1}{d\eta^2}+\left[-\frac{1}{4}+\frac{R}{\eta}\right]V_1=0, \tag{21}$$

where $R=\frac{k(\mathrm{d}_1\gamma-\mathrm{d}_2)}{2\gamma_1\gamma^2}$.

Solution of (21) satisfying the condition $V(\mathrm{z})\to 0$ as $\mathrm{z}\to\infty$, i.e., $V_1(\eta)\to 0$ as $\eta\to\infty$ may be taken as

$V_1=A_3W_{R,\frac{1}{2}}(\eta)$,

where A_3 is arbitrary constant and $W_{R,\frac{1}{2}}(\eta)$ is Whittaker function (Whittaker and Watson[39]).

Hence the solution of half-space may be written as

$$v=v_1(\text{say})=\frac{D_3W_{R,\frac{1}{2}}(\eta)}{e^{\frac{\gamma z}{2}}}e^{ik(\mathrm{x}-\mathrm{ct})}, \tag{22}$$

Where $D_3 = \frac{A_3}{\sqrt{a_1'}}$.

5. Boundary Conditions

The appropriate boundary conditions are as follows:

(i) Stress of layer vanishes at $z = -H$, i.e.,

$$\sigma_{23} = 0. \tag{23}$$

(ii) Displacement and stress components are continuous at $z = 0$, i.e.,

$$v_0 = v_1 \tag{24}$$

and $\sigma_{23} = \tau_{23}$ (25)

Using the above boundary conditions in equations (10) and 22), we obtained the following three equations

$$D_1(\mathrm{R}-\mathrm{Q}\zeta_1)\,\mathrm{e}^{ik\zeta_1 H} + D_2(\mathrm{R}-\mathrm{Q}\zeta_2)\,\mathrm{e}^{ik\zeta_2 H} = 0, \tag{26}$$

$$D_1 + D_2 - D_3 W_{R,\frac{1}{2}}\left[\frac{2\gamma_1 k}{\gamma}(1+\gamma\,\mathrm{z})\right]_{z=0} = 0, \tag{27}$$

$$D_1(\mathrm{R}-\mathrm{Q}\zeta_1)\mathrm{ik} + D_2(\mathrm{R}-\mathrm{Q}\zeta_2)\mathrm{ik} - \mathrm{D}_3\,\frac{a_1'}{\mu_T}\left\{\frac{d}{dz}\left[\frac{W_{R,\frac{1}{2}}\left\{\frac{2\gamma_1 k}{\gamma}(1+\gamma\,\mathrm{z})\right\}}{e^{\frac{\gamma z}{2}}}\right]\right\}_{z=0} = 0 \tag{28}$$

Eliminating D_1, D_2 and D_3 from equations (26)-(27) and expanding the Whittaker function upto linear terms, the dispersion equation for Love waves in fibre reinforced medium is obtained as

$$\tan\left[\frac{kH}{Q}\sqrt{R^2 + Q\left(\frac{c^2}{c_0^2} - P\right)}\right] = \frac{a_1'}{\mu_T}\,\frac{\gamma_1 - \frac{\gamma}{2k} - \frac{2\gamma_1 A}{1 + \frac{2\gamma_1 k A}{\gamma}}}{\sqrt{R^2 + Q\left(\frac{c^2}{c_0^2} - P\right)}} \tag{29}$$

where $A = \frac{1-R}{2}$.

Equation (29) gives the dispersion equation of Love waves in a fibre-reinforced layer over a heterogeneous orthotropic half-space.

6. Particular Cases

Case I: If $a_1 = 1, a_2 = a_3 = 0$, then $P \to \frac{\mu_L}{\mu_T}, Q \to 1$ and $R \to 0$, the equation (29) reduces to

$$\tan\left[\frac{kH}{Q}\sqrt{\left(\frac{c^2}{c_0^2}-\frac{\mu_L}{\mu_T}\right)}\right] = \frac{a_1'}{\mu_T}\frac{\gamma_1 - \frac{\gamma}{2k} - \frac{2\gamma_1 A}{1+\frac{2\gamma_1 kA}{\gamma}}}{\sqrt{\left(\frac{c^2}{c_0^2}-\frac{\mu_L}{\mu_T}\right)}}.$$

This is the dispersion equation of Love waves in the presence of heterogeneous orthotropic half-space with initial stress.

