Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.19, No.1&2, January-June, 2020, 13-20

# FIVE DIMENSIONAL BIANCHI TYPE-I VISCOUS FLUID SPACE-TIME WITH VACUUM ENERGY DENSITY (Λ) IN GENERAL RELATIVITY

**Mahesh Kumar Yadav, R.K. Gangele and Sanjeev Kumar**
Department of Mathematics and Statistics, Dr. Harisingh Gour Vishwavidyalaya, Sagar – 470003 (M.P.), India
Email: yadav1976mk@gmail.com, rkgangele23@gmail.com, sanjeev.1501.rajpoot@gmail.com

**Abstract**: In this paper, we have investigated the five dimensional Bianchi type-I viscous fluid cosmological model with vacuum energy density in general relativity. To get a deterministic model, we use the conditions between metric potentials with the fact that the scalar expansion is proportional to shear scalar. The co-efficient of bulk viscosity ($\zeta$) is taken constant. The physical and geometrical behavior of our models are discussed.

**Keywords:** Five dimensional Bianchi type-I model, Viscous fluid model, Hubble parameter, General relativity.

## 1. Introduction

To obtain a static model of the universe, the cosmological constant was introduced into Einstein's field equations (Mohajan [9]). The Study of higher dimensional space-time is important, as the cosmos at its early stage of evolution might have had a higher-dimensional era (Singh et al. [19]). Hence for dynamically evolving (4 + k) dimensional manifolds (k being the number of extra dimensions) the five dimensional space-time may be a better choice. Freund [8], Appelquist and Chodos [2], Randjbar-Daem et al. [12] investigated the effect of bulk viscosity and time varying gravitational parameter G on the evolution of a five-dimensional model of the universe within the framework of Lyra geometry. Rahman [13] found that if dimensional reduction of the extra space takes place then the five dimensional universe naturally reduces into an effective four dimensional one. Adhav et al. [1] says that the inclusion of the dark energy into the system with higher dimensions gives rise to an accelerated expansion of the model. Samanta and Debata [16] discussed that the higher dimensional theory at very early stages of the evolution of the universe might be very useful. They found that in the isotropic Bianchi type-I higher dimensional model, cosmic strings do not survive. Bali et al. [4] examined homogeneous and anisotropic Bianchi type-I cosmological model with viscous stiff matter and time-varying cosmological term Λ which scales with Hubble parameter H. Reddy and Naidu [14] studies the model with Nambu string, p-string and Reddy string in the theory in five dimensions. These models are free from initial singularities and these are expanding, anisotropic, shearing, non-rotating and decelerate in the standard way in five dimensional cosmological models generated by a cloud of strings. Samanta et al. [15] constructed five different five-dimensional string cosmological model.

Bali and Singh [5] discussed locally rotationally symmetric Bianchi Type-I massive string cosmological models with bulk viscosity and time varying cosmological term (Λ) (vacuum energy density). Banerjee et al.[6] noted that in higher dimensional cosmology physically realistic models are those in which dimensional reduction occurs when the standard dimensions expand and the extra dimensions shrink with time to zero or Planckian length. Singh et al. [18] presented five dimensional cosmological models in Lyra geometry and shown that empty universe model yield a power law relation between A and B, whereas Rahman and Bera [11] have assumed this type of relation. Pahwa [10] discussed the effect of the extra dimensions is indirectly seen as the accelerated expansion of the scale factor in normal dimensions. Some five dimensional bulk viscous string cosmological models in Lyra geometry proposed by Sen and Dunn [17]. Bali and Bola [3] studied the Bianchi Type-I massive string cosmological model with bulk viscosity and vacuum energy density and found the vacuum energy density (Λ) decreases with time and the model has no singularity.

This motivates us to study five dimensional Bianchi type-I viscous fluid space-time in general relativity.

The organization of the paper is as follows: The field equations of Bianchi type-I are established in section 2. In section 3, we present the solution of the field equations. The physical and geometrical properties of the model are discussed in section 4.

## 2. The Metric and Field Equations

We consider the five dimensional Bianchi type-I line element in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 + D^2du^2, \tag{1}$$

where A, B, C and D are the functions of cosmic time t only. Here the extra co-ordinate is taken to be space like.

