Journal of Rajasthan Academy of Physical Sciences
ISSN: 0972-6306; URL: http://raops.org.in
Vol.20, No.3&4, July-December, 2021, 229-242

BIANCHI TYPE-III COSMOLOGICAL MODELS WITH CONSTANT DECELERATION PARAMETER FOR PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

M.K. Yadav, M. Hembram and R.K. Gangele
Deptt. of Mathematics and Statistics, School of Mathematical and Physical Sciences,
Dr. Harisingh Gour Vishwavidyalaya, Sagar, M.P. 470003, India
E-mail: yadav1976mk@gmail.com, mhmath2019@gmail.com, and
rkgangele23@gmail.com

Abstract: In this paper, we have studied Bianchi type-III cosmological models with constant deceleration parameter. We have studied the cases when $k = 0$ and $k \neq 0$. To obtain a determinate model, we have assumed shear (σ) is proportional to expansion (θ) which leads to $B = lC^n$, where B and C are metric potential and l and n are constants. Geometrical and kinematical properties of the model are also presented.

Keywords: Bianchi Type-III cosmological models, deceleration parameter, perfect fluid.

1. Introduction

It is known that in the presence of dark energy, our universe behaves as an expanding universe. But in the presence of dark matter, it behaves as a decelerating one [14]. To obtain cosmological solutions we solve Einstein field equation, which is a set of non-linear differential equations. Berman [4,5,6] proposed a method to solve these equations by using the law of variation for Hubble's parameter. Singh and Beesham [24] have incorporated the same deceleration parameter techniques for solutions.

In the simplest case the Hubble law yields a constant value for the deceleration parameter. The variation of Hubble parameter is consistent with the observation, when it is represented in the form of scale factor. Maharaj and Naidoo [13] have consistently applied the non-variable deceleration parameter under the frame work of alternative theory of gravity. Johari and Kalyani [10] have studied cosmological models with constant deceleration parameter in Brans-Dicke theory.

The deceleration parameter listed by Hulke and Singh [9] with respect to their values according to universe behavior is shown below,

- $q > 0$: Decelerating expansion
- $q = 0$: Expansion with constant rate

- $-1 < q < 0$: Accelerating power law expansion
- $q = -1$: Exponential expansion / de Sitter expansion
- $q < -1$: Super exponential expansion.

In the large scale structure the spatially homogeneous and anisotropic cosmological models play an impotant role to determine the behavior of the universe. Vinutha et al. [25] have shown that the equation of state parameter (ω) varies accordingly to cosmic time t and their model is consistent with the observational data. Singh and Agrawal [23] have studied Bianchi type-III cosmological model for a scalar-tensor theory. Most of the researchers applied the method of Berman's law in various context of general relativity to solve the Einstein field equation [1,3,4,6,7,22]. Pradhan and Vishwakarma [17] obtain exact solutions of LRS Bianchi type I inhomogeneous cosmological model by using constant deceleration parameter. They [17] also studied the cosmological models in the framework of Lyra's geometry. Singh and Kumar [20] have applied the Berman's law to Bianchi Type-II space time with perfect fluid and scalar field. Many more authors have also applied the method of Berman's law to obtained the Einstein's Field Equations and solve them various anisotropic cosmological models.Assmptotic behavior of the models are described by MacCallum [12].

A logarithmic parameterization of q(z) have been discussed by Manon and Das [14] to probe the evolution history of the universe. Bali and Dave [3] have obtained magnetized Bianchi type III string cosmological models for massive string in and bulk viscous fluid string cosmological model in general relativity. Rahaman et al. [18] have discussed two cosmological models namely Bianchi-I and Kantowski-Sachs models with constant deceleration parameter within the framework of Lyra geometry. Yadav et al. [26] have solved a cosmic string cosmological model for Bianchi type-III in presence of bulk viscous fluid. Singh et al. [22] have discussed Bianchi type-III Cosmological model with variables G and Λ in the presence of perfect fluid. Adhav et al. [1] obtained an exact solution of the vacuum Brans-Dicke field equations for a spatially homogeneous and anisotropic metric. Akarsu and Kılınç [2] have studied LRS Bianchi type I models with anisotropic dark energy where a special law is assumed for the deviation from isotropic Equation of State. It is consistent with the assumption on the conservation of the energy-momentum tensor of the Dark Energy. Chandel and Ram [8] have studied anisotropic Bianchi type-III perfect fluid cosmological models in $f(R,T)$ theory of gravity. Reddy et al. [19] have obtained a dark energy model with the equation of state parameter in $f(R,T)$ gravity in Bianchi type-III space time in presence of a perfect fluid source. Pawar and Sahare [16] have utilized the power law relation $C = t^m$, where $m > 0$ to solve the cosmological model and obtained that the universe is expanding with a constant deceleration parameter. Vinutha and Kavya [25] have studied quadratic functional form in f(R,T) gravity and observed that the model considered by them is asymptotically exact de Sitter solution, and also find that the physical parameters values are consistent with the current observational data.