Case II: If $\mu_L = \mu_T = \mu_0$, i.e., the upper layer is isotropic with rigidity μ_0, then by equation (29) we get

$$\tan\left[\frac{kH}{Q}\sqrt{\left(\frac{c^2}{c_0^2}-1\right)}\right] = \frac{a_1'}{\mu_0}\frac{\gamma_1 - \frac{\gamma}{2k} - \frac{2\gamma_1 A}{1+\frac{2\gamma_1 kA}{\gamma}}}{\sqrt{\left(\frac{c^2}{c_0^2}-1\right)}},$$

which is the dispersion equation of Love waves in an isotropic homogeneous layer lying over a heterogeneous orthotropic half-space under initial stress.

Case III: When the half-space is homogeneous, i.e., $\alpha \to 0, \beta \to 0$ and $\gamma \to 0$, the dispersion equation (29) becomes

$$\tan\left[\frac{kH}{Q}\sqrt{R^2 + Q\left(\frac{c^2}{c_0^2}-P\right)}\right] = \frac{a_1'}{\mu_T}\frac{\sqrt{\frac{a_3'}{a_1'} - \frac{c^2}{c_1^2} - \frac{P_0}{2a_1'}}}{\sqrt{R^2 + Q\left(\frac{c^2}{c_0^2}-P\right)}}.$$

Case IV: In this case, the half-space is isotropic, i.e., $a_1' = a_3' = \mu_1$, homogeneous and free from initial compression, then the dispersive equation (29) takes the form

$$\tan\left[\frac{kH}{Q}\sqrt{R^2+Q\left(\frac{c^2}{c_0^2}-P\right)}\right]=\frac{\mu_1}{\mu_T}\frac{\sqrt{1-\frac{c^2}{c_1^2}}}{\sqrt{R^2+Q\left(\frac{c^2}{c_0^2}-P\right)}},$$

which is the dispersion equation in a fibre-reinforced layer over an isotropic homogeneous half-space without initial stress.

Case V: If $\mu_L=\mu_T=\mu_0$, i.e., the upper layer is isotropic with rigidity μ_0 and the lower half-space is homogeneous, isotropic, i.e., $a_1'=a_3'=\mu_1$ and free from initial stress, equation (29) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_0^2}-1}\right]=\frac{\mu_1}{\mu_0}\frac{\sqrt{1-\frac{c^2}{c_1^2}}}{\sqrt{\frac{c^2}{c_0^2}-1}},$$

This is the well known Love wave equation (Love [29]) in a homogeneous isotropic layer over a homogeneous isotropic half-space.

7. Numerical Results and Discussion

For numerical discussion we used the following relevant parameters in fibre-reinforced layer and orthotropic half-space.

(i) For fibre-rein-forced layer (Hool and Kinne [20])

$$\mu_L=5.66\times10^9\,N/m^2,\ \mu_T=2.46\times10^9\,N/m^2,\ \rho=7800kg/m^3$$

(ii) For the orthotropic half-space (Gubbins [17])

$$a_1'=5.82\times10^{10}\,N/m^2,\ a_3'=3.99\times10^{10}\,N/m^2,\ \rho=4500kg/m^3.$$

To show the effect of different heterogeneity parameters, initial stress and reinforcement on nature of wave motion we have plotted dimensionless phase velocity $\frac{c}{c_0}$ against dimensionless wave number kH on the propagation of Love waves in fibre-reinforced medium. The numerical calculations of phase velocities have been computed from equation (29) for different values of these parameters. The variations are shown in Figures 2 to 8. In all these we have noticed that the phase velocity decreases with the increase in dimensionless wave number.

