Journal of Rajasthan Academy of Physical Sciences
ISSN: 0972-6306; URL: http://raops.org.in
Vol.20, No.3&4, July-December, 2021, 257-268

BIANCHI TYPE VI_h STRING COSMOLOGICAL MODEL FOR STIFF PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

Vaishali Gore[1], D.S. Chouhan[2] and Arvind Sharma[3]
[1,2,3]Department of Mathematics, Sir Padampat Singhania University, Udaipur,
Chittorgarh Rd, Bhatewar, Rajasthan-313601, India
E-mail: [1]vaishali.gore@spsu.ac.in, [2]ds.chouhan@spsu.ac.in,
[3]sharma.arvind@spsu.ac.in

Abstract

The large scale distribution of galaxies in our universe shows that the material distribution can be satisfactorily distributed by perfect fluid and there is existence of another exotic fluid, a stiff fluid with an equation of state $p = \rho$. In this paper we have studied Bianchi type VI_h string cosmological model for stiff perfect fluid distribution in general relativity. To obtain the deterministic solution of Einstein's field equations, we assume the condition $A = \beta B^n$. Some physical and geometrical parameters of the model are discussed.

Keywords: Bianchi-VI_h space time, perfect fluid, cosmic string, general relativity.

MSC 2010 No.: 83C05, 83F05

1. Introduction

In the universe one of the greatest cosmological mysteries is structure formation. The observational data by the luminosity of type-Ia supernovae and cosmic microwave background radiation of the universe described accelerated universe and energy density of the universe is in the form of dark energy [9, 14, 20]. Also there is another exotic fluid which is stiff fluid with equation of state $p = \rho$ where p is the pressure and ρ is density of stiff fluid. The equation of state parameter of this fluid assumes the largest value (equal to 1) compatible with cause and effect because the speed of sound in this fluid is equal to the speed of light. Zeldovich was the first who studied the model with stiff fluid first [32]. The accelerating phase and the various observational cosmological facts of the present universe are well explained by Bianchi cosmological model. Bali R. and Poonia Laxmi studied Bianchi Type VI0 inflationary cosmological model in general relativity [3]. Also, it is believed that for the large structure formation in the universe strings may be one of the source [13, 31]. In recent years there has been a keen interest in the study of string cosmological models in general theory of relativity. These models play an important role in the study of early stage universe. In earlier study, exact solution of string cosmology in various Bianchi space time have been studied by Bali R and Singh Deo Karan[6], Bali R. and Pradhan A. [4], Tikekar R and Patel L.K. [27], Bhattacharjee et al. [7], Krori et al [17]. Tikekar R. and Patel L.K. studied some exact solutions in Bianchi VI_0 string cosmology [26]. Bijan Saha studied Bianchi type-VI model with cosmic strings in the presence of a magnetic field [8]. Sharma A et al. [25], Jain K et al. [15], Ladke L.S., et al. [18], Gore et al. [12] have investigated string cosmological models in different Bianchi space time.

Bali and Pareek investigated Bianchi Type III magnetized massive string cosmological model for perfect fluid distribution in general relativity [2]. Bali and Sharma [5] investigated Bianchi type-I stiff fluid magnetized cosmological model in general relativity. Bali R. et al. [1] investigated Bianchi Type I Magnetized Barotropic Perfect Fluid Cosmological Model in General Relativity. Das et al. [11] prescribed most general solution for perfect fluid cosmological universe.

Roy and Prasad [23] have obtained solutions, which generalize inhomogeneous Bianchi type VIh cosmological models with perfect fluid. Tripathi et al. [29], Mishra et al. [19], Santhi et al. [30] investigated Bianchi type VIh model in different aspects.

Motivated by the previous investigations, in this paper we have focused on the Bianchi type VIh string cosmological model for stiff perfect fluid distribution in general relativity. The scheme of the paper is as follows. Section 2 deals with the Bianchi type VI_h metric and field equations in the presence of string for stiff perfect fluid. In section 3 we have discussed the solution of the field equations of the model with special solution for $n = 1/2$ and physical parameters whereas in section 4 concluding remarks are given.

2. The metric and field equations

Consider Bianchi type VI_h metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2hx} dy^2 + C^2 e^{2hx} dz^2 \tag{1}$$

where metric potentials A, B and C are functions of ‘t’ only.