Motivated by the above, we have studied Bianchi Type-III cosmological model with constant deceleration parameter in General Relativity.

2. Field Equations

We consider the general Bianchi Type-III space time, in the following form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2\alpha x}dy^2 + C^2(t), \tag{1}$$

where A, B, and C are function of t only and α is a positive constant.

The energy momentum tensor for perfect fluid, is defined by [11]

$$T_i^j = (\rho + p)v_i v^j + p g_i^j \ , \tag{2}$$

where $v_i v^i = -1, and\ v^i = (0,0,0,1)$.

Einstein's field equation, is written as follows,

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j \ , \tag{3}$$

here, in the gravitational units, we have taken $c = 1, and\ G = 1$.

The Einstein's field equation (3), for the metric (1) takes the following form

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi p, \tag{4}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -8\pi p, \tag{5}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\alpha^2}{A^2} = -8\pi p, \tag{6}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{\alpha^2}{A^2} = 8\pi\rho, \tag{7}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0. \tag{8}$$

Equation (8) gives us

$$A = mB, \tag{9}$$

where m is a constant of integration.

By using equation (9) in equations (4), (5), (6) and (7), we obtain

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi p, \tag{10}$$

$$\frac{2B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{\alpha^2}{m^2 B^2} = -8\pi p, \tag{11}$$

$$\frac{2B_4 C_4}{BC} + \left(\frac{B_4}{B}\right)^2 - \frac{\alpha^2}{m^2 B^2} = 8\pi\rho. \tag{12}$$

The average scale factor $a(t)$, and the Hubble parameter are given by,

$a^3(t) = ABC$, and $H = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$. (13)

The deceleration parameter (q), is defined in the form of H as

$q = -1 - H_4/H^2 = k - 1$, (14)

Here 4 represents ordinary differentiation with respect to time t.

If, $k < 1$ then the universe inflates, but in case $k > 1$, then the universe decelerates.

To identify the effective equation of state for matter, Berman [7] suggested a special law of variation for Hubble parameter, which is

$H = \delta a^{-k}(t)$ (15)

where $\delta \geq 0,\ k > 0$ are constants, and $a(t)$ represents the average scale factor of the model.

The above cited Berman's law is an extended form of the de Sitter as given by Narlikar [15], and power-law expansions [21].

The Hubble parameter, $\mathrm{H} = a_4/a$, Hence equation (15) gives

$a(t) = \{k(\delta t + d)\}^{\frac{1}{k}}$, (16)

here d is a constant of integration.

2.1 Non-Singular origin case: ($\boldsymbol{k = 0}$)

Now putting the value k = 0 in (15), keeping in view equation (16) we have

$H = \delta$. (17)

To get a determinate solution, we assume that shear (σ) is proportionate to expansion (θ) as given by Bali [4]. This leads to

$B = lC^n$. (18)

where *l* and *n* are constant.

We obtained the metric potential values using equations (9), (13) and (18) as

$A = K_1 e^{(3\delta nt/2n+1)}$, (19)

$B = K_2 e^{(3\delta nt/2n+1)}$, (20)

$C = \left\{\frac{K_2}{l} e^{(3\delta nt/2n+1)}\right\}^{1/n}$, (21)

here, K_1, and K_2 are constant of integration, and d is considered as unity without loss of generality.

The relation between average scale factor and red shift [15] is,

$a(t) = a_0(1+z)^{-1}$, (22)

a_0 represents current scale factor, with the help of above relation between scale factor and redshift we express the energy density and pressure in form of redshift by considering $a_0 = 1$. From equations (10)-(12), we get pressure and density if, $\alpha^2 = m^2 e^{\left(\frac{3n\delta t}{2n+1}\right)} . \left(\frac{3n\delta}{2n+1}\right)^2 (2n^2 - n - 1)$ exist.

