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LATE-TIME ACCELERATION IN BIANCHI TYPE V PERFECT FLUID COSMOLOGICAL MODELS

Abhay Singh[1], Prashant S. Baghel[2] and J. P. Singh[2]
[1]Department of Mathematics, Technocrats Institute of Technology and Science, Bhopal – 462022, India
Email: abhaysingh5784@gmail.com
[2]Department of Mathematical Sciences, A.P.S. University, Rewa 486003, India
Email: jpsinghmp@gmail.com, prashantsbaghel@gmail.com

Abstract: Perfect fluid Bianchi type-V cosmological models with time varying cosmological parameter Λ are investigated for a specific Hubble parameter. Exact solutions of Einstein's field equations yield models of the universe which represent initially decelerating and late-time accelerating expansion.

Keywords: Bianchi type V · Hubble Parameter · Variable cosmological term ·Deceleration parameter.

1. Introduction

Observational evidences from the Hubble diagram of SNe Ia Supernovae [11,14,15,24,25], Cosmic Microwave Background Radiation (CMBR) and Baryon Acoustic Oscillation (BAO) [1,5,7,10,21,22,23] favour the scenario that our universe is currently expanding with acceleration. This currently observed acceleration could be brought about by some exotic field with an effective negative pressure. Astronomical observations show that out of total energy budget, our universe at present contains approximately 4% only of radiation and baryonic matter [13,16,26]. While about 26% is the non-baryonic dark matter, the rest of our universe content, which is about 70% is the exotic component known as dark energy. This dominant component with repulsive gravitation leads to the present accelerating expansion of the universe. There are many theoretical models to describe the nature of dark energy but the first choice of candidates for this dark energy has been the cosmological constant Λ. In Einstein's field equations, this term has the concept of intrinsic energy density of vacuum [8]. However, this choice suffers the fine-tuning problem: why the observed value of vacuum energy density is very far below that is predicted from particle physics. The cosmological constant Λ has also been interpreted in terms of Higgs scalar field [28]. It has also been proposed as a function of time [12]. Many researchers suggested the idea of a decreasing vacuum energy by taking into account the conservation of the matter and vacuum taken together. Hence a varying cosmological term Λ with cosmic expansion in the framework of general relativity has been investigated by a number of researchers. We consider the following four Λ-decay scenarios which have been discussed in literature:

Case1: $\Lambda \sim H^2$

Case2: $\Lambda \sim H$

Case3: $\Lambda \sim \frac{\ddot{S}}{S}$

Case4: $\Lambda \sim \rho$.

Here H, S and ρ are respectively the Hubble parameter, average scale factorand matter energy density.

In this paper, we intend to discuss the above mentioned Λ-decay scenarios in the background of Bianchi type-V space-time with perfect fluid distribution. Exact solutions of Einstein's field equations have been obtained by considering a specific form of Hubble parameter *H* that yields a deceleration parameter *q* which has positive value in the early universe, and it becomes negative at late times. We have discussed cosmological consequences of the models obtained.

2 Metric and Field Equations

We consider Bianchi type-V space-time in orthogonal form represented by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2\alpha x}\{B^2(t)dy^2 + C^2(t)dz^2\}, \tag{1}$$

where α is constant.

Energy-momentum tensor T_{ij} for the perfect fluid distribution has the form

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} \tag{2}$$

where ρ is the matter density, p, the isotropic pressure and v_i is the four velocity vector of the fluid satisfying $v_i \; v^i$ = -1. In commoving coordinates $v^i = \delta_4^i$, we have

$$T_1^1 = T_2^2 = T_3^3 = p, T_4^4 = -\rho. \tag{3}$$

Einstein's field equations (in gravitational units $8\pi G$ = c = 1) with time-dependent cosmological term Λ are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} + \Lambda(t)g_{ij}. \tag{4}$$

For the line element (1), the field equations (4) lead to

$$p - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \tag{5}$$

$$p - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{C}}{AC}, \tag{6}$$

$$p - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB}, \tag{7}$$

$$\rho + \Lambda = -\frac{\alpha^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}, \tag{8}$$

$$2\frac{\dot{A}}{A} = \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{9}$$

where overhead dot (.) indicates time derivatives.