The energy momentum tensor for the cloud of string in the presence of perfect fluid distribution is given by

$$T_i^j = (\rho + p) v_i v^j + p g_i^j - \lambda x_i x^j \tag{2}$$

where

p being the pressure, ρ the density

λ is string tension density

x_i is the unit space like vector specifying the direction of string satisfying the conditions

$$v_i v^i = -x_i x^i = -1, \text{ and } v^i x_i = 0 \tag{3}$$

The flow vector v^i satisfying

$$g_{ij}v^iv^j = -1 \tag{4}$$

The co-ordinates are considered to be co moving so that $v^1 = 0 = v^2 = v^3$ & $v^4 = 1$ for the line element (1)

Choose x^i parallel to x-axis so that x^i = (A^{-1}, 0, 0, 0)

The Einstein's field equation (in gravitational units $c = 1, 8\pi G = 1$)

$$R_i^j - \frac{R}{2} g_i^j = -T_i^j \tag{5}$$

For the line element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{h^2}{A^2} = \lambda - p \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{h^2}{A^2} = -p \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{h^2}{A^2} = -p \tag{8}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{h^2}{A^2} = \rho \tag{9}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{10}$$

3. Solution of the field equations

The field equations (6) – (10) are a system of five equations with six unknown parameters $A, B, C, \lambda, p, \rho$.

To obtain explicit solution of the model one additional constraint is required.

We assume the condition $A = \beta B^n$ (11)

Equation (10) leads to

$$C = \alpha B \tag{12}$$

where α is constant of integration

For simplicity we assume $\alpha = 1$

$\therefore B = C$

Now, equations (6), (7) and (8) becomes

$$\frac{2B_{44}}{B} + \frac{{B_4}^2}{B^2} + \frac{h^2}{A^2} = \lambda - p \tag{13}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{h^2}{A^2} = -p \tag{14}$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{h^2}{A^2} = \rho \tag{15}$$

For Stiff Fluid

$$p - \rho = 0 \tag{16}$$

Adding equation (14) and (15) we get,

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + 3\frac{A_4 B_4}{AB} + \frac{{B_4}^2}{B^2} - \frac{2h^2}{A^2} = 0 \tag{17}$$

After using the condition $\frac{A_4}{A} = n\frac{B_4}{B}$ in equation (17)

Equation (17) leads to

$$(n+1)\frac{B_{44}}{B} + (n+1)^2 \frac{{B_4}^2}{B^2} - \frac{2h^2}{A^2} = 0 \tag{18}$$

$$2ff' + 2(n+1)\frac{B_4^2}{B} = \frac{4h^2}{(n+1)\beta^2}\frac{1}{B^{2n-1}}$$

Where $B_4 = f(B), B_{44} = ff', f' = \frac{df}{dB}$

$$f^2 = \frac{h^2}{\beta^2(n+1)} B^{-2(n-1)} + CB^{-2(n+1)}$$

Where C is the constant of integration.

$$\frac{dB}{\sqrt{\frac{h^2}{\beta^2(n+1)} B^{-2(n-1)} + CB^{-2(n+1)}}} = t + M$$

Where M is the constant of integration.

Using suitable transformation the line element (1) reduces to

$$ds^2 = \frac{-dT^2}{\frac{h^2}{\beta^2(n+1)}B^{-2(n-1)}+CB^{-2(n+1)}} + T^2(\beta^2 T^n dx^2 + e^{-2hx}dy^2 + e^{2hx}dz^2) \tag{19}$$

Where $B = T$

Special Solution: $\boldsymbol{n = \frac{1}{2}}$

With the condition $A = \beta B^{1/2}$ equation (18) reduces to

$$B_4^2 = \frac{2}{3}\frac{h^2}{\beta^2}B + L\,B^{-3}$$

where L is the constant of integration.

To get the solution in terms of cosmic time t, we assume that $L = 0$

$$B_4 = \sqrt{\frac{2}{3}}\frac{h}{\beta}B^{1/2}$$

$$B = \frac{1}{4}\left(\sqrt{\frac{2}{3}}\frac{h}{\beta}t + c_1\right)^2$$, where c_1 is the constant of integration.

$$B = \frac{T^2}{4} \tag{20}$$

$$where, \quad T = \sqrt{\frac{2}{3}}\frac{h}{\beta}t + c_1$$

$$\therefore dt = \sqrt{\frac{3}{2}}\frac{\beta}{h}dT$$

The line element (1) becomes

$$ds^2 = \frac{-3\beta^2}{2h^2}dT^2 + \frac{T^4}{16}\left(\beta^2 T^{-2}dx^2 + e^{-2hx}dy^2 + e^{2hx}dz^2\right) \tag{21}$$

3.1 Physical and Geometrical Features

The model (21) starts with a big bang at $T = 0$ and rotation (ω) is identically zero. i.e. $\omega = 0$