We obtained pressure and energy density with the help of equations (10)-(12), and (19)-(21),

$$\rho = \frac{1}{8\pi}\left(\frac{3n\delta}{2n+1}\right)^2 \left[\frac{2+n}{n} - \frac{(2n^2-n-1)}{e^{\left(\frac{3n\delta t}{2n+1}\right)}}\right] \tag{23}$$

$$p = \frac{1}{8\pi}\left(\frac{3n\delta}{2n+1}\right)^2 \left[-3 - \frac{(n+1-2n^2)}{e^{\left(\frac{3n\delta t}{2n+1}\right)}}\right] \tag{24}$$

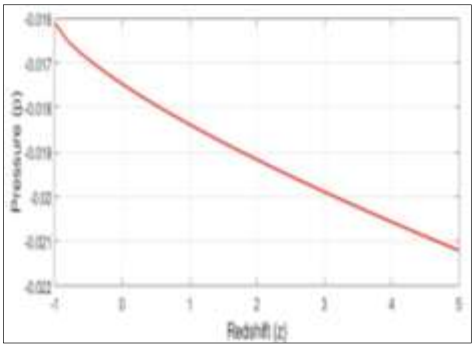
After putting equation (22) in equations (23), and (24) we get,

$$\rho = \frac{1}{8\pi}\left(\frac{3n\delta}{2n+1}\right)^2 \left[\frac{2+n}{n} - (2n^2 - n - 1)L^2(1+z)^{\frac{3n}{2n+1}}\right], \tag{25}$$

$$p = \frac{1}{8\pi}\left(\frac{3n\delta}{2n+1}\right)^2 \left[-3 + (2n^2 - n - 1)L^2(1+z)^{\frac{3n}{2n+1}}\right], \tag{26}$$

we have taken, $L^2 = \left(\frac{a_1}{a_0}\right)^{\frac{3n}{2n+1}}$, where a_1, and a_0 are constant.

The variation of pressure (p), and density (ρ) with respect to redshift (z) shown below,



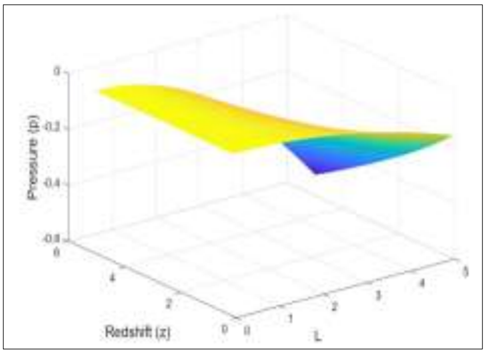
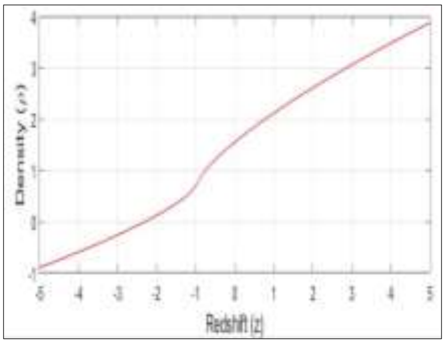


Fig.1 Plot of pressure (p) versus redshift (z) n=0.48, L=2.5, $\delta = 0.5$, **Fig.2** Plot of pressure (p) versus redshift (z) n=0.48, $\delta = 0.5$

From figs. 1, we observed that pressure (p) of the model goes down with negative sign. w. r. t. redshift (z), which indicates that the universe of the model taken, accelerates the expansion of universe. Fig. 4 also verify for different values of L (0 < L < 1) in the form of 3-D.



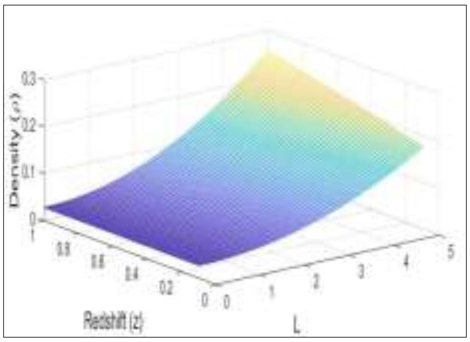


Fig.3 Plot of energy density (ρ) versus redshift (z) n=0.48, L=2.5, $\delta = 0.5$, **Fig.4** Plot of energy density (ρ) versus redshift (z) n=0.48, L=2.5, $\delta = 0.5$

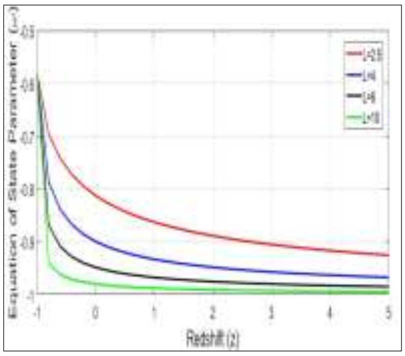
From figs. 3 and 4 the variation of density (ρ) w. r. t. redshift (z) is increasing, i.e. the density of universe gets positive, which is the sign. of expansion of universe. Here we also represents density for different values of L (0 < L < 5) in fig. 4 the form of 3-D.