From equations (5)-(9), we obtain

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 0. \tag{10}$$

When Λ is constant, we recover the usual equation of continuity for matter.

Equation (10) shows that in order to satisfy the energy conservation, a decaying vacuum term necessarily leads to matter production.

The average scale factor S for the Bianchi type-Vmodel (1) is defined as

$$S = (ABC)^{\frac{1}{3}}. \tag{11}$$

The volume scale factor *V* is given by

$$V = S^3 = ABC. \tag{12}$$

From equations (5)-(7) together with (9), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S}, \tag{13}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} - \frac{k}{S^3}, \tag{14}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} + \frac{k}{S^3}, \tag{15}$$

where k is a constant.

Generalized Hubble parameter H and generalized declaration parameter q are

$$H = \frac{\dot{S}}{S} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{16}$$

and

$$q = -\frac{\dot{H}}{H^2} - 1, \tag{17}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors along spatial directions x, y and z respectively.

We introduce volume expansion θ and shear scalar σ as usual

$$\theta = v^i_{;i} \tag{18}$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{19}$$

where σ_{ij} is the shear tensor defined by

$$\sigma_{ij} = \frac{1}{2}(v_{i;k}h^k_j + v_{j;k}h^k_i) - \frac{1}{3}\theta h_{ij} \tag{20}$$

Here $h_{ij} = g_{ij} + v_i v_j$ is the projection tensor. Semicolon (;) stands for covariant derivative. For the line element (1) expressions for the dynamical quantities come out to be

$$\theta = 3\frac{\dot{S}}{S}, \tag{21}$$

$$\sigma^1_1 = H_1 - H, \tag{22}$$

$$\sigma^2_2 = H_2 - H, \tag{23}$$

$$\sigma_3^3 = H_3 - H\ , \tag{24}$$

$$\sigma_4^4 = 0\,. \tag{25}$$

Shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\left[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2\right] \tag{26}$$

Therefore

$$\sigma = \frac{k}{S^3}. \tag{27}$$

Equations (5)-(10) can be rewritten in terms of H, σ and q as

$$p - \Lambda = \frac{\alpha^2}{S^2} + (2q-1)H^2 - \sigma^2, \tag{28}$$

$$\rho + \Lambda = -\frac{3\alpha^2}{S^2} + 3H^2 - \sigma^2, \tag{29}$$

$$\dot{\rho} + \dot{\Lambda} + 3H(\rho + p) = 0. \tag{30}$$

From equation (29), we observe that in Bianchi type-V space-time, energy density ρ is smaller than the corresponding energy density in Bianchi type-Ispace time indicating that Bianchi type V is a low density universe. Also, from (29), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{\rho}{\theta^2} - 3\frac{\alpha^2}{R^2\theta^2} - \frac{\Lambda}{\theta^2}. \tag{31}$$

Therefore, $0 < \dfrac{\sigma^2}{\theta^2} < \dfrac{1}{3}$ for $\Lambda > 0$. Thus a positive Λ puts restriction on the upper limit of anisotropy whereas a negative Λ provides more room for the anisotropy.

From equations (28) and (29), we have

$$\frac{d\theta}{dt} = -\frac{1}{2}(\rho + 3p) - 2\sigma^2 - \frac{\theta^2}{3} + \Lambda. \tag{32}$$

which is Raychaudhari equation for given distribution. We observe that a positive Λ will slow down the rate of decrease.