We use the expression for the Ricci tensor in the following form:

$$R_{ij} = \frac{\partial^2 \ln\sqrt{-g}}{\partial x^i\,\partial x^j} - \frac{\partial\,\Gamma^l_{ij}}{\partial\, x^l} + \Gamma^m_{in}\,\Gamma^n_{jm} - \Gamma^l_{ij}\,\frac{\partial \ln\sqrt{-g}}{\partial x^l}\,. \tag{2}$$

The energy-momentum for viscous fluid distribution is given by

$$T^j_i = (\in + p)v_i\,v^j + pg^j_i - \eta\left(v_{i;}{}^{j} + v^j{}_{;i} + v^j v^l v_{i;l} + v_i\,v^l\,v^j_{;l}\right) - \left(\zeta - \frac{2}{3}\eta\right)v^l_{;l}\,(g^j_i + v_i\,v^j)\,, \tag{3}$$

Where, $\zeta$ and $\eta$ the coefficient of viscosity, p be the pressure and $v^i$ is the flow vector together with

$$g_{ij}\ v^i v^j = -1. \tag{4}$$

The coordinates are considered to be co-moving, thus

$$= v^2 = v^3 = v^4 = 0\ and\ v^5 = 1. \tag{5}$$

The Einstein's field equations are

$$R_{ij} - \frac{1}{2} g_{ij}\ R + \Lambda\, g_{ij} = -8\pi\, T_{ij}\ . \tag{6}$$

For metric (1), the field equation takes the following forms

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{D_{44}}{D} + \frac{B_4 C_4}{BC} + \frac{B_4 D_4}{BD} + \frac{C_4 D_4}{CD} + \Lambda = -8\pi \left[ p - 2\eta \frac{A_4}{A} - \left( \zeta - \frac{2}{3}\,\eta \right) \theta \right], \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{D_{44}}{D} + \frac{A_4 C_4}{AC} + \frac{A_4 D_4}{AD} + \frac{C_4 D_4}{CD} + \Lambda = -8\pi \left[ p - 2\eta \frac{B_4}{B} - \left( \zeta - \frac{2}{3}\,\eta \right) \theta \right], \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{D_{44}}{D} + \frac{A_4 B_4}{AB} + \frac{A_4 D_4}{AD} + \frac{B_4 D_4}{BD} + \Lambda = -8\pi \left[ p - 2\eta \frac{C_4}{C} - \left( \zeta - \frac{2}{3}\,\eta \right) \theta \right], \tag{9}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} + \Lambda = -8\pi \left[ p - 2\eta \frac{D_4}{D} - \left( \zeta - \frac{2}{3}\,\eta \right) \theta \right], \tag{10}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{A_4 D_4}{AD} + \frac{B_4 C_4}{BC} + \frac{B_4 D_4}{BD} + \frac{C_4 D_4}{CD} + \Lambda = 8\pi\epsilon, \tag{11}$$

where the sub indices 4 in A, B, C and D denotes ordinary differentiation with respect to t.

To discuss the kinematical properties of models, we also evaluate the average Hubble parameter (H), the expansion scalar (θ), the shear scalar (σ) and the anisotropy parameter ( $\bar{A}$ ) for the metric. These quantities are

$$H\ \frac{1}{4} \sum_{i=1}^{4} \mathrm{H_i}\ , \tag{12}$$

$$\theta = 4\,H = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} + \frac{D_4}{D}\,, \tag{13}$$

$$\sigma^2 = \frac{1}{2}\ \sigma^{ij}\,\sigma_{ij} = \frac{1}{2} \left( \sum_{i=1}^{4} {H_i}^2 - \frac{\theta^2}{4} \right), \tag{14}$$

$$\bar{A} = \frac{1}{4} \sum_{i=1}^{4} \left( \frac{\Delta H_i}{H} \right)^2, \tag{15}$$

where $\mathrm{H_1} = \frac{A_4}{A}$ , $\mathrm{H_2} = \frac{B_4}{B}$, $\mathrm{H_3} = \frac{C_4}{C}$ and $\mathrm{H_4} = \frac{D_4}{D}$ and $\Delta = \mathrm{H_i} - \mathrm{H}$.