Figure 2 gives the dispersion curves of Love waves as function of dimensionless wave number in fibre-reinforced medium over a heterogeneous orthotropic half-space. Dispersion curves are plotted for different values of heterogeneity parameter $\frac{\alpha}{k}$ associated with initial stress. The value of $\frac{\alpha}{k}$ for curve no.1, no.2, no.3 and no.4 has been taken as 0.1, 0.2, 0.3 and 0.4 respectively. From these curves it can be realized that phase velocity increases with the decrease of $\frac{\alpha}{k}$. The curves become closer to each other when the value of $\frac{\alpha}{k}$ decreases. So the heterogeneity parameter $\frac{\alpha}{k}$ has much dominance at large values.

Figure 3 manifests the effect of heterogeneity parameter $\frac{\beta}{k}$ associated with density on the phase velocity of Love waves. In this figure the value of $\frac{\beta}{k}$ has been taken as 0.2, 0.4, 0.6 and 0.8 for curve no.1, no.2, no.3 and no.4 respectively. From this figure it has been observed that the increasing value of heterogeneity parameter decreases the phase velocity for a particular frequency. Also the curves are little far apart from each other at higher phase velocity, i.e., the heterogeneity parameter $\frac{\beta}{k}$ has much dominant effect at higher phase velocity and lower wave number.

Figure 4 represents the effect of heterogeneity parameter $\frac{\gamma}{k}$ associated with shear moduli on the phase velocity of Love waves. The value of $\frac{\gamma}{k}$ for curve no.1, no.2, no.3 and no.4 has been considered as 0.05, 0.10, 0.15 and 0.20 respectively. These curves show that the phase velocity of Love waves decreases with the increase of $\frac{\gamma}{k}$. Also it has negligible effect for the higher magnitude.

Figure 5 gives a variation of velocity of Love waves for the variation of compressive initial stress $\frac{P_0}{2a_1'}$ of the half-space in the presence of reinforced parameters. For curve no.1, no.2, no.3 and no.4, the value of $\frac{P_0}{2a_1'}$ has been taken as 0.2, 0.4, 0.6 and 0.8 respectively. It is observed that the phase velocity of Love waves increases with an increase in the compressive initial stress. It has also been noticed that dominant effect on phase velocity is visible at higher magnitude of $\frac{P_0}{2a_1'}$.

In Figure 6, an attempt has been made to study the effect of initial stress parameter in the half-space when the reinforced parameters are neglected in the upper layer, i.e.,

$a_1 = a_3 = 0$. The value of initial stress parameter $\frac{P_0}{2a_1'}$ for curve no.1, no.2, no.3 and no.4, has been taken as 0.2, 0.4, 0.6 and 0.8 respectively. The curves of this figure also show that the phase velocity of Love waves increases with the increase of initial stress parameter $\frac{P_0}{2a_1'}$ in the absence of reinforcement of the upper layer. Here also, the initial stress parameter has much dominance at large values.

Figure 7 gives the dispersion curves for different values of reinforced parameters in the presence of initial stress of the half-space. The values of a_1^2 and a_3^2 for curve no.1, no.2, no.3 and no.4 have been taken as 0.25, 0.30, 0.35, 0.40 and 0.75, 0.70, 0.65, 0.60 respectively. The figure shows that the effect of reinforcement is very prominent on the propagation of Love waves. The phase velocity increases with the decrease of a_1^2 and increase of a_3^2 at a particular frequency.

In Figure 8, the effect of reinforced parameters a_1^2 and a_3^2 in the absence of initial stress of the half-space on Love wave propagation has been shown. For curve no.1, no.2, no.3 and no.4, the values of a_1^2 and a_3^2 have been taken as 0.25, 0.30, 0.35, 0.40 and 0.75, 0.70, 0.65, 0.60 respectively. It shows that such parameters have remarkable effect on Love wave propagation. This figure confirms that the phase velocity increases with the decrease of a_1^2 and increase of a_3^2 at a particular wave number.