3. Solutions and Discussion

For the solution of Einstein's field equations, we normally assume a form for the matter content of the universe or a space-time admitting Killing vector symmetries. One can also generate the solutions by applying a law of variation for Hubble's parameter H. Berman [6] proposed Hubble parameter H related the average scale factor for FRW models that yields a constant value of deceleration parameter which has either positive or negative value. Thus, the universe is either decelerating or accelerating throughout the evolution. It is to mention that our model of the universe should have a deceleration in the early phase of matter era for the structure formation followed by late-time accelerated phase suggested by the observations in the last decades [11,14,15,24,25]. Therefore, the deceleration parameter q should have sign flip i.e. positive value in the early phase of the universe and negative value at late times. We assume the Hubble parameter H in the form

$$H = m + n\coth t, \tag{33}$$

where m(> 0) and n(> 0) are constants. It gives rise to a model of the universe with an early deceleration and late-time acceleration. Singh [19,20], Bali, et al. [3], Bali and Singh [4] have also investigated law of variation of Hubble parameter/average scale factor describing models of the universe with initial deceleration and late-time acceleration. For this assumption, we obtain scale factor *S*, spatial volume *V*, expansion scalar θ and deceleration parameter *q* as

$$S = (\sinh t)^n e^{mt}, \tag{34}$$

$$V = (\sinh t)^{3n} e^{3mt}, \tag{35}$$

$$\theta = 3(m + n\coth t), \tag{36}$$

$$q = -1 + \frac{n}{(m\sinh t + n\cosh t)^2}. \tag{37}$$

We observe that scale factor *S*is zero t = 0 and expansion scalar θ is infinite at *t = 0*, which shows that the universe begins with a big-bang. Volume expansion θ decreases as time t increases. At *t = 0*, $q = -1 + \frac{1}{n} > 0$ provided 0< n < 1 and for $t = \infty, q = -1$. Thus, the model universe represents initial decelerating and late time accelerating expansion.

For the model, anisotropy

$$\frac{\sigma}{\theta} = \frac{k}{3(m + n\coth t)(\sinh t)^{3n} e^{3mt}}. \tag{38}$$

We observe that $\frac{\sigma}{\theta}$ is infinitely large at initial moment provided $\frac{1}{3} < n < 1$. It decreases with cosmic time to vanish for $t \to \infty$. Therefore, the model exhibits isotropic behaviour at late times of its evolution.

To determine ρ; p and Λ explicitly from the above two equations (28) and (29), we require one more relation. In cosmology, it is common to assume that ρ and *p* are related by a linear equation of state

$$p = \omega\rho, \; 0 \le \omega \le 1. \tag{39}$$

Then matter density ρ and cosmological term Λ are obtained as

$$(1+\omega)\rho = 2n \operatorname{csch}^2 t - \frac{2k^2}{(\sinh t)^{6n} e^{6mt}} - \frac{2\alpha^2}{(\sinh t)^{2n} e^{2mt}}, \tag{40}$$

$$\Lambda = 3(m + n\coth t)^2 - \frac{2n \operatorname{csch}^2 t}{1+\omega} - \frac{(1-\omega)2k^2}{(1+\omega)(\sinh t)^{6n} e^{6mt}} - \frac{(1+3\omega)\alpha^2}{(1+\omega)(\sinh t)^{2n} e^{2mt}}. \tag{41}$$

We observe that the model has singularity at $t = 0$. It evolves from its singular state at $t = 0$ with ρ and Λ infinite. For large values of t, matter density ρ becomes zero but cosmological term $\Lambda \approx 3(m+n)^2$. Thus, our modeltends asymptotically to a deSitter universe with $H = \sqrt{\frac{\Lambda}{3}} = m + n$ for large values of t.

We can also determine ρ, *p* and Λ by assuming one of the following cases of the phenomenological decay of Λ.

3.1 Case 1:

We assume

$$\Lambda = 3\beta H^2 \tag{42}$$

where β is constant [9,27]. For this choice, we obtain matter density ρ, isotropic pressure p and cosmological term Λ as

$$\rho = 3(1-\beta)(m + n\coth t)^2 - \frac{k^2}{(\sinh t)^{6n} e^{6mt}} - \frac{3\alpha^2}{(\sinh t)^{2n} e^{2mt}}, \tag{43}$$

$$p = \frac{\alpha^2}{(\sinh t)^{2n} e^{2mt}} - 3(1-\beta)(m + n\coth t)^2 + 2n\,\mathrm{csc}\,h^2 t - \frac{k^2}{(\sinh t)^{6n} e^{6mt}}, \tag{44}$$

