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ACCELERATING CONCIRCULARLY FLAT SPACE TIME

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Abstract: In the present paper, we study FRW-Cosmological Model of universe filled with dark matter for concircularly flat spacetime. Solution of Einstein field equations with variable (Λ) has been obtained by parametrization of Hubble parameter H in terms of cosmic time t. The pressure, energy density, and Λ are found to be decreasing function of cosmic time t. The physical and kinematical properties of model are also discussed.

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1. Introduction

In general theory of relativity, the curvature tells how space and matter behaves. Concircular curvature tensor in differentiable manifolds and general relativity plays an important role in studying the property of spacetime. The conformal transformation $\widetilde{g_{ij}} = \psi^2 g_{ij}$ does not transform a geodesic circle into geodesic circle. The transformation$\psi_{;i;j} = \varphi g_{ij}$ which transform a geodesic circle into geodesic circle is known as concircular transformation and the geometry that deals with concircular transformation is known as concircular geometry. This transformation is first studied by Yano [12]. Ahsan and Siddiqui [1] studied the results on concircular curvature tensor and fluid spacetimes. The concircular tensor C^*_{hijk} in n-dimensional space time manifold is given by

$$C^*_{hijk} = R_{hijk} - \frac{R}{n(n-1)}(g_{hk}g_{ij} - g_{ik}g_{hj}) \quad (1)$$

where R_{hijk}, R_{ij} is the Riemann curvature tensor and the Ricci-tensor respectively. A space for which C^*_{hijk} vanishes at each point is said to be of concircularly flat manifold. Siddqui and Ahsan [9] has also studied the conhormonic curvature tensor and its significance in relativity. FRW-space time and its physical and kinematical properties has been studied by many authors for different physical and geometrical aspects. Motivated by [1], we study the accelerating FRW-cosmological model for concircularly flat space time by assumming the source of matter distribution as dark matter fluid. In the present paper, our objectiveis to study certain new aspects of FRW-spacetime and its physical behaviour.

The cosmological constant was introduced by Einstien to make pressure and density positive in the formation of field equation to study universe. Cosmological model with variable Λ are becoming popular because it resolves the static model of Einstien's field equation in a smooth way. In order to solve the cosmological constant problem, variable Λ introduced such a way that it was large in the early universe and then it decayed with evolution (Dolgov[4]). Recently many authors worked on FRW-cosmology [5, 3] and [10].

Our discussion of concircular tensor and some basic equations for FRW-space time are presented in next section(2). In section(3) the cosmological solution of field equations has been discussed.

2. Basic Equations

2.1 Concircular tensor in FRW-space time

The FRW - space time metric is described by

$$ds^2 = -dt^2 + a^2(t)[(1-kr^2)^{-1}dr^2 + r^2d\theta^2 + r^2sin^2\theta d\phi^2] \quad (2)$$

where a is cosmic scale factor and function of t and k is curvature index with the value$-1,0,+1$ for open, flat and closed universe respectively. The spatial proper volume $V$and the Hubble's parameter H of the present model are defined as

$$V(t) = 2\pi^2a^3, H(t) = \frac{\dot{a}(t)}{a(t)} \quad (3)$$

Einstein tensor $R_{ij} - \frac{1}{2}Rg_{ij}$ and vanishing divergence of energy momentum tensor T_{ij} imply that Λ is constant. In general relativity and cosmology with variable Λ authors either introduce new terms into left hand side of field equation to cancel the condition of non - zero divergence of Λg_{ij}[2, 10] or describes Λ as source of matter and transfer it to right hand side of the Einstien's field equation [13], in such a way that the energy - momentum conservation is considered as $\tilde{T}_{ij};_k = 0$ where $\tilde{T}_{ij} = T_{ij} - \frac{\Lambda}{8\pi}g_{ij}$. The above two consideration, are similar for a given theory [6].

$$\tilde{T}_{ij} = (\rho + p)u_iu_j + (p - \frac{\Lambda}{8\pi})g_{ij} \quad (4)$$

The equation of state of dark matter is as follows

$$p = \omega\rho \quad (5)$$

where ρ and p are energy density and pressure of dark matter respectively.