## 3. Solutions of the Field Equations

The field equations (7) - (11) are five non-linear differential equations with ten unknown parameters A, B, C, D, p, ϵ, **η**, ζ, θ and Λ. We need some additional conditions to obtain explicit solution of the system. In our study the co-efficient of bulk viscosity ($\zeta$) is taken as a constant. Also we assume that A = m B, D = BC and $\propto \eta$ , where m is constant.

Applying these in (7) – (11), we obtain

$$2\ \frac{B_{44}}{B} + 2\ \frac{C_{44}}{C} + \left( \frac{B_4}{B} \right)^2 + \left( \frac{C_4}{C} \right)^2 + 5\ \frac{B_4 C_4}{BC} + \Lambda = -8\pi \left[ p - 2\eta \frac{B_4}{B} - \left( \zeta - \frac{2}{3}\,\eta \right) \theta \right], \tag{16}$$

$$3\ \frac{B_{44}}{B}+\frac{C_{44}}{C}+3\left(\frac{B_4}{B}\right)^2+4\ \frac{B_4C_4}{BC}+\ \Lambda\ =-8\pi\ \left[p-2\eta\frac{C_4}{C}-\left(\zeta-\frac{2}{3}\ \eta\right)\theta\right], \tag{17}$$

$$2\ \frac{B_{44}}{B}+\ \frac{C_{44}}{C}+\left(\frac{B_4}{B}\right)^2\ +2\ \frac{B_4C_4}{BC}+\ \Lambda\ =-8\pi\ \left[p-2\eta\left(\frac{C_4}{C}\ +\frac{B_4}{B}\right)-\left(\zeta-\frac{2}{3}\ \eta\right)\theta\right], \tag{18}$$

$$3\ \left(\frac{B_4}{B}\right)^2+\left(\frac{C_4}{C}\right)^2+5\ \frac{B_4C_4}{BC}+\Lambda\ =\ 8\pi\epsilon\ . \tag{19}$$

Solving these field equations, we get

$$A=m\ B=m\ k_4\ [(\alpha\ +1\ )(k_5t\ +\ k_6\ )]^{\frac{k_3}{\alpha+1}}, \tag{20}$$

$$B=k_4\ [(\alpha\ +1\ )(k_5t\ +\ k_6\ )]^{\frac{k_3}{\alpha+1}}, \tag{21}$$

$$C=[(\alpha\ +1\ )(k_5t\ +\ k_6\ )]^{\frac{1}{\alpha+1}}, \tag{22}$$

$$D\ =k_4\ [(\alpha\ +1\ )(k_5t\ +\ k_6\ )]^{\frac{k_3+1}{\alpha+1}}, \tag{23}$$

where, $\alpha=(16\,\pi\, l+1\,)(3\,k_3+\ 2)$, $k_3$, $k_4$, $k_5$ and $k_6$ are constants.

Hence the cosmological model (1) reduces to

$$ds^2\ =-dt^2+\ m^2\ k_4^2\ [(\alpha\ +1\ )(k_5t\ +\ k_6\ )]^{\frac{2k_3}{\alpha+1}}dx^2\ +\ k_4^2\ [(\alpha\ +1\ )(k_5t\ + k_6\ )]^{\frac{2k_3}{\alpha+1}}dy^2+\ [(\alpha\ +1\ )(k_5t\ +\ k_6\ )]^{\frac{2}{\alpha+1}}\ dz^2\ + k_4^2\ [(\alpha\ +1\ )(k_5t\ +\ k_6\ )]^{\frac{2(k_3+1)}{\alpha+1}}\ du^2. \tag{24}$$

