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EFFECT OF INITIAL STRESS AND IMPERFECT INTERFACE ON LOVE WAVES PROPAGATION IN PRESTRESSED ORTHOTROPIC LAYER COATED OVER A PRESTRESSED ORTHOTROPIC SEMI-INFINITE SPACE

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Abstract: This paper contains the dispersion equation of Love waves propagating in a prestressed orthotropic layer coated over a prestressed orthotropic semi-infinite space. Cases for welded contact and spring contact are considered separately. The layer's surface is assumed to be traction-free. The comparison of the velocity curve for medium with initial stress and without initial stress has been examined graphically. The effect of initial stress on the velocity of Love waves with spring contact has been shown graphically.

Keywords: orthotropic, spring contact, Love waves, velocity, welded contact.

1. Introduction

Surface waves are the most destructive type of elastic wave and travel at low velocity. Love waves are surface waves in which particles vibrate in a horizontal direction. The interior of earth can be studied from the information which can collected through waves propagating in different composite mediums. The stress which exists in the absence of external body forces in a body is known as initial stress. Physical factors like cold working, variations in gravity, and the slow process of creep develop initial stress in the medium. Also, the earth is an elastic medium under high initial stress. Therefore, it becomes very important to consider the effect of initial stress on the propagation of waves in the body or over the surface of the earth. The propagation of Love waves in prestressed inhomogeneous layers due to point sources is discussed by Chattopadhyay and Kar [3]. Vishwakarma et al. [15] derived a dispersion relation for Love waves propagating in an orthotropic layer which is perfectly bonded with a layered anisotropic-porous material semi-infinite space. Saha et al. [13] examined Love waves in a prestressed heterogeneous orthotropic layer. The effect of initial stress on Love wave phase velocity is investigated by Pandit and Kundu [12]. Contact of the layer with orthotropic semi-infinite space is considered as welded in this paper. Gupta et al. [7] investigated the effect of rigid boundary and initial stress on the phase velocity of Love waves propagating in a layered semi-infinite space. The effect of initial stress on the phase velocity of Love waves propagating in a layer in welded contact with semi-infinite space is discussed by Kundu et al. [10], Gupta et al. [6] and more. Gupta et al. [6] observed that anisotropy, initial stress, and porosity affect Love wave velocity. Love wave propagation in a porous layer coated over a semi-infinite space is investigated by Chhattarj et al. [2], Madan et al. [11], Vaishnav et al. [14], Wang [16] and more. Due to faults or thermal mismatch, some defects may occur at the time of manufacturing some materials. These defects give an imperfect interface. Problems that explore the effect of elastic constants and material gradients on the phase velocity of surface waves are very helpful. Goyal and Sahu [5] examined Love waves propagation in a piezo layered substrate in which an imperfect interface is considered. In this paper, it is observed that phase velocity decreases as wave number increases. Chaudhary et al. [4] investigated the Love wave propagation in the piezoelectric material layer. This layer is considered in imperfect bonding with elastic semi-infinite space in this paper. Jin et al. [9] examined the effect of imperfect bonding of the piezoelectric layer with the substrate on the propagation of Love waves. Hua et al. [8] analyze the propagation of Love waves in layered graded composites in which imperfect bonding is considered.

The crust of earth contains soft and hard materials, that may have orthotropic property and the thermal or mechanical properties of orthotropic materials are unique. These facts inspire us to examine Love waves in an orthotropic medium. Also there is discontinuity of layer(crust) and half-space(mantle) inside the earth. So the effect of this discontinuity on the velocity of Love waves may help geologists to estimate the cause of earthquake. In the above papers, Love waves propagation in a prestressed orthotropic layer over a prestressed orthotropic semi-infinite space is not discussed. In this paper, the propagation of Love waves in a prestressed orthotropic layer lying over an orthotropic semi-infinite space with a perfect and imperfect interface is discussed. The effect of free surface and initial stress on phase velocity is examined.

2. Basic theory and problem formulation

The equation of motion in orthotropic initially stressed medium is given by (Biot [1])

$$Q_1 \frac{\partial^2 v_2}{\partial z^2} + \left(Q_3 - \frac{P}{2}\right)\frac{\partial^2 v_2}{\partial x^2} = \rho \frac{\partial^2 v_2}{\partial t^2} \tag{1}$$

where S_{11} is initial stress component in x direction, $P = -S_{11,}$ ρ is density, v_2 is component of displacement and Q_1, Q_3 are the incremental elastic coefficients.