For physically realistic scenario the energy density must be positive, $\rho \geq 0$, so by using equations (23) and (25), we must have

$$t \geq \frac{1}{\delta} ln\left(\frac{L^2(2n^3-n^2-n)}{n(n+2)}\right)^{2n+1/3n}, \text{ or } z \leq \left(\frac{(n+2)}{L^2(2n^3-n^2-n)}\right)^{2n+1/3n} - 1 \qquad (27)$$

The EoS parameter, which is defined as $\omega = p/\rho$, yields

$$\omega = \left[\frac{-3 - \frac{(n+1-2n^2)}{e^{\left(\frac{3n\delta t}{2n+1}\right)}}}{\frac{2+n}{n} - \frac{(2n^2-n-1)}{e^{\left(\frac{3n\delta t}{2n+1}\right)}}}\right] = \left[\frac{-3n+(2n^3-n^2-n)L^2(1+z)^{\frac{3n}{2n+1}}}{2+n-(2n^3-n^2-n)L^2(1+z)^{\frac{3n}{2n+1}}}\right] \qquad (28)$$



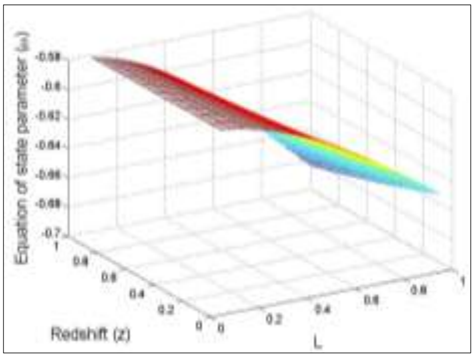


Fig.5 Plot of Equation of states Parameter (ω) versus redshift (z) n=0.48,L=(2.5, 4, 6, 10) $\delta = 0.5$, **Fig.6** Plot of Equation of states Parameter (ω) versus redshift (z) n=0.48, $\delta = 0.5$

In the above plotted figs. 5 and 6, the EoS (ω) of our model appear phantom for $z \geq -1$, and exist phantom as well as quintessence for $z \leq -1$, fig. 5 represents for four different values of L whereas fig. 6 represents for (0 < L < 1) in 3-D form.

When, $z = 0$ means present universe, and when $t = 0$ is current time, these two condition shows the present value of EoS parameter, $\omega_0\ (z = 0)$, is

$$\omega_0 = \left[\frac{-3n+(2n^3-n^2-n)L^2}{2+n-(2n^3-n^2-n)L^2}\right] \tag{29}$$

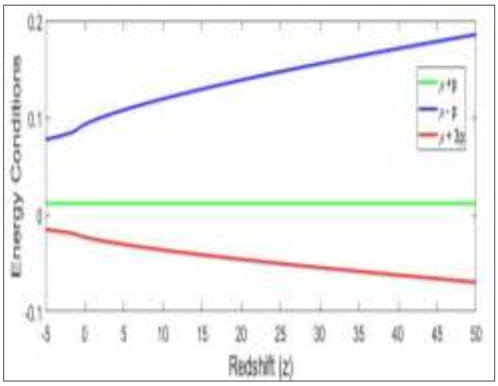
2.1.1 Energy Conditions

For the present model, the energy conditions **[7]** $\rho + p \geq 0, \rho - p \geq 0$ and $\rho + 3p \geq 0$ leads to

$$\rho + p = \frac{1}{4\pi}\left(\frac{3n\delta}{2n+1}\right)^2\left(\frac{1-n}{n}\right) \geq 0, \tag{30}$$

$$\rho - p = \frac{1}{4\pi}\left(\frac{3n\delta}{2n+1}\right)^2\left[\frac{1+2n}{n} - (2n^2 - n - 1)l^2(1+z)^{\frac{3n}{2n+1}}\right] \geq 0, \tag{31}$$

$$\rho + 3p = \frac{1}{4\pi}\left(\frac{3n\delta}{2n+1}\right)^2\left[\frac{1-4n}{n} + (2n^2 - n - 1)l^2(1+z)^{\frac{3n}{2n+1}}\right] \geq 0, \tag{32}$$