The field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\tilde{T}_{ij} \quad (6)$$

The concircular curvature tensor in local coordinates in spacetime is

$$C^*_{hijk} = R_{hijk} - \frac{R}{12}(g_{hk}g_{ij} - g_{ik}g_{hj}) \quad (7)$$

Thus, for a concircularly flat spacetime we have $C^{*}_{hijk} = 0$ which leads to

$$R_{hijk} = \frac{R}{12}(g_{hk}g_{ij} - g_{ik}g_{hj})$$

Now contracting it with indices*h* and *j* by g^{hk}

$$g^{hk}R_{hijk} = \frac{R}{12}(g^{hk}g_{hk}g_{ij} - g^{hk}g_{ik}g_{hj})$$

$$R_{ij} = \frac{R}{12}(4g_{ij} - g_{ij})$$

$$Rg_{ij} = 4R_{ij} \tag{8}$$

In view of (8) equations (6) become

$$R_{ij} = 8\pi\tilde{T}_{ij} \tag{9}$$

which gives the following equation :

$$3a\ddot{a} = -(8\pi\rho + \Lambda)a^2 \tag{10}$$

$$a\ddot{a} + 2\dot{a}^2 + 2k = (8\pi p - \Lambda)a^2 \tag{11}$$

From above, we have the relation :

$$\frac{d}{dt}\left(\frac{\dot{a}}{a}\right) = ka^{-2} - 4\pi(\rho + p) \tag{12}$$

Here over dot denote differentiation with respect to cosmic time t.

3. Cosmological Solution

In order to solve the field equation, we have considered the following parametrization of Hubble's parameter H function of cosmic time t [8].

$$H(t) = \beta t \tag{13}$$

Where $\beta > 0$. The Deceleration parameter q is define by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{14}$$

From (3) and (13) we obtained

$$a(t) = Ce^{\beta\frac{t^2}{2}} \tag{15}$$

Where C is constant

Now using (15) in (14), we get

$$q = -1 - \frac{1}{\beta t^2} \tag{16}$$

This shows that the deceleration parameter is negative for this model. using (15) into the field equation (10) and (11) and solving with (5) we find the energy density, pressure and Λ

$$\rho = \frac{[Ce^{-\beta t^2} - \beta]}{1+\omega} \tag{17}$$

$$p = \omega \frac{[Ce^{-\beta t^2} - \beta]}{1+\omega} \tag{18}$$

$$\Lambda = -3 - \frac{3}{\beta t^2} - 8\pi \frac{[Ce^{-\beta t^2} - \beta]}{1+\omega} \tag{19}$$

From equation (15),the exact concircularly flat model of universe is

$$ds^2 = -dt^2 + Ce^{\beta\frac{t^2}{2}}[(1-kr^2)^{-1}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2] \tag{20}$$

The expansion expression Θ and spatial volume V for the model are given by

$$\Theta(t) = u^i_{;i} = 3\beta t \tag{21}$$

$$V(t) = 2\pi^2[Ce^{\beta\frac{t^2}{2}}] \tag{22}$$

The red shift is given by

$$z = e^{\beta\frac{t_0^2 - t_1^2}{2}} - 1, t_0 > t_1 \tag{23}$$

and Hubble's time is

$$H_0^{-1} = (\beta t_0)^{-1} \tag{24}$$

t_0 being the present age of the universe.

4. Physical behaviour of the Model

From the above observation it is clear that the standard big bang model shows initial singularity but our model are initially without singularity. At initial stage i.e. at t = 0, density and pressure are positive when c > β. As $t \to 0$ the energy density, pressure, cosmological constant become negative and the spatial volume become finite while the standard big-bang model tells about very high energy density. As $t \to \infty$ the energy density, pressure, cosmological constant become negative and spatial volume become infinite showing expanding behavior of universe. The negative energy density and pressure shows violation of positive energy condition. The negative value of cosmological constant indicates initial stage of evolution of universe as mentioned by Pedram et al.[7].

5. Concluding Remarks

In this paper, we have discussed the accelerating concircularly flat space time and we find a new solution in case of universe filled with dark matter. In order to determine complete cosmological solution, we have considered parametrization of Hubble' parameter as function of cosmic time t and obtained the interesting result that for this model initially there is no singularity and energy density, pressure, cosmological constant are finite and as $t \to \infty$ the cosmological parameters energy density, pressure and cosmological constant become negative.

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