Consider a prestressed orthotropic semi-infinite space $z \geq 0$ superimposed by a prestressed orthotropic layer $-h \leq z \leq 0$ of width h (Fig. 1). Let the surface of the layer be traction-free. Quantities associated to semi-infinite space and layer have alike notation but prominent by bar when relate with the layer. Here we consider the antiplane-strain problem in such a way that

$$v_2 = v_2(x, z, t), \bar{v}_2 = \bar{v}_2(x, z, t)$$

where $v_2, \bar{v}_2$ are the displacement components, t is time.

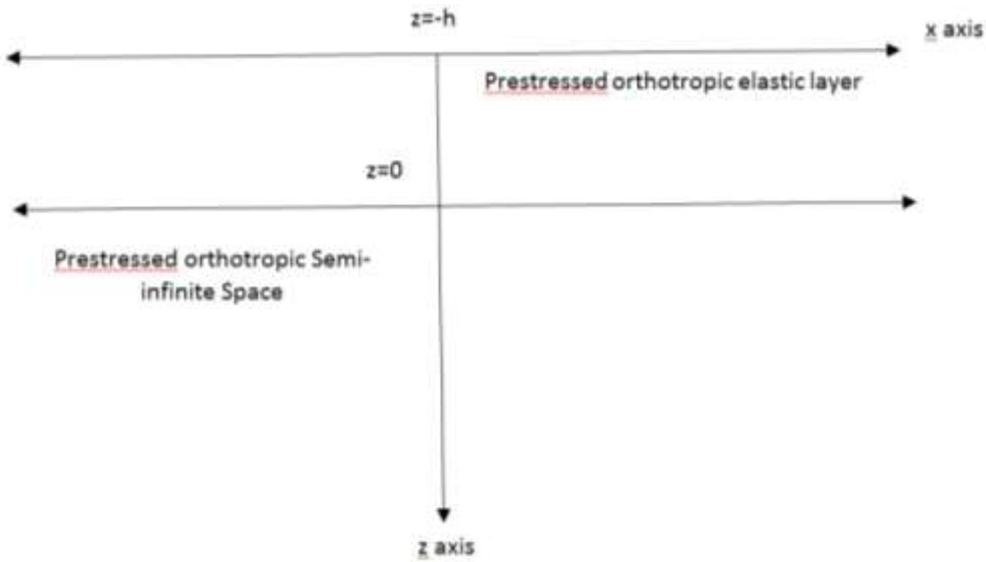

Fig. 1. Geometry of the problem.

Let $P = -S_{11}, \bar{P} = -\bar{S}_{11}, P_H = \frac{P}{Q_3}, P_L = \frac{\bar{P}}{\bar{Q}_3}, e_1 = \frac{Q_3 - \frac{P}{2}}{Q_3}, e_2 = \frac{Q_1}{Q_3}, r_u = \frac{\bar{Q}_3}{Q_3}, c_2 = \sqrt{\frac{Q_3}{\rho}},$

$$\bar{c}_2 = \sqrt{\frac{\bar{Q}_3}{\bar{\rho}}}, r_v = \frac{c_2}{\bar{c}_2}, c_T = \frac{kQ_3}{k_T}, C = \frac{c^2}{\bar{c}_2^2}, s_{23} = \frac{\sigma_{23}}{Q_3}, \bar{s}_{23} = \frac{\bar{\sigma}_{23}}{\bar{Q}_3}, v = \frac{v_2}{h}, \bar{v} = \frac{\bar{v}_2}{h}, \quad (2)$$

where k_T is stiffness constant; σ_{23} is component of stress in half-space; $\bar{\sigma}_{23}$ component of stress in layer.

Stresses in orthotropic initially stressed medium (in dimensionless form) are given by

$$\bar{s}_{23} = \bar{e}_2 \frac{\partial \bar{v}}{\partial z}, \quad (3)$$

The Equation of motion (in dimensionless form) is,

$$\bar{e}_1 \frac{\partial^2 \bar{v}}{\partial x^2} + \bar{e}_2 \frac{\partial^2 \bar{v}}{\partial z^2} = \frac{1}{\bar{c}_2^2} \frac{\partial^2 \bar{v}}{\partial t^2} \quad (4)$$

Displacements of the Love waves in the layer, that satisfy Equation (4), are given by

$$\bar{v} = \bar{V}(x_3) e^{ik(x-ct)} \quad (5)$$

where $x_3 = kz$ and

$$\bar{V}(x_3) = G_1 \cosh(m x_3) + G_2 \sinh(m x_3)\ , \quad (6)$$

where $m^2 = \frac{\bar{e}_1 - C}{\bar{e}_2}$ (7)