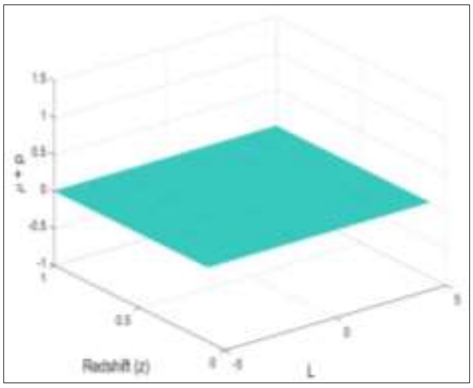
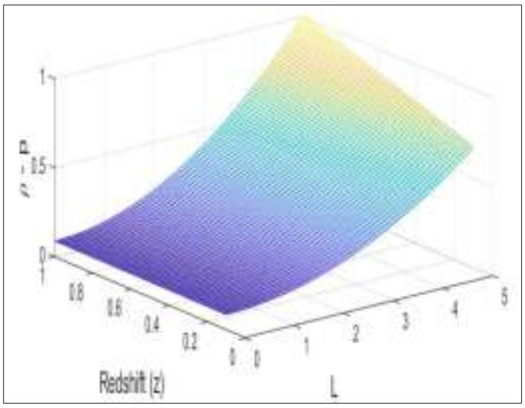


Fig. 7 Plot of energy conditions versus redshift (z) n=0.48, L=2.5, = 0.5 , **Fig. 8** Plot $(\rho + p)$ versus redshift (z) n=0.48, $\delta = 0.5$



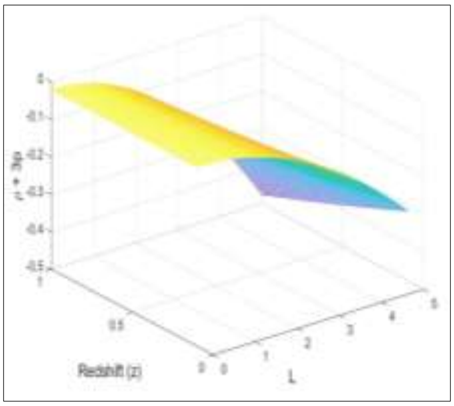


Fig. 9 Plot $(\rho - p)$ versus redshift (z) n=0.48, $\delta = 0.5$, **Fig. 10** Plot $(\rho + 3p)$ versus redshift (z) n=0.48, $\delta = 0.5$

From fig. 7, we can easily see that $\rho + p \geq 0$ and $-p \geq 0$, hence the null energy condition (NEC) and dominant energy condition (DEC) satisfied for perfect fluid distribution, here we can also see that $\rho + 3p \ \leq 0$, which means the violation of strong energy condition (SEC), therefore our model (universe) accelerates. Figs. 8, 9 and 10 also represents the same in 3-D formation for different values of L.

2.1.2 Geometrical behaviour of the model

The rate of the expansion along the x, y, and z-axes are

$$H_x = \frac{A_4}{A} = \frac{3n\delta}{2n+1} = H_y, \tag{33}$$

$$H_z = \frac{C_4}{C} = \frac{3\delta}{2n+1}, \tag{34}$$

Equations (33), and (34) shows that all directional Hubble parameter is free from cosmic time t, means gives constant values.

The expansion scalar (θ), shear scalar (σ) and the Anisotropy parameter (A_m) is defined as,

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = 3\delta, \tag{35}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{3}\left[\left(\frac{A_4}{A}\right)^2 + \left(\frac{B_4}{B}\right)^2 + \left(\frac{C_4}{C}\right)^2 - \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} - \frac{A_4C_4}{AC}\right] = 3\left(\frac{\delta(n-1)}{2n+1}\right)^2, \tag{36}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 2\left(\frac{n-1}{2n+1}\right)^2. \tag{37}$$

2.1.3 Scalar Field model

The energy density and pressure of a minimally coupled normal ($\epsilon = 1$) or phantom ($\epsilon = -1$) scalar field, ϕ with self-interacting potential $V(\phi)$ are given [24] by,

$$\rho = \frac{1}{2}\epsilon{\phi_4}^2 + V(\phi), \tag{38}$$

$$p = \frac{1}{2}\epsilon{\phi_4}^2 - V(\phi), \tag{39}$$

substituting (38), and (39) in (25), and (26) we get the expressions for the kinetic energy, and scalar potential,