$$\Lambda = 3\beta(m + n\coth t)^2. \tag{45}$$

This model starts expanding with a big-bang at t = 0 where ρ, p and Λ all diverge. In the limit of large times (i.e. $t \to \infty$), $\rho \to 3(1-\beta)(m+n)^2$, $p \to -3(1-\beta)(m+n)^2$ and $\Lambda \to 3(m+n)^2$. Matter density ρ and cosmological term Λ decrease in the course of expansion to become constant at late times. For the large values of *t*, $\rho + p \to 0$. Therefore, the model is dominated by vacuum energy at late times.

3.2 Case 2:

In this case, we consider

$$\Lambda = \Lambda_0 H, \tag{46}$$

where Λ_0 is constant [17,18].

For this assumption, we get matter density ρ, pressure p and vacuum energy density Λ as

$$\rho = 3(m + n\coth t)^2 - \frac{k^2}{(\sinh t)^{6n} e^{6mt}} - \frac{3\alpha^2}{(\sinh t)^{2n} e^{2mt}} - \Lambda_0 (m + n\coth t), \tag{47}$$

$$p = 2n\,\mathrm{csc}\,h^2 t + \frac{\alpha^2}{(\sinh t)^{2n} e^{2mt}} + \Lambda_0 (m + n\coth t) - 3(m + n\coth t)^2 - \frac{k^2}{(\sinh t)^{6n} e^{6mt}}, \tag{48}$$

$$\Lambda = \Lambda_0 (m + n\coth t). \tag{49}$$

This model also has singularity at t = 0. The model starts evolving with a big-bang from its singular state at t = 0 with ρ, p and Λ all infinite. In the limit of large times (i.e. $t \to \infty$), $\rho + p \to 0$ and $\Lambda \to \Lambda_0 (m+n)$. Therefore, at late times, vacuum energy dominates.

3.3 Case 3:

We assume

$$\Lambda = \Lambda_0 \frac{\ddot{S}}{S}, \tag{50}$$

where $\Lambda_0 > 0$ is constant as considered by Arbab [2]. For this choice, we obtain

$$\rho = (3-\Lambda_0)(m+n\coth t)^2 + \Lambda_0 n \operatorname{csc} h^2 t - \frac{k^2}{(\sinh t)^{6n} e^{6mt}} - \frac{3\alpha^2}{(\sinh t)^{2n} e^{2mt}}, \tag{51}$$

$$p = \frac{\alpha^2}{(\sinh t)^{2n} e^{2mt}} - (3-\Lambda_0)(m+n\coth t)^2 + n(2-\Lambda_0)\operatorname{csc} h^2 t - \frac{k^2}{(\sinh t)^{6n} e^{6mt}}, \tag{51}$$

$$\Lambda = \Lambda_0 (m+n\coth t)^2 - n\Lambda_0 \operatorname{csc} h^2 t. \tag{53}$$

The model has singularity at t = 0. At t = 0, ρ, p and Λ are all infinite. For the large values of t, $\rho \to (3-\Lambda_0)(m+n)^2$, $p \to -(3-\Lambda_0)(m+n)^2$ and $\Lambda \to \Lambda_0 (m+n)^2$. We observe that the cosmological term Λ is very large at initial times. It reduces to genuine constant at late times. For large values of t, $\rho + p \to 0$ indicating the dominance of vacuum energy.

3.4 Case 4:

In this case, we consider

$$\Lambda = \Lambda_0 \rho, \tag{55}$$

where $\Lambda_0 > 0$ is constant [27]. For this assumption, we obtain

$$(1+\Lambda_0)\rho = 3(m+n\coth t)^2 - \frac{k^2}{(\sinh t)^{6n} e^{6mt}} - \frac{3\alpha^2}{(\sinh t)^{2n} e^{2mt}}, \tag{56}$$

$$p = -\left(\frac{3\Lambda_0}{1+\Lambda_0}\right)(m+n\coth t)^2 + 2n \operatorname{csc} h^2 t - \frac{3(\Lambda_0 - 2)\alpha^2}{(1+\Lambda_0)(\sinh t)^{2n} e^{2mt}} - \frac{(\Lambda_0 + 2)k^2}{(1+\Lambda_0)(\sinh t)^{6n} e^{6mt}}, \tag{57}$$

$$(1+\frac{1}{\Lambda_0})\Lambda = 3(m+n\coth t)^2 - \frac{k^2}{(\sinh t)^{6n} e^{6mt}} - \frac{3\alpha^2}{(\sinh t)^{2n} e^{2mt}}. \tag{58}$$