**4. The Physical and Geometrical Properties**

The directional Hubble parameters and the average Hubble parameter are

$$H_1=\ H_2\ =\ \frac{k_5k_3}{(\alpha+1)(k_5t+k_6)}, \tag{25}$$

$$H_3\ =\ \frac{k_5}{(\alpha+1)(k_5t+k_6)}, \tag{26}$$

$$H_4\ =\ \frac{k_5(\ k_3+1)}{(\alpha+1)(k_5t+k_6)}, \tag{27}$$

$$H\ =\ \frac{k_5(3\ k_3+2)}{4\ (\alpha+1)(k_5t+k_6)}. \tag{28}$$

The expansion (θ) for the model (24) is

$$\theta\ =4H=\ \frac{k_5(3\ k_3+2)}{(\alpha+1)(k_5t+k_6)}, \tag{29}$$

and the shear scalar (σ ) is

$$\sigma^2\ =\ \frac{k_5^2(\frac{3}{4}K_3^2-k_3\ +1)}{2\left((\alpha+1)(k_5t+k_6)\right)^2}. \tag{30}$$

The Anisotropic parameter is

$$\overline{A} = \frac{3\,k_3^2 - 4k_3 + 16}{(3\,k_3 t + 2)^2}. \tag{31}$$

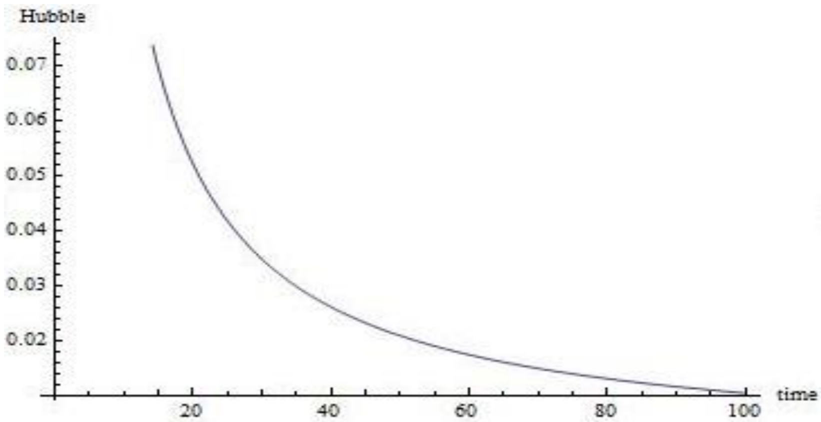


**Fig.1** Hubble parameter versus cosmic time. Here$k_5 = 1;\ \alpha = 0.2\ and\ k_3 = 1$

The energy density of the model (24) is

$$8\pi\epsilon \ = \ k_5^2(3\,k_3^2 + \ 5\,k_3 + \ 1)\big((\alpha+1)(k_5 t + k_6)\big)^{-2} + \Lambda. \tag{32}$$

The pressure of the model (24) becomes

$$8\pi p \ = k_5^2[(\alpha+1)(k_5 t + k_6)]^{-2}\Big[-3k_3^2 + \ 2\,\alpha(\,k_3 + \ 1) - 3k_3 - \ 1 - \frac{32}{3}\pi l(3k_3 + 2)\Big] + \ 8\pi k_5\,\zeta(3k_3 + \ 2)[(\alpha+1)(k_5 t + k_6)]^{-1} \ - \ \Lambda\,. \tag{33}$$

The energy conditions give by Ellis [7] are (i) ($\epsilon$ +p) > 0 and (ii) ($\epsilon$ +3p) > 0, which leads to

$$k_5^2[(\alpha+1)(k_5 t + k_6)]^{-2}\Big[2\,k_3 + \ 2\,\alpha(\,k_3 + \ 1) - \frac{32}{3}\pi l(3k_3 + \ 2)\Big] + \ 8\pi k_5\,\zeta(3k_3 + 2)[(\alpha+1)(k_5 t + k_6)]^{-1} > 0, \tag{34}$$

and

$$k_5^2[(\alpha+1)(k_5 t + k_6)]^{-2}[-6\,k_3^2 - 4k_3^2 + \ 6\,\alpha(\,k_3 + \ 1) - \ 32\pi l(3k_3 + \ 2)] + 24\pi k_5\,\zeta(3k_3 + \ 2)[(\alpha+1)(k_5 t + k_6)]^{-1} > 2\Lambda\,. \tag{35}$$