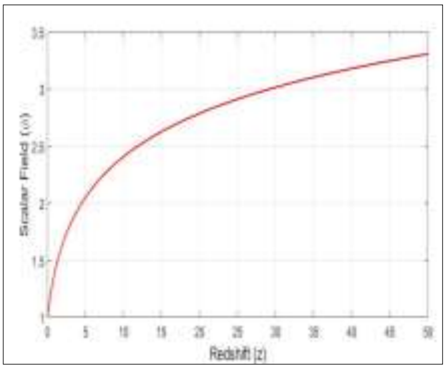
$$\frac{1}{2}\epsilon{\phi_4}^2 = \frac{1}{4\pi}\left(\frac{3n\delta}{2n+1}\right)^2\left(\frac{1-n}{n}\right), \tag{40}$$

$$V(\phi) = \frac{1}{2\pi}\left(\frac{3n\delta}{2n+1}\right)^2\left[\frac{1+2n}{n} - (2n^2 - n - 1)L^2(1+z)^{\frac{3n}{2n+1}}\right]. \tag{41}$$

The scalar potential with an infinite value which decreases accordance with the evolution, and vanishes at late times. However kinetic energy always flat throughout the evolution. For, $\epsilon = 1$, integrating equation (40), we find the expression for the minimally coupled scalar field,

$$\phi = \phi_0 \pm \sqrt{\left(\frac{1-n}{n\pi}\right)} \cdot \frac{1}{2\delta}\ln\left(\frac{1}{z+1}\right), \tag{42}$$

The obtained result is physically consistent only for negative sign, and representation of the graph shown below,



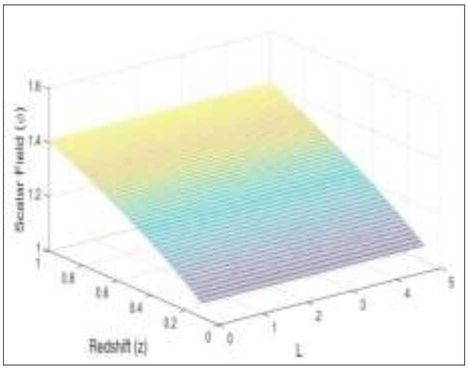


Fig. 11 Plot Scalar field (ϕ) versus redshift (z) n=0.48, $\delta = 0.5$, **Fig. 12** Plot Scalar field (ϕ) versus redshift (z), $\delta = 0.5$

Here, the scalar field (ϕ) increases at the evolution of the universe with redshift (z).

2.2 Singular origin case ($k \neq 0$)

Now putting the value $k \neq 0$ in (15), keeping in view equation (16) we have $H = \delta[k(\delta t + d)]^{-1}$, where d is constant of integration, substituting these in (9), (13) and (18), we get

$$A = K_3(\delta t + d)^{(3n/(2n+1)k)} \tag{43}$$

$$B = K_4(\delta t + d)^{(3n/(2n+1)k)} \tag{44}$$

$$C = \left[\frac{K_4}{l}(\delta t + d)^{(3n/(2n+1)k)}\right]^{1/n} \tag{45}$$

For $k \neq 0$, the energy density and pressure from (10)-(12), after using equations (43)-(45)

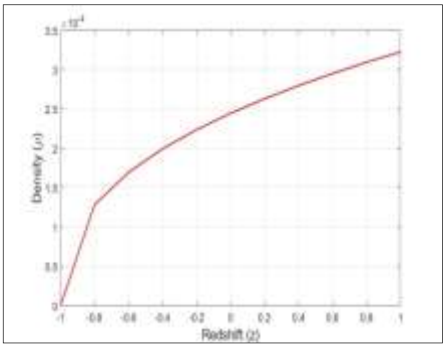
$$\rho = \frac{1}{8\pi}\left[\frac{3\delta^2}{(2n+1)k}(\delta t + d)^{-2}\right].\left[\frac{(-9n^2+9n+3)}{(2n+1)k} + n - 1\right], \tag{46}$$

$$p = -\frac{1}{8\pi}\left[\frac{3\delta^2}{(2n+1)k}(\delta t + d)^{-2}\right].\left[2 + n - \frac{3n^2+3n+3)}{(2n+1)k}\right]. \tag{47}$$

The equations (46), and (47) using equation (22) becomes,

$$\rho = \frac{1}{8\pi}\left[\frac{3\delta^2 k}{(2n+1)}.\frac{(1+z)^{2k}}{(a_0)^{2k}}\right].\left[\frac{(-9n^2+9n+3)}{(2n+1)k} + n - 1\right] \tag{48}$$

$$p = -\frac{1}{8\pi}\left[\frac{3\delta^2}{(2n+1)}.\frac{(1+z)^{2k}}{(a_0)^{2k}}\right].\left[2 + n - \frac{3n^2+3n+3)}{(2n+1)k}\right] \tag{49}$$