The model starts expanding with a big-bang at t = 0. At t = 0, ρ ,p and Λ are all infinitely large. For large values of t, $\rho \to \frac{3(m+n)^2}{1+\Lambda_0}$, $p \to \frac{-3\Lambda_0 (m+n)^2}{1+\Lambda_0}$ and $\Lambda \to \frac{3\Lambda_0 (m+n)^2}{1+\Lambda_0}$. We observe that the cosmological term Λ is a decreasing function of time. Being very large at initial epoch it reduces to a small value for large t. This is in agreement with the recent results from supernovae Ia observations. Matter density ρ tends to a genuine constant for large values of t. For illustrative purposes evolutionary behaviour of some cosmological parameters are shown graphically [Figs. 1-2].

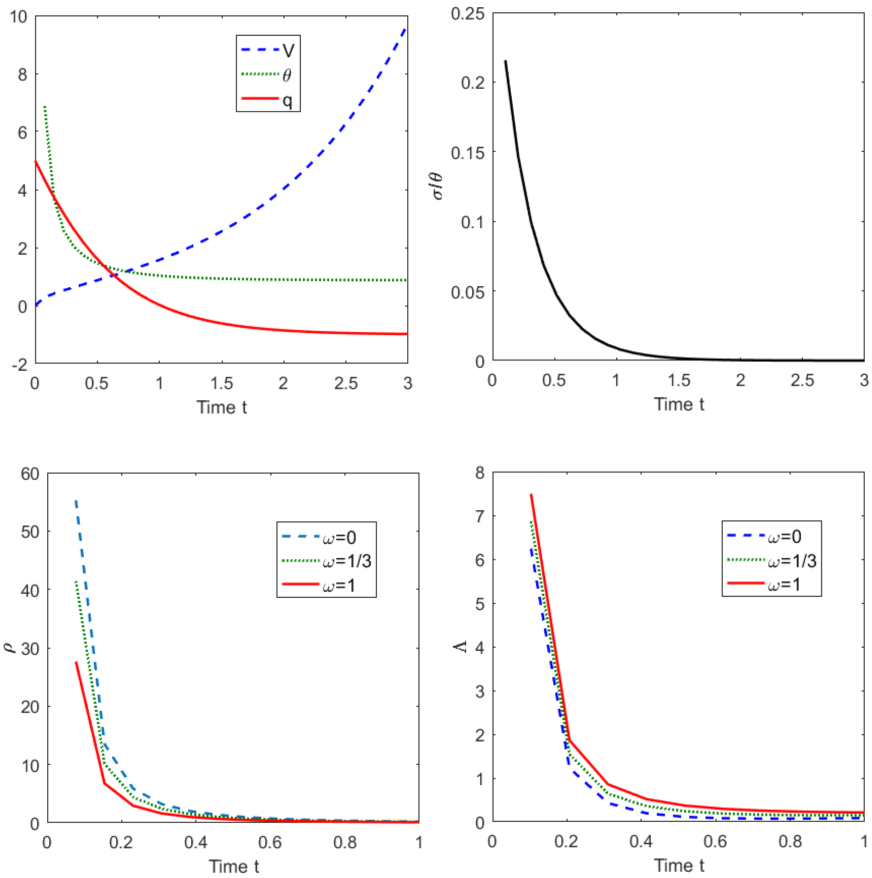

Fig. 1 Variation of spatial volume V, expansion scalar θ, deceleration parameter q, expansion anisotropy σ/θ, energy density ρ and vacuum energy density Λ with cosmic time t.