Using Equations (5) and (6) in Equation (3) leads to

$\bar{s}_{23}$=$k\bar{S}(x_3)e^{ik(x-ct)}$, (8)

where

$\bar{S}(x_3) = m\bar{e}_2(G_1\sinh(mx_3) + G_2\cosh(mx_3),$ (9)

In the semi-infinite space z>0, displacement of Love waves is given by,

$v = V(x_3)e^{ik(x-ct)}, x_3 = kz$ (10)

where $V(x_3) = Ge^{-gx_3}$, where, $g^2 = \frac{e_1 - \frac{c}{r_v^2}}{e_2}$ (11)

Introducing Equations (10) and (11) into Equation (3) without bars lead to

s_{23}=$kS(x_3)e^{ik(x-ct)}$, (12)

In which

$S(x_3) = -e_2 gGe^{-gx_3}$. (13)

In this paper, we will derive dispersion relation for welded and spring contact.

3. Dispersion equation for welded contact

Let the surface z=-h is traction-free, therefore

$\bar{s}_{23} = 0$ at z=-h (14)

Let layer and semi-infinite space are in perfect bonding with each other, thus

$v = \bar{v}, \bar{s}_{23} = s_{23}$ at z=0. (15)

Using Equations (5) - (13) in Equations (14) and (15) leads to

$-G_1 sinh(me)+G_2 cosh(me) = 0,$

$G_1 - G = 0,$

$m\bar{e}_2 G_2 + ge_2 G = 0$ (16)

If a solution of system Equation (16) is non-trivial, the determinant must vanish i.e.,

$m\bar{e}_2\tanh(me) + \mathrm{g}e_2 = 0$ (17)

This equation is the dispersion equation for Love waves propagating in prestressed orthotropic layer coated over a prestressed orthotropic semi-infinite space having welded contact between them.

4. Dispersion equation for spring contact

As the surface z=-h is considered traction-free, therefore

$\bar{s}_{23} = 0$ at z = -h (18)

Since at z=0, layer and semi-infinite space are in spring contact. Thus,

$$s_{23} = \frac{k_N}{\bar{q}}(v - \bar{v}),\ \bar{s}_{23} = s_{23} \text{ at z=0.} \tag{19}$$

Substituting Equations (5) - (11) in Equations (18) and (19), we have

$$-G_1 sinh(me) + G_2 cosh(me) = 0,$$

$$-G_1 + (1 + ge_2 c_T)G = 0,$$

$$m\bar{e}_2 G_2 + \frac{ge_2}{r_u} G = 0 \tag{20}$$

If a solution of system Equation (20) is non-trivial, the determinant must vanish i.e.,

$$(1 + c_T e_2 g)m\bar{e}_2 \tanh(me) + \frac{ge_2}{r_u} = 0 \tag{21}$$

This equation is the dispersion equation for Love waves propagating in prestressed orthotropic layer coated over a prestressed orthotropic semi-infinite space having spring contact between them.

5. Numerical results and discussion

Here we used the olivine material for layer and topaz material for half-space. The dimensionless parameters are considered as: $e_1 = 1 - \frac{P_H}{2}, \bar{e}_1 = 1 - \frac{P_L}{2}, e_2 = 0.83, \bar{e}_2 = 0.71, r_u = 0.35, r_v = 1.57$. MATLAB programming language is used for plotting the graphs.

In Fig. 2 the effect of initial stress $P_H(0.3,0.4,0.5)$ and $P_L(0.3856,0.4956,0.6)$, on phase velocity of Love waves for welded contact is shown.

In Fig. 3 the effect of initial stress $P_H(0.1,0.2,0.3)$ and $P_L(0.1356,0.2612,0.3856)$, on phase velocity of two modes for welded contact is shown. From Figs 2 and 3, it is found that velocity decreases as layer thickness increases.

In Fig. 4 effect of initial stress $P_H(0.1,0.3,0.5)$ and $P_L(0.1356,0.3856,0.6)$, on phase velocity for spring contact is shown. It is observed that as initial stress increases, velocity of Love waves decreases.

In Fig. 5 effect of initial stress $P_H(0.1,0.5)$ and $P_L(0.1356,0.6)$, on phase velocity for spring contact is shown. The variation of the second and third modes of velocity is observed more against dimensionless layer thickness as compared to the first mode.