**5. The Special Model**

In particular case, the proper choice of coordinates and constants (i.e., choosing $k_5$ = 1and $k_6$ = 0) the model (33) reduces to

$$ds^2 \ = \ -\,dt^2 + \ m^2\,k_4^2\,[(\alpha\ +1\,)t]^{\frac{2k_3}{\alpha+1}}\,dx^2 \ + \ k_4^2\,[(\alpha\ +1\,)t]^{\frac{2k_3}{\alpha+1}}dy^2 + \ [(\alpha\ + 1\,)t]^{\frac{2}{\alpha+1}}\,dz^2 + k_4^2\,[(\alpha\ +1\,)t]^{\frac{2(k_3+1)}{\alpha+1}}\,du^2. \tag{36}$$

The Hubble parameter (H), expansion (θ) and shear (σ) of the model (36) are given by

$$H = \frac{(3\,k_3 + 2)}{4\,(\alpha + 1)t}, \tag{37}$$

$$\theta = 4H = \frac{(3\,k_3 + 2)}{(\alpha + 1)\,t}, \tag{38}$$

$$\sigma^2 = \frac{(\frac{3}{4}k_3^2 - k_3 + 1)}{2(\,(\alpha + 1)\,t)^2}. \tag{39}$$

The Anisotropic parameter is given as

$$\overline{\mathrm{A}} = \frac{3\,k_3^2 - 4k_3 + 16}{(3\,k_3 t + 2)^2}. \tag{40}$$

The expressions for density and pressure for the model (36) are given by

$$8\pi\epsilon = (3\,k_3^2 + 5\,k_3 + 1)\big((\alpha + 1)\,t\big)^{-2} + \Lambda\,, \tag{41}$$

and

$$8\pi p = [(\alpha + 1)\,t\,]^{-2}\left[-3k_3^2 + 2\,\alpha(\,k_3 + 1) - 3k_3 - 1 - \frac{32}{3}\pi l(3k_3 + 2)\right] + 8\pi\,\zeta(3k_3 + 2)[(\alpha + 1)\,t\,]^{-1} - \Lambda\,. \tag{42}$$

The energy condition (i) leads to

$$[(\alpha + 1)\,t\,]^{-2}\left[2\,k_3 + 2\,\alpha(\,k_3 + 1) - \frac{32}{3}\pi l\,(3k_3 + 2)\right] + 8\pi\,\zeta(3k_3 + 2)[(\alpha + 1)\,t\,]^{-1} > 0, \tag{43}$$

and the energy condition (ii) leads to

$$[(\alpha + 1)\,t\,]^{-2}[-6\,k_3^2 - 4k_3^2 + 6\,\alpha(\,k_3 + 1) - 32\pi l(3k_3 + 2)] + 24\pi\,\zeta(3k_3 + 2)\,[(\alpha + 1)\,t\,]^{-1} > 2\Lambda\,. \tag{44}$$

## 6. Conclusion

In this paper, we have investigated a five dimensional Bianchi type-I space-time with a viscous fluid in the framework of general relativity. Our models (24) and (36) are expanding, sheering and non-rotating. It is observed that for $t \to 0$ the value of θ and σ both diverge, and for large value of t, the parameters, scalar expansion (θ) and shear scalar (σ) vanish.

The value of Hubble parameter (H) for the model (24) decreases when time increases and finally vanishes at $t \to \infty$ (Fig.1). The energy density also decreases as time increases. The energy density becomes zero for late time, thus the model results in an empty universe. The pressure (p) of the model decreases when time increase. The energy conditions (i) $(\epsilon + p) > 0$ and (ii) $(\epsilon + 3p) > 0$ are satisfied.

The model (36) starts with a big-bang at t = 0, where - 1 < α < 1 and the expansion in the model decreases as time increases. The expansion in the model stops at $t \to \infty$. The shear scalar is non-zero for t > 0 and becomes infinitely large as $t \to \infty$. Thus the universe is not shear-free at infinite time. When t → 0 then $\epsilon \to \infty$, $p \to \infty$ and when $t \to \infty$ then $\epsilon \to \Lambda$, $p \to -\Lambda$. Hence the model represents a realistic scenario.

