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SOME BIANCHI TYPE IX DUST FLUID TILTED COSMOLOGICAL MODELS WITH BULK VISCOSITY IN GENERAL RELATIVITY

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Abstract. Bianchi type IX dust fluid tilted cosmological models with bulk viscosity are discussed. To get the deterministic model, we have considered the condition $A = B^n$ where A and B are metric potentials and n is a constant. Physical aspects of the model with and without viscosity are discussed.

Key words: Bianchi type IX, tilted, bulk viscosity

1. Introduction

In recent years, there has been considerable interest in spatially homogeneous, non isotropic cosmological models [24]. These are the so-called Bianchi models, there are two major distinct cases that arise : Orthogonal universe, in which the matter moves orthogonally to the hypersurface of homogeneity and tilted universe, in which the fluid flow vector is not normal to the hypersurfaces of homogeneity [16]. Bradley and Sviestine [8] in the investigation have shown that heat flow is expected for tilted universes. The tilted cosmological models with heat flux have been investigated by number of authors viz. Banerjee and Sanyal [6], Coley [9], Roy and Prasad [23], Roy and Banerjee [22], Bali and Sharma [5], Bali and Meena [4], Bali and Kumawat [1,2,3].

The general anisotropic Bianchi IX model was first investigated by Belinskii and Khalatnikov [7] later by Misner [19] and Doroshkevich et al. [10]. Recently Ghate, Sontakke [14] have investigated Bianchi type IX radiating cosmological model in self creation cosmology.

Dissipative effects including both the bulk and shear relativistic theory of non-equilibrium thermodynamics was developed by Eckart [12] to study the effect of bulk viscosity. Misner [20,21] have investigated the effect of viscosity on the evolution of cosmological models. Heller and Klimek [15] have investigated viscous fluid cosmological models without initial singularity. Recently Dubey et al. [11] have investigated Bianchi type I space time with bulk and shear viscosity and Tiwari and Tiwari [25] have investigated Bianchi type V cosmological model with viscous fluid.

In this paper, we have investigated Bianchi type IX dust fluid tilted cosmological models with bulk viscosity. To get the deterministic model, we have considered a condition $A = B^n$ where A and B are metric potentials. To get the solution in terms of cosmic time t, we consider n = 2. The physical and geometrical features of the model in the presence and absence of bulk viscosity are discussed.

2. Field Equations and Solutions

We consider the Bianchi type IX metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2\sin^2 y + A^2\cos^2 y)dz^2 - 2A^2\cos y\, dx\, dz \quad (1)$$

Energy momentum tensor for heat conduction is given by Ellis [13] and for bulk viscosity given by Landau and Lifshitz [17], is given by

$$T_i^j = (\in + p) v_i v^j + p g_i^j + q_i v^j + v_i q^j - \zeta\theta (g_i^j + v_i v^j) \quad (2)$$

together with

$$g_{ij} v^i v^j = -1 \quad (3)$$

$$q_i v^i = 0 \quad (4)$$

$$q_i q^i > 0 \quad (5)$$

The fluid flow vector v^i has the components $\left(\frac{\sinh\lambda}{A}, 0, 0, \cosh\lambda\right)$ satisfying (3), λ being the tilt angle.

The Einstein's field equation

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j \quad (6)$$

for the metric (1) leads to

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{3}{4}\frac{A^2}{B^4} = -8\pi[(\in + p)\sinh^2\lambda + p + 2A q^1 \sinh\lambda - K\cosh^2\lambda] \quad (7)$$

$$\frac{A_4 B_4}{AB} + \frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{A^2}{4B^4} = -8\pi(p - K) \quad (8)$$

$$\frac{2A_4 B_4}{AB} - \frac{A^2}{4B^4} + \frac{B_4^2}{B^2} + \frac{1}{B^2} = -8\pi[-(\in + p)\cosh^2\lambda + p - 2Aq^1 \sinh\lambda + K\sinh^2\lambda] \quad (9)$$

$$(\in + p)\,A\sinh\lambda\cosh\lambda + A^2 q^1 \frac{\cosh 2\lambda}{\cosh\lambda} - KA\sinh\lambda\cosh\lambda = 0 \quad (10)$$

where $\zeta\,\theta = K$

We assume that the universe is filled with dust fluid distribution which leads to

$$p = 0 \quad (11)$$

also assume that

$$A = B^n \quad (12)$$

when n is the constant.