Fig. 6 shows the variation in velocity against dimensionless layer thickness for welded contact with initial stress ($P_H(0.5)$, $P_L(0.6)$), without initial stress and spring contact with initial stress ($P_H(0.5)$, $P_L(0.6)$), without initial stress.

Significant effect of initial stress is observed from figures. Also it is observed that discontinuity of layer and half-space affect the velocity of these waves and found that the study of Love waves in a prestressed orthotropic layer lying over an orthotropic semi-infinite space with imperfect interface may help geologists to estimate the cause of earthquake.

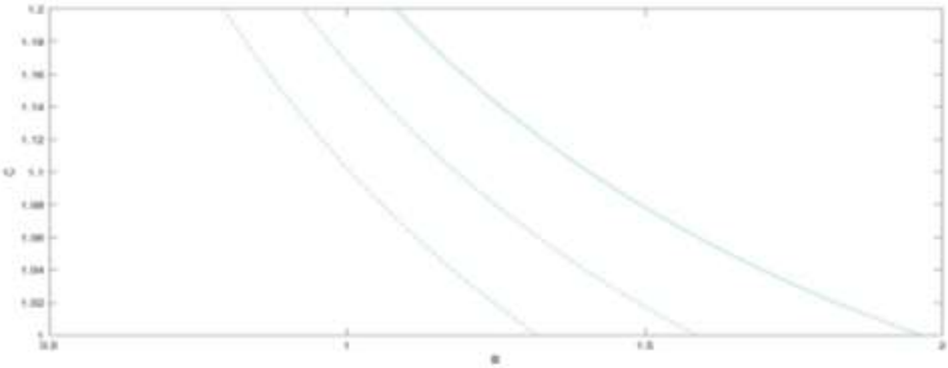

Fig 2. Variation in velocity for $P_H = 0.3$ and $P_L = 0.3856$,(denoted by solid line), $P_H = 0.4$ and $P_L = 0.4956$, (denoted by dash-dotted line), $P_H = 0.5$ and $P_L = 0.6$, (denoted by dashed line) for welded contact

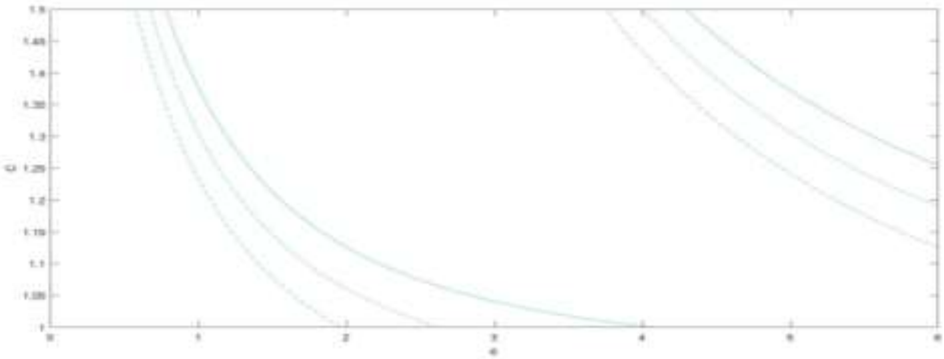

Fig. 3. Variation in velocity for $P_H = 0.1$ and $P_L = 0.1356$,(denoted by solid line), $P_H = 0.2$ and $P_L = 0.2612$, (denoted by dash-dotted line), $P_H = 0.3$ and $P_L = 0.3856$, (denoted by dashed line) for welded contact.

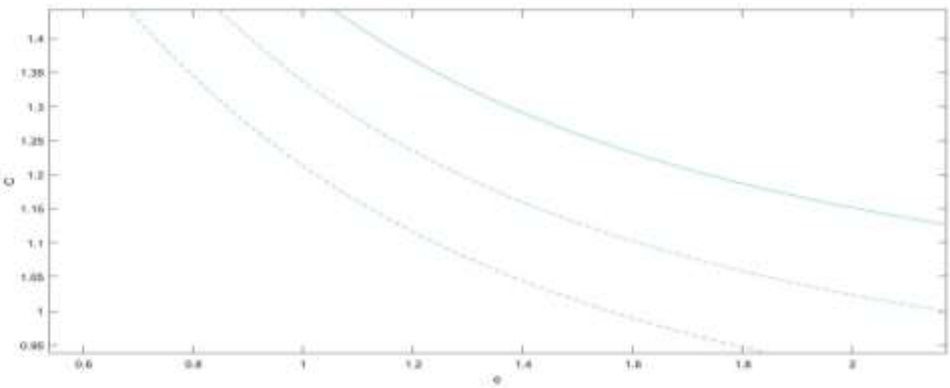

Fig 4. Variation in velocity for $P_H = 0.1$ and $P_L = 0.1356$,(denoted by solid line), $P_H = 0.3$ and $P_L = 0.3856$, (denoted by dashed-dotted line), $P_H = 0.5$ and $P_L = 0.6$, (denoted by dashed line) for spring contact.