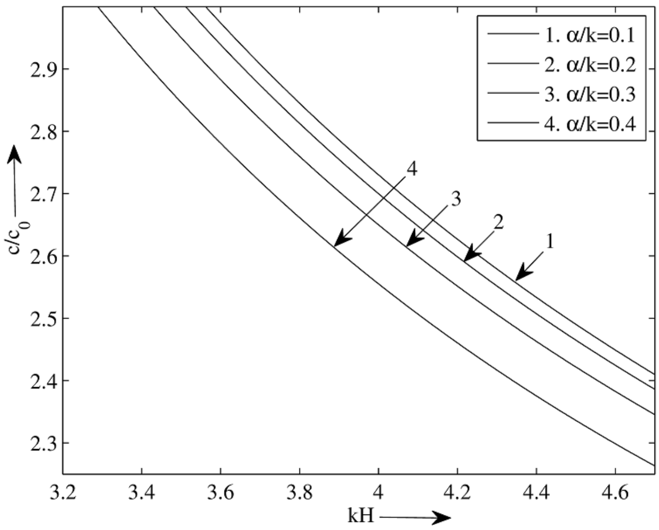

Figure 2: Dimensionless phase velocity as function of dimensionless wave number of Love waves for different values of $\frac{\alpha}{k}$ and for $\frac{\beta}{k} = 0.1, \frac{\gamma}{k} = 0.6, a_1^2 = 0.4, a_3^2 = 0.6, \frac{P_0}{2a_1'} = 0.6.$

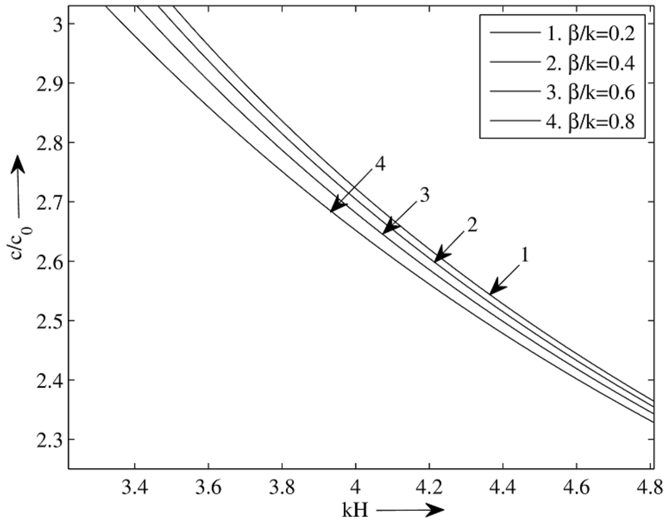

Figure 3**:** Dimensionless phase velocity as function of dimensionless wave number of Love waves for different values of $\frac{\beta}{k}$ and for $\frac{\alpha}{k} = 0.1, \frac{\gamma}{k} = 0.6, a_1^2 = 0.4, a_3^2 = 0.6, \frac{P_0}{2a_1'} = 0.6.$

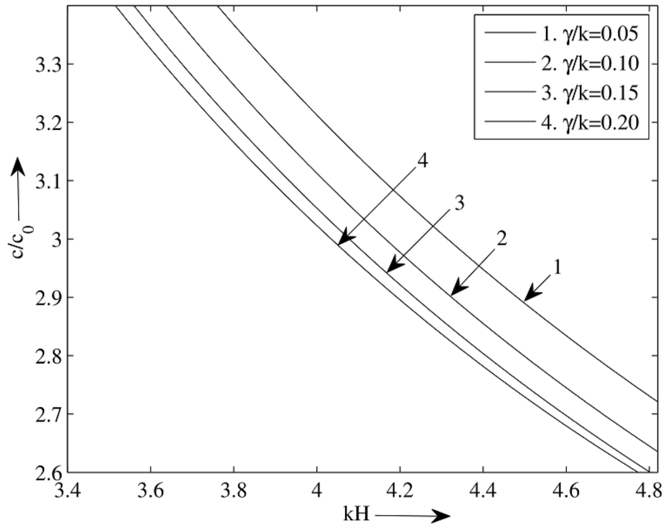

Figure 4**:** Dimensionless phase velocity as function of dimensionless wave number of Love waves for different values of $\frac{\gamma}{k}$ and for $\frac{\alpha}{k} = 0.1, \frac{\beta}{k} = 0.6, a_1^2 = 0.4, a_3^2 = 0.6, \frac{P_0}{2a_1'} = 0.6.$