4. Conclusions

Evolution of Bianchi type-V cosmological model is considered for the perfect fluid distribution with dark energy represented by varying cosmological parameter Λ(t). By assuming a law of variation for Hubble parameter H, we have obtained a single analytical expression for deceleration parameter q which transits from its positive phase to the negative one yielding a cosmological model which represents decelerating expansion in the early epoch followed by accelerating expansion at late times. Phenomenological decay laws of cosmological term Λ(t) for the models have been discussed. We observe that the cosmic expansion in the model is driven by big bang impulse. The models represent shearing, non-rotating and expanding universe. The cosmological term Λ tend to a genuine constant asymptotically. Some of the models tend to deSitter universe for large values of cosmic time t. We also find that late time universe is dominated by dark energy, a clear evidence for accelerated expansion in agreement with observations. The anisotropy in the models damps out during cosmic evolution and models turn out to be isotropic for large values of t. The solutions presented in this paper may be useful in the study of dynamical and physical behaviour of homogeneous and anisotropic cosmological models with time varying cosmological term Λ(t).

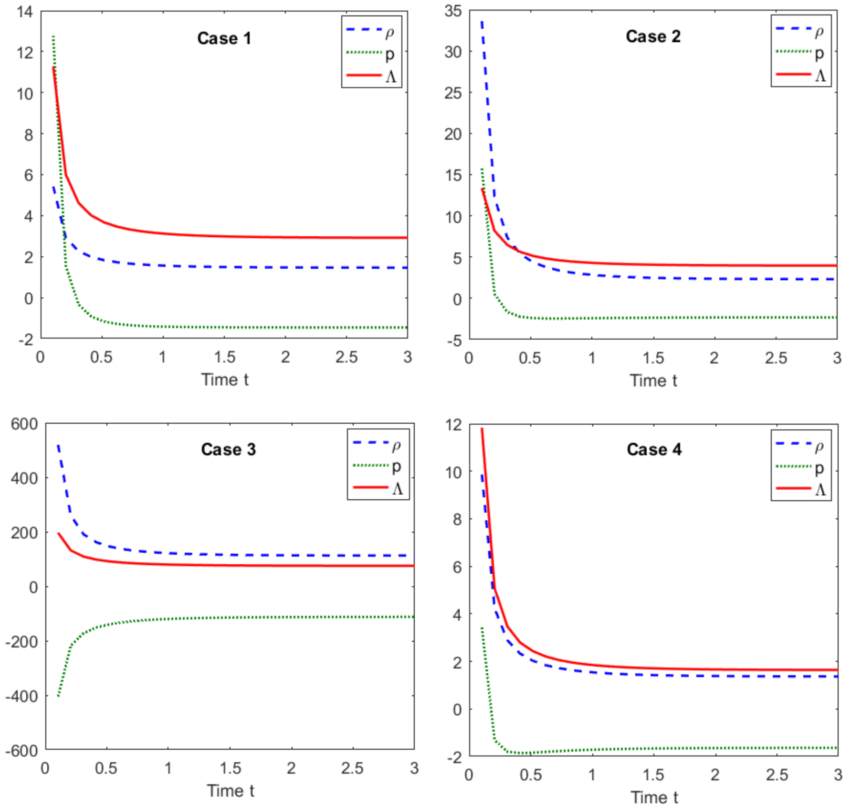

Fig. 2 Plots of matter energy density ρ, pressure p and vacuum energy density Λ with cosmic time t in various cases.

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References

[1] Allen, S. W., et al. (2004). Constraints on dark energy from Chandra observations of the largest relaxed galaxy clusters, Mon. Not. Roy. Astron. Soc. **353**, 457-467.

[2] Arbab, A. I. (2003). Cosmic acceleration with a positive cosmological constant, Class, Quantum Grav. **20**, 93-100.

[3] Bali, R., Tinkar, S. and Singh, P. (2010). Bianchi Type III Cosmological Models for Barotropic Perfect Fluid Distribution with Variable G and Λ, Int. J. Theor. Phys. **49**, 1431-1438.