Using (8) and (11) ,we have

$$\frac{A_4 B_4}{AB} + \frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{A^2}{4B^4} = 8\pi K \quad (13)$$

Equations (12) and (13) lead to

$$(n+1)\frac{B_{44}}{B} + n^2 \frac{B_4^2}{B^2} = 8\pi K - \frac{B}{4}^{2n-4} \quad (14)$$

If we take n=2 (15)

Equation (14) becomes ,

$$2B_{44} + \frac{8}{3B} B_4^2 = (\frac{32\pi K - 1}{6})\,B \quad (16)$$

We assume that

$$B_4 = f\,(B) \quad (17)$$

Thus $B_{44} = f\,f^{'}$

Using equations (16) and (17), we have

$$\frac{df^2}{dB} + \frac{8}{3}\frac{f^2}{B} = \left(\frac{32\pi K - 1}{6}\right)B \quad (18)$$

On solving equation (18), we have

$$f^2 = B_4 = \left(\frac{32\pi K - 1}{28}\right)B^2 + NB^{-8/3}$$

$$or\ \frac{B^{4/3}}{\sqrt{\alpha B^{14/3}+N}}\,dB = dt \tag{19}$$

Where N is a constant of integration and $\alpha = \left(\frac{32\pi K - 1}{28}\right)$

On solving equation (19), we have

$$B = \left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{3/7} \tag{20}$$

Therefore, the metric (1) takes the form

$$ds^2 = -dt^2 + \left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{12/7} dx^2 + \left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7} dy^2 +$$
$$\left[\left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\sin^2 y + \left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{12/7}\cos^2 y\right]dz^2$$
$$-2\left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{12/7}\cos y\,dx\,dz \tag{21}$$

Using the transformations

$$\left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\} = \frac{a\sin\sqrt{K}\tau}{\sqrt{K}} \tag{22}$$

$$\text{And}\ \alpha = \left(\frac{32\pi K - 1}{28}\right) = lK \tag{23}$$

Where l is the constant.

The metric (21) in the absence of bulk viscosity reduces to the form

$$ds^2 = -\frac{9a^2}{49N}d\tau^2 + (a\tau)^{12/7}dx^2 + (a\tau)^{6/7}dy^2 + \left[(a\tau)^{6/7}\sin^2 y + (a\tau)^{12/7}\cos^2 y\right]dz^2$$
$$-2(a\tau)^{12/7}\cos y\,dx\,dz \tag{24}$$

3. Some Physical and Geometrical Features

The matter density ($\in$), the tilt angle (λ), the expansion (θ), the flow vector (v^i) and heat conduction vector (q^i) for the metric (21) are given by

$$8\pi \in = \frac{10}{3}\alpha \coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right) + 2\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7} - \left(\frac{7+16\pi K}{6}\right) \tag{25}$$

$$\cosh^2\lambda = \frac{\left[(32\pi K+1) - 20\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right) - 4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]}{\left[\left(\frac{64\pi K-8}{3}\right) - \frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]} \tag{26}$$

$$\sinh^2\lambda = \frac{\left[\left(\frac{(32\pi K+11)}{3}\right) + \frac{20}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right) - 4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]}{\left[\left(\frac{64\pi K-8}{3}\right) - \frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]} \tag{27}$$

$$\theta = \frac{8\sqrt{\alpha}\coth\left(\frac{7}{3}\sqrt{\alpha}t\right)\left(\begin{array}{l}\left[\left(\frac{64\pi K-8}{3}\right) - \frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right] \\ \left[\frac{35}{3}\alpha\cos ech^2\left(\frac{7}{3}\sqrt{\alpha}t\right) + \left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right] + \\ \left[(32\pi K+1) - 20\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right) - 4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right] \\ \left[\left(\frac{352\pi K-29}{9}\right) - \frac{380}{9}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]\end{array}\right)}{\begin{array}{l}2\left[(32\pi K+1) - 20\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right) - 4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]^{1/2} \\ \left[\left(\frac{64\pi K-8}{3}\right) - \frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]^{3/2}\end{array}} \tag{28}$$