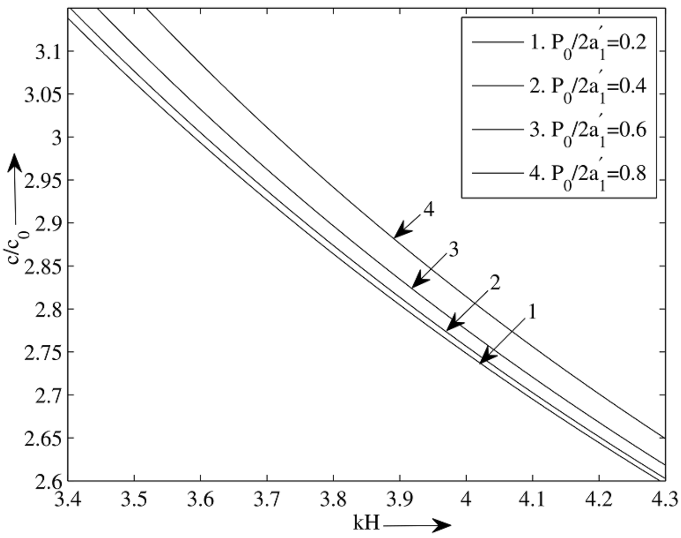

Figure 5: Dimensionless phase velocity as function of dimensionless wave number of Love waves for different values of $\frac{P_0}{2a_1'}$ and for $\frac{\alpha}{k}=0.1, \frac{\beta}{k}=0.1, \frac{\gamma}{k}=0.4, a_1^2=0.4, a_3^2=0.6.$

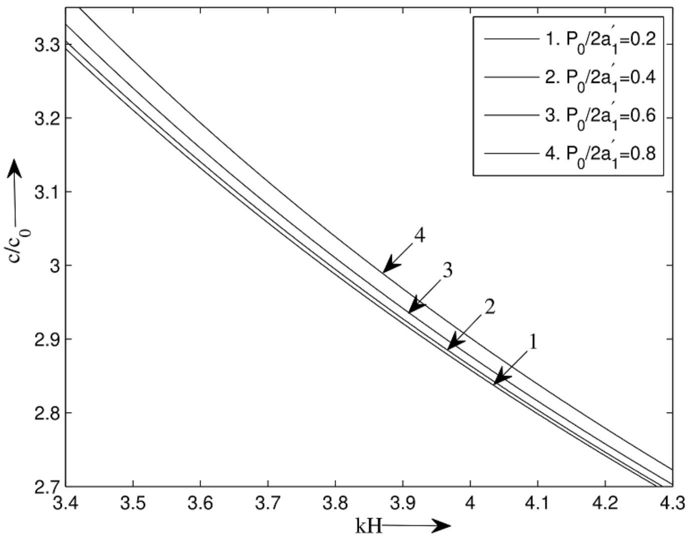

Figure 6: Dimensionless phase velocity as function of dimensionless wave number of Love waves for different values of $\frac{P_0}{2a_1'}$ and for $\frac{\alpha}{k}=0.1, \frac{\beta}{k}=0.1, \frac{\gamma}{k}=0.4, a_1=a_3=0.$