[4] Bali, R. and Singh, P. (2012). Bulk Viscous Bianchi Type I Barotropic Fluid Cosmological Model with Varying Λ and Functional Relation on Hubble Parameter, Int. J. Theor. Phys. **51**, 772-776.

[5] Bennett, C. L., et al. (2003). First-year Wilkinson Microwave Anisotropy (WMAP) observations: Preliminary maps and basic results, Astrophys. J. Suppl. Ser. **148**, 1-27.

[6] Berman, M. S. (1983). A special law of variation for Hubble's parameter, Nuovo Cimento **74**B, 182-186.

[7] Bernardis, P. de, et al. (2000). A flat Universe from high-resolution maps of the cosmic microwave background radiation, Nature, **404**, 955-959.

[8] Carroll, S. M. (2001). The Cosmological Constant, Living Rev. Rel. 4, 1-50.

[9] Carvalho, J.C., Lima J. A.S. and Waga, I. (1992). Cosmological consequences of a time-dependent Λ term: Phys. Rev. **D46**, 2404-2407.

[10] Eisenstein, D. J., et al. (2005). Detection of the Baryon Acoustic Peak in the Large Scale Correlation Function of SDSS Luminous Red Galaxies, Astrophys. J. **633**, 560 -574.

[11] Knop, R.A., et al. (2003). New Constraints on Ω_M, Ω_Λ, and w from an Independent Set of Eleven High-Redshift Supernovae Observed with HST, Astrophys. J. **598**, 102-154.

[12] Linde, A.D. (1974). Is the Lee constant a cosmological constant? JETP Lett. **19**, 183-184.

[13] Perlmutter, S., et al. (1998). Discovery of a Supernova Explosion at Half the Age of the Universe and its Cosmological Implications, Nature, **391**, 51-54.

[14] Riess, A. G., et al. (1998). Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astron. J. **116**, 1009-1038.

[15] Riess, A. G., et al. (2004). Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, Astron. J. **607**, 665-687.

[16] Riess, A. G., et al. (2007). New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z > 1$: Narrowing Constraints on the Early Behaviour of Dark Energy, Astrophys. J. **659**, 98-121.

[17] Schutzhold, R. (2002). On the cosmological constant and the cosmic coincidence problem, Int. J. Mod. Phys. A**17**, 4359-4364.

[18] Schutzhold, R. (2002a). Small cosmological constant from the QCD trace anomaly? Phys. Rev. Lett. **89**, 081302 (1-4).

[19] Singh, J. P. (2008). A cosmological model with both deceleration and acceleration, Astrophys. Space Sci. **318**, 103-107.

[20] Singh, J. P. (2009). Bianchi V Cosmology with a Specific Hubble Parameter, Int. J. Theor. Phys. **48**, 2041-2049.

[21] Spergel, D. N., et al. (2003). First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations Determination of Cosmological Parameters, Astrophys. J. Suppl. Ser. **148**, 175-194.

[22] Spergel, D. N., et al. (2007). Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, Astrophys. J. Suppl. Ser. **170**, 377-408.

[23] Stompor, R., et al. (2001). Cosmological implications of the MAXIMA-I high resolution Cosmic Microwave Background anisotropy measurement, Astrophys. J. **561**, L7-L10.

[24] Tegmark, M., et al. (2004). Cosmological Parameters from SDSS and WMAP, Phys. Rev. D**69**, 103501-103529.

[25] Tegmark, M., et al. (2004a). The Three-Dimensional Power Spectrum of Galaxies from the Sloan Digital Sky Survey, Astrophys. J. 606, 702-740.

[26] Tonry, J. L., et al. (2003). Cosmological Results from High-z Supernovae, Astrophys. J. 594, 1-24.

[27] Vishwakarma, R. G. (2001). Consequences on variable Λ-models from distant type Ia supernovae and compact radio sources, Class Quantum Grav. **18**, 1159-1172.

[28] Wagoner, R. V. (1970). Scalar-Tensor Theory and Gravitational Waves, Phys. Rev. D **1**, 3209-3216.