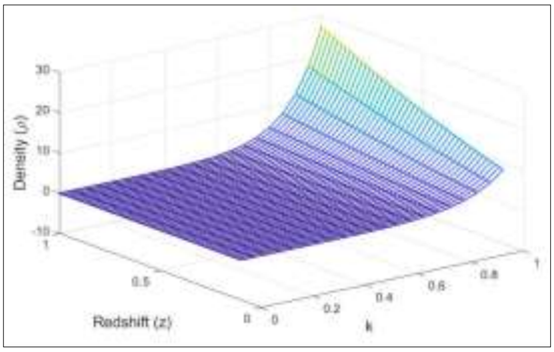
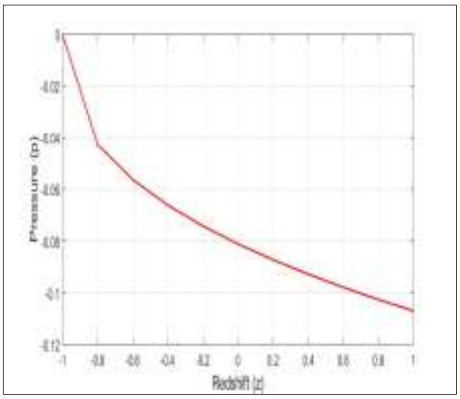


Fig. 13 Plot energy density (ρ) versus redshift (z) n=0.48, $\delta = 0.5$, k=0.2, a=0.01, **Fig. 14** Plot energy density (ρ) versus redshift (z) n=0.48, $\delta = 0.5$, k=0.2, a=0.01

For the singular case $k \neq 0$) we noted that the energy density also gives positive values for $(-1 < z < 1)$ from fig. 13 and fig. 14 also justify the universe expand for different values of k.



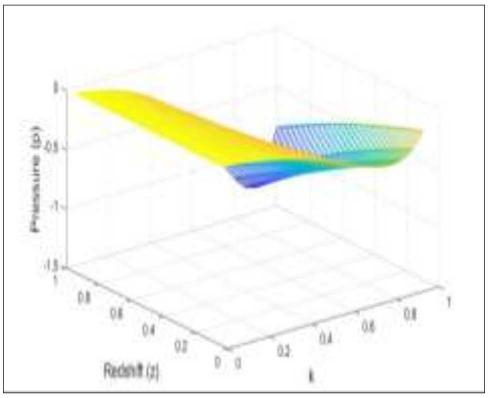


Fig. 15 Plot pressure (p) versus redshift (z) n=0.48, $\delta = 0.5$, k=0.2, a=0.01, **Fig. 16** Plot pressure (p) versus redshift (z) n=0.48, $\delta = 0.5$, k=0.2, a=0.01

In the above shown fig. 15 and 16, the pressure was highly positive at late times and goes negatively for $z > -1$. The variation of pressure from increasing to decreasing shows the expanding nature of the universe.

2.2.1 Energy Conditions

$$\rho + p = \frac{1}{8\pi}\left[\frac{3\delta^2 k}{(2n+1)}\frac{(1+z)^{2k}}{{a_0}^{2k}}\right].\left[\frac{(-12n^2+6n)}{(2n+1)k} + 2n + 1\right] \geq 0, \tag{50}$$

$$\rho - p = \frac{1}{8\pi}\left[\frac{3\delta^2 k}{(2n+1)}\frac{(1+z)^{2k}}{{a_0}^{2k}}\right].\left[\frac{(-6n^2+12n+6)}{(2n+1)k} - 3\right] \geq 0, \tag{51}$$

$$\rho + 3p = \frac{1}{8\pi}\left[\frac{3\delta^2 k}{(2n+1)}\frac{(1+z)^{2k}}{{a_0}^{2k}}\right].\left[\frac{(-18n^2-6)}{(2n+1)k} + 4n + 5\right] \geq 0, \tag{52}$$

and the equation of state parameter is,

$$\omega = \left[\frac{(2+n)(2n+1)k-(3n^2+3n+3)}{(-9n^2+9n+3)+(n-1)(2n+1)k}\right]. \tag{53}$$