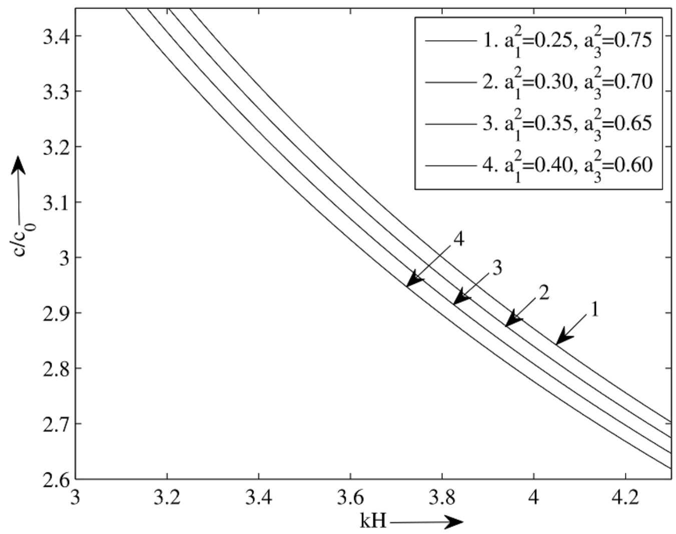

Figure 7: Dimensionless phase velocity as function of dimensionless wave number of Love waves for different values of a_1^2 and a_3^2 and for $\frac{\alpha}{k}=0.1, \frac{\beta}{k}=0.1, \frac{\gamma}{k}=0.4, \frac{P_0}{2a_1'}=0.6.$

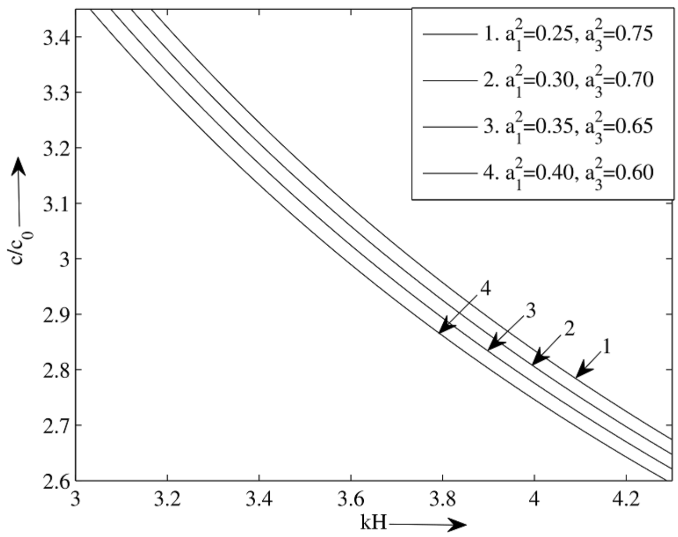

Figure 8: Dimensionless phase velocity as function of dimensionless wave number of Love waves for different values of a_1^2 and a_3^2 and for $\frac{\alpha}{k}=0.1, \frac{\beta}{k}=0.1, \frac{\gamma}{k}=0.4, \frac{P_0}{2a_1'}=0.$

8. Conclusions

An analytic approach is used to investigate the propagation of Love waves in a fibre-reinforced medium over a heterogeneous orthotropic half-space under initial stress. The method of separation of variables is applied to find the displacements in the media and generalized dispersion relation. Some special cases of interest have been deducted from the dispersion equation. When the reinforcement of the upper layer and the initial stress, orthotropy, heterogeneity in the half-space are neglected, the dispersion equation obtained is in agreement with the standard equation of Love waves. For graphical representation, MATLAB software has been used to generalize the results. From the above discussions we may conclude that

(i) Dimensionless phase velocity $\frac{c}{c_0}$ of Love waves increases with the decreases of non-dimensional wave number kH in all the figures.

(ii) An increase in heterogeneity associated with initial stress, density and shear moduli decreases the phase velocity of Love waves.

(iii) The effect of initial stress on the phase velocity of Love waves is significant. In the presence or absence of reinforcement of the layer, the phase velocity of Love waves increases when the initial stress parameter increases.

(iv) The reinforced parameters have also pronounced influence on the propagation of Love waves. The phase velocity of Love waves increases with the decrease of a_1^2 and increase of a_3^2.

There are some hard and soft rocks inside the Earth which show reinforced property and the reinforced materials are basic construction materials, so the wave propagation in reinforced medium plays an important role in civil engineering.

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