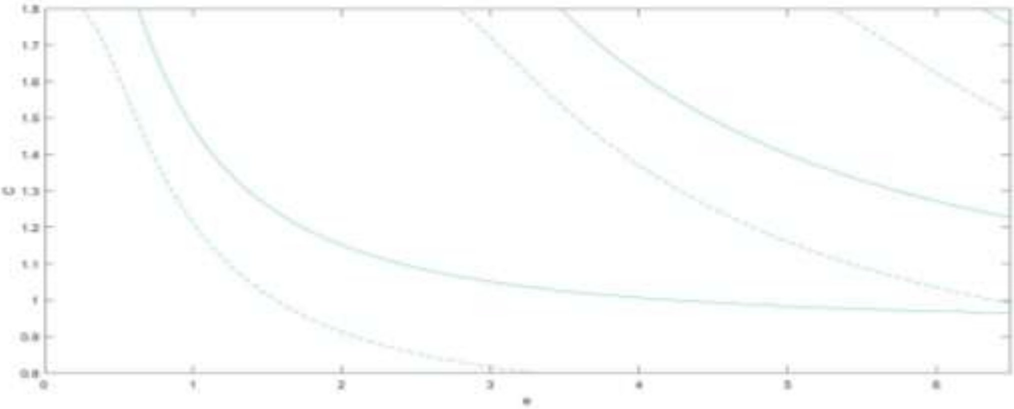

Fig. 5. Variation in velocity for $P_H = 0.1$ and $P_L = 0.1356$,(denoted by solid line), $P_H = 0.5$ and $P_L = 0.6$, (denoted by dotted line) for spring contact.

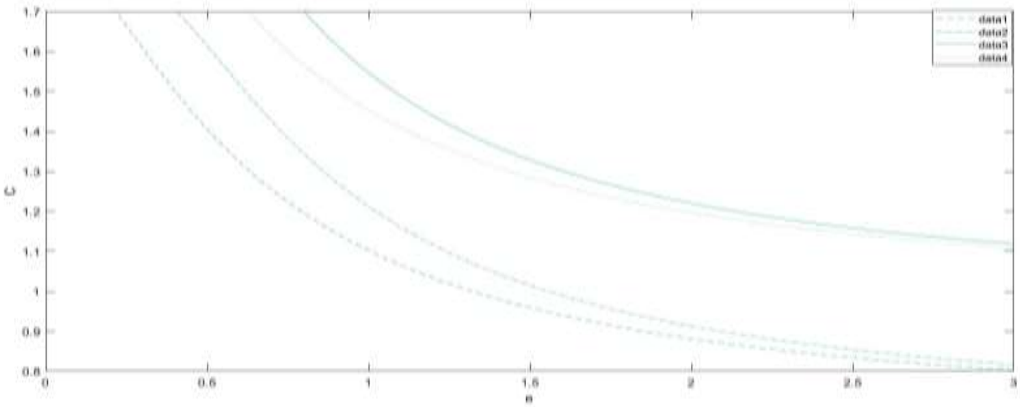

Fig. 6. Variation in velocity for welded contact, $P_H = 0.5$ and $P_L = 0.6$ (denoted by dashed line), $P_H = 0$ and $P_L = 0$, (denoted by dotted line), for spring contact, $P_H = 0.5$ and $P_L = 0.6$, (denoted by dashed-dotted line), $P_H = 0$ and $P_L = 0$, (denoted by solid line).

These figures show that:

1. The Love wave velocity decreases as dimensionless layer thickness increases for both welded and spring contact.
2. Love wave velocity decreases as the value of initial stress increases.
3. The velocity of the second and third modes shows more variation against dimensionless layer thickness as compared to first mode.
4. The velocity curve for welded contact lies below the velocity curve for spring contact.
5. Initial stress strongly affects the velocity of Love waves.

6. Conclusion

The dispersion equation of Love waves in a prestressed orthotropic layer coated over a prestressed orthotropic semi-infinite space with welded and spring contact is derived. A comparison of the velocity curves for welded and spring contact is shown graphically. In this paper, a traction-free model is considered. It is concluded that initial stress and an imperfect interface significantly affect the velocity of Love waves.

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