2.2.2 Geometrical Behaviour of the model

Physical behavior of the model

$$H_x = \frac{A_4}{A} = \frac{3n\delta(\delta t+d)^{-1}}{(2n+1)k} = H_y \tag{54}$$

$$H_z = \frac{C_4}{C} = \frac{3\delta(\delta t+d)^{-1}}{(2n+1)k} \tag{55}$$

The expansion scalar, θ

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{3\delta}{k(\delta t+d)}, \tag{56}$$

and the shear scalar, σ

$$\sigma^2 = 3\left(\frac{\delta(n-1)}{(2n+1)(\delta t+d)}\right)^2. \tag{57}$$

2.2.3 Scalar Field model

$$\epsilon{\phi_4}^2 = \frac{1}{8\pi}\left[\frac{3\delta^2 k}{(2n+1)}\frac{(1+z)^{2k}}{{a_0}^{2k}}\right].\left[\frac{(-12n^2+6n)}{(2n+1)k} + 2n + 1\right] \tag{58}$$

$$V(\phi) = \frac{1}{4\pi}\left[\frac{3\delta^2 k}{(2n+1)}\frac{(1+z)^{2k}}{{a_0}^{2k}}\right].\left[\frac{(-6n^2+12n+6)}{(2n+1)k} + -3\right] \tag{59}$$

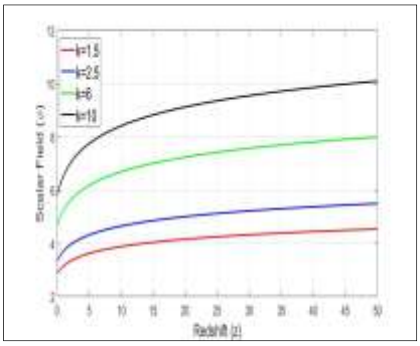
For, $\epsilon = 1$, integrating equation (58), we find the expression for the minimally coupled scalar field,

$$\phi = \ln(\phi_0) \pm \sqrt{\frac{1}{8\pi}\left[\frac{3\delta^2 k}{(2n+1)}\right]\left[\frac{(-12n^2+6n)}{(2n+1)k} + 2n + 1\right]}\left(\frac{1}{\delta}\right)\ln(\delta t + d), \tag{60}$$

where d is a constant of integration, we can also expressed scalar field in the form of redshift as,

$$\phi = \ln(\phi_0) \pm \sqrt{\frac{1}{8\pi}\left[\frac{3\delta^2 k}{(2n+1)}\right]\left[\frac{(-12n^2+6n)}{(2n+1)k} + 2n + 1\right]}\left(\frac{1}{\delta}\right)\ln\left(\frac{\left(\frac{a_0}{1+z}\right)^k}{k}\right). \tag{61}$$

Here only negative sign is compatible for physical consistency.



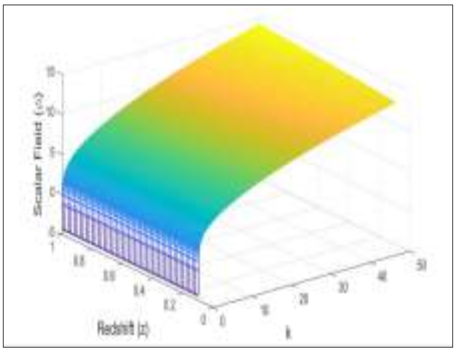


Fig. 17 Plot scalar field (ϕ) versus redshift (z) n=0.48, $\delta = 0.5$, k=1.5, $a_0 = 0.01$, **Fig. 18** Plot scalar field (ϕ) versus redshift (z) n=0.48, $\delta = 0.5$, $a_0 = 0.01$

From equation (57) we have plotted figs. 17 and 18 represents that the scalar field increasing w. r. t. redshift (z) it corresponds to expanding nature of the universe.

3. Conclusion

In this paper, we have studied the Bianchi Type-III cosmological models with constant deceleration parameter. The kinematics properties of the model are also discussed. The model with $k = 0$, EoS decreasing with the increasing of redshift, and for $k \neq 0$, increases EoS with the increasing of redshift follows the recent [24] observations. The equation of state parameter ω is found to be time dependent for $k = 0$, and independent of time for $k \neq 0$. It is observed that $\omega \cong -0.58 < -\frac{1}{3}$ as time approaches negative infinity for $k = 0$. The condition for the expanding Universe is $k > 0$ and all the physically viable models of the expanding Universe exist only for this condition. The model has been clarified and explicit forms of scale factors have been obtained in each case. Which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the Universe. This class of solutions is compatible with the recent observations of Supernovae Ia that require the present Universe to be accelerating.

Acknowledgement: The authors are thankful to the Referee for valuable comments and suggestions.

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