$$v^1 = \frac{\left[\left(\frac{(32\pi K+11)}{3}\right) + \frac{20}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right) - 4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]^{1/2}}{\left[\left(\frac{64\pi K-8}{3}\right) - \frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]^{1/2}\left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}} \tag{29}$$

$$v^4 = \frac{\left[(32\pi K+1)-20\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]^{1/2}}{\left[\left(\frac{64\pi K-8}{3}\right)-\frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]^{1/2}} \tag{30}$$

$$\frac{1}{8\pi}\left[\left(\frac{(64\pi K+7)}{6}\right)-\frac{10}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-2\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]$$

$$\left[\left(\frac{(32\pi K+11)}{3}\right)+\frac{20}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]^{1/2}$$

$$q^1 = \frac{\left[(32\pi K+1)-20\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]}{\left[\left(\frac{(126\pi K+14)}{3}\right)-\frac{40}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-8\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]} \tag{31}$$

$$\left[\left(\frac{64\pi K-8}{3}\right)-\frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]^{1/2}\left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}$$

$$\frac{1}{8\pi}\left[\left(\frac{(64\pi K+7)}{6}\right)-\frac{10}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-2\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]$$

$$\left[\left(\frac{(32\pi K+11)}{3}\right)+\frac{20}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]$$

$$q^4 = \frac{\left[(32\pi K+1)-20\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]^{1/2}}{\left[\left(\frac{(126\pi K+14)}{3}\right)-\frac{40}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)-8\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]} \tag{32}$$

$$\left[\left(\frac{64\pi K-8}{3}\right)-\frac{80}{3}\alpha\coth^2\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]^{1/2}$$

The components of shear tensor (σ_{ij}) are given by

$$\sigma_{11}=\frac{\begin{gathered}2\sqrt{\alpha}\coth\left(\frac{7}{3}\sqrt{\alpha}t\right)\left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{12/7}\\ \left[(32\pi K+1)-20\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]^{1/2}\\ \left(\begin{gathered}\left[(32\pi K+1)-20\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]\\ \left[\left(\frac{832\pi K-44}{9}\right)-\frac{800}{9}\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]+4\left[\left(\frac{64\pi K-8}{3}\right)-\frac{80}{3}\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]\\ \left[\frac{35}{3}\alpha\cos ech^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)+\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]\end{gathered}\right)\end{gathered}}{3\left[\left(\frac{64\pi K-8}{3}\right)-\frac{80}{3}\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]^{5/2}} \tag{33}$$

$$\sigma_{14}=-\frac{\begin{gathered}2\sqrt{\alpha}\coth\left(\frac{7}{3}\sqrt{\alpha}t\right)\left\{\sqrt{\frac{N}{\alpha}}\sinh\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\\ \left[\left(\frac{32\pi K+11}{3}\right)+\frac{20}{3}\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]^{1/2}\\ \left(\begin{gathered}\left[(32\pi K+1)-20\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)-4\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]\\ \left[\left(\frac{832\pi K-44}{9}\right)-\frac{800}{9}\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]+4\left[\left(\frac{64\pi K-8}{3}\right)-\frac{80}{3}\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]\\ \left[\frac{35}{3}\alpha\cos ech^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)+\left\{\sqrt{\frac{\alpha}{N}}\cos ech\left(\frac{7}{3}\sqrt{\alpha}t\right)\right\}^{6/7}\right]\end{gathered}\right)\end{gathered}}{3\left[\left(\frac{64\pi K-8}{3}\right)-\frac{80}{3}\alpha\coth^{2}\left(\frac{7}{3}\sqrt{\alpha}t\right)\right]^{5/2}} \tag{34}$$