It is observed that

$$\lim_{t\to\infty} \frac{\sigma}{\theta} = \frac{1}{(3K_3+2)} \sqrt{\frac{1}{2}\left(\frac{3}{4}\, k_3^2 - k_3 + 1\right)} \neq 0 . \qquad (45)$$

The condition (45) indicates that the model (24) does not approach isotropy. At the time of evolution, the anisotropy of the universe is constant. Thus the Universe remains anisotropic throughout the evolution.

**Acknowledgements:** The authors are thankful to referee for valuable comments and suggestions.

**References**

[1] Adhav, K.S., Bansod, A.S., Munde, S.L. and Desale, M.S. (2011). Higher dimensional Bianchi type-V cosmological models with perfect fluid and dark energy, International Journal of Theoretical Physics, **50**, 2573-2581.

[2] Appelquist, T. and Chodos, A. (1983). Quantum effects in Kaluza-Klein theories, Physical Review Letter, **50**, 141-145.

[3] Bali, R. and Bola, S.C. (2011). Bianchi Type-I Massive String Cosmological Model with Bulk Viscosity and Vacuum Energy Density, International Journal of Applied Science and Mathematics, **3(6)**, 2394-2894.

[4] Bali, R., Singh, P. and Singh, J.P. (2012). Viscous Bianchi Type-I Universe with Stiff Matter and Decaying Vacuum Energy Density, International Scholarly Research Network, **2012**, 11 pages, Article ID 704612,

[5] Bali, R. and Singh, S. (2016). Locally Rotationally Symmetric Bianchi Type-I Massive String Cosmological Models with Bulk Viscosity and Decaying Vacuum Energy Density , Advances in Astrophysics, **1(2)**, 113-121.

[6] Banerjee, A., Bhui, B. and Chatterjee, S. (1990). Bianchi type-I cosmological models in higher dimensions, The Astrophysical Journal, **358**, 23-27.

[7] Ellis, G.R.F. (1971). General relativity and cosmology edited by R K Sachs, Academic Press, **125**, 117.

[8] Freund, P.O.G. (1982). Kaluza-Klein cosmologies, Nuclear Physics B, **209**, 146- 156.

[9] Mohajan, H.K. (2017). Development of Einstein's static cosmological model of the universe, Journal of Scientific Achievements, **2(7)**, 18-29.

[10] Pahwa, I. (2014). Higher dimensional cosmological models: An alternative explanation for late time cosmic acceleration, Journal of Physics, Conference Series, **01**, 484-489.

[11] Rahman, F. and Bera, J. (2001). Higher dimensional cosmological model in Lyra geometry, International Journal of Modern Physics D, **10,** 729-733.

[12] Randjbar-Daemi, S., Salaus, A. and Strathdee, J. (1984). On Kaluza-Klein cosmology, Physics Letter B, **135**, 388-392.

[13] Rahaman, F., Bhui, B.C. and Bhui, B. (2006). Cosmological Model with a Viscous Fluid in a Kaluza-Klein Metric, Astrophysics and Space Science, **301**, 47-49.

[14] Reddy, D.R.K. and Naidu, R.L. (2007). Five dimensional string cosmological models in a scalar-tensor theory of gravitation, Astrophysics and Space Science, **307**, 395-398.

[15] Samanta, G.C., Biswal, S.K. and Mohanty, G. (2011). Some Five-dimensional Bianchi type-III string cosmological models in general relativity, Bulgarian Journal of Physics, **38**, 380-389.

[16] Samanta, G.C., and Debata, S. (2012). Five dimensional Bianchi type-I string cosmological models in Lyra manifold, Journal of Modern Physics, **3**, 180-183.

[17] Sen, D.K. and Dunn, K.A. (1971). A scalar tensor theory of gravitation in a modified Riemannian manifold, Journal of Mathematical Physics, **12**, 578-586.

[18] Singh, G.P., Kotambar, S. and Anirudh (2003). Higher dimensional cosmological model in Lyra geometry: Revisited, International Journal of Modern Physics D, **12**, 853-860.

[19] Singh, G.P., Deshpande, R.V. and Singh, T. (2004). Higher dimensional cosmological model with variable gravitational constant and bulk viscosity in Lyra geometry, Pramana journal of physics, **63**, 937-945.