The trace free condition for σ_{ij} is given by

$$\sigma_{ij}\mathrm{v}^{\mathrm{i}}=0$$

This leads to

$$\sigma_{11}v^1 + \sigma_{14}v^4 = 0 \tag{35}$$

Which is satisfied for given values of σ_{11}, σ_{14}, v^1 and v^4

In absence of viscosity, the above mentioned quantities lead to

$$8\pi \in = \frac{\left[2(a\tau)^{8/7} - \left(\frac{15}{12}\right)(a\tau)^2 + \frac{10}{3}N\right]}{(a\tau)^2} \tag{36}$$

$$\cosh^2\lambda = \frac{\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]}{\left[2(a\tau)^2 + \frac{80}{3}N\right]} \tag{37}$$

$$\sinh^2\lambda = \frac{\left[4(a\tau)^{8/7} - 4(a\tau)^2 - \frac{20}{3}N\right]}{\left[2(a\tau)^2 + \frac{80}{3}N\right]} \tag{38}$$

$$\theta = \frac{4\sqrt{N}\left(\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]\left[2(a\tau)^2 + \frac{380}{9}N\right] - \left[2(a\tau)^2 + \frac{80}{3}N\right]\left[(a\tau)^{8/7} + \frac{35}{3}N\right]\right)}{\left[2(a\tau)^2 + \frac{80}{3}N\right]^{3/2}\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]^{1/2}(a\tau)} \tag{39}$$

$$v^1 = \frac{\left[4(a\tau)^{8/7} - 4(a\tau)^2 - \frac{20}{3}N\right]^{1/2}}{(a\tau)^{6/7}\left[2(a\tau)^2 + \frac{80}{3}N\right]^{1/2}} \tag{40}$$

$$v^4 = \frac{\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]^{1/2}}{\left[2(a\tau)^2 + \frac{80}{3}N\right]^{1/2}} \tag{41}$$

$$q^1 = \frac{\frac{1}{8\pi}\left[\frac{3}{2}(a\tau)^2 - 2(a\tau)^{8/7} - \frac{10}{3}N\right]\left[4(a\tau)^{8/7} - 4(a\tau)^2 - \frac{20}{3}N\right]^{1/2}\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]}{(a\tau)^{20/7}\left[8(a\tau)^{8/7} - 6(a\tau)^2 + \frac{40}{3}N\right]\left[2(a\tau)^2 + \frac{80}{3}N\right]^{1/2}} \tag{42}$$

$$q^4 = \frac{\frac{1}{8\pi}\left[\frac{3}{2}(a\tau)^2 - 2(a\tau)^{8/7} - \frac{10}{3}N\right]\left[4(a\tau)^{8/7} - 4(a\tau)^2 - \frac{20}{3}N\right]\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]^{1/2}}{(a\tau)^2\left[8(a\tau)^{8/7} - 6(a\tau)^2 + \frac{40}{3}N\right]\left[2(a\tau)^2 + \frac{80}{3}N\right]^{1/2}} \tag{43}$$

$$\sigma_{11} = \frac{2\sqrt{N}(a\tau)^{5/7}\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]^{1/2}\left(\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]\left[2(a\tau)^2 + \frac{800}{9}N\right] - 4\left[2(a\tau)^2 + \frac{80}{3}N\right]\left[(a\tau)^{8/7} + \frac{35}{3}N\right]\right)}{3\left[2(a\tau)^2 + \frac{80}{3}N\right]^{5/2}} \tag{44}$$

$$\sigma_{14} = -\frac{2\sqrt{N}\left[4(a\tau)^{8/7} - 4(a\tau)^2 - \frac{20}{3}N\right]^{1/2}\left(\left[4(a\tau)^{8/7} - 2(a\tau)^2 + 20N\right]\left[2(a\tau)^2 + \frac{800}{9}N\right] - 4\left[2(a\tau)^2 + \frac{80}{3}N\right]\left[(a\tau)^{8/7} + \frac{35}{3}N\right]\right)}{(a\tau)^{1/7}\,3\left[2(a\tau)^2 + \frac{80}{3}N\right]^{5/2}} \tag{45}$$

Thus $\sigma_{11}v^1 + \sigma_{14}v^4 = 0$ (46)

4. Discussion

In presence of viscosity, the matter density ($\in$) $\to \infty$ when t $\to$ 0 where $(32\pi K - 1) > 0$ and N > 0. The model (21) has point type singularity at t = 0 (MacCallum [18]) and the model (21) starts with a big bang at t = 0.

In absence of viscosity, the matter density ($\in$) $\to \infty$ when (τ) $\to$ 0. The model (24) has point type singularity at τ = 0 (MacCallum [18]). The model (24) starts with a big bang at τ = 0 and the expansion in the model decreases as time increases.

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