The Hubble parameter is given by

$$H = \frac{5}{3}\sqrt{\frac{2}{3}}\frac{h}{\beta T} \qquad (22)$$

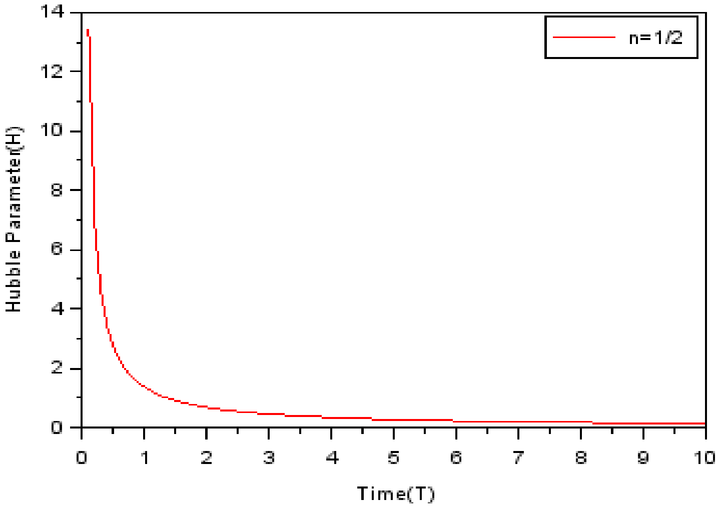

Fig.1 The plot of Hubble Parameter (*H*) versus Time (*T*)

Figure 1 shows that the Hubble parameter value decreases as time increases i.e. $\mathrm{H} \to 0$ as $T \to \infty$

The deceleration parameter (*q*) for the model is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \qquad (23)$$

$$q = \frac{d}{dt}\left(\frac{3\sqrt{3}\,\beta\,T}{5\sqrt{2}\,h}\right) - 1$$

$$q = -\,0.4$$

The value of deceleration parameter is in the range of observed value. The universe is accelerating at present as observed in recent observations of Type Ia supernova [21, 24, 28, 22, 10]. From this data the deceleration parameter of the universe is in the range $-1 \le q \le 0$

The expansion (θ), shear (σ), pressure (p), density (ρ), string tension density (λ), Volume (*V*) and Hubble Parameter (H) for the model (21) are given by

$$\theta = 5\sqrt{2/3}\,\frac{h}{\beta T} \qquad (24)$$

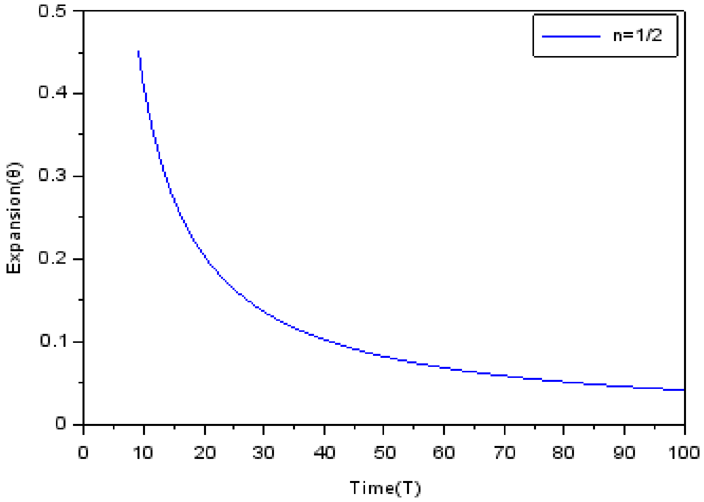

Fig.2 The plot of expansion (*θ*) versus Time (*T*)

Figure 2 depicts that the expansion in the model (30) decreases as time increases.

Shear of the model (21) is given by

$$\sigma = \frac{\sqrt{2}}{3}\frac{h}{\beta}.\frac{1}{T} \tag{25}$$

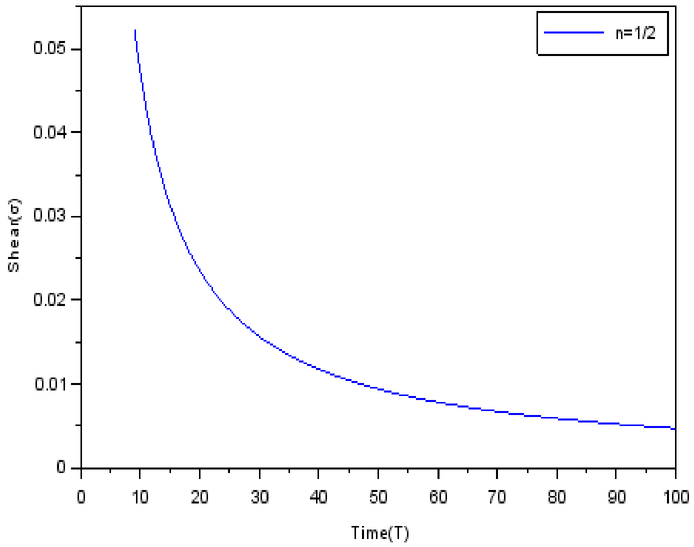

Fig.3 : The plot of Shear (σ) versus Time(T)

Figure 3 shows that the Shear (σ) is decreasing function of time (*T*) and approaches to zero as time (*T*) approaches to infinity.

$$\frac{\sigma}{\theta}=\frac{1}{5\sqrt{3}}\neq 0$$

$$p=\frac{16h^2}{3\beta^2T^2}=\rho \tag{26}$$

$$V=\beta\left(\frac{T}{2}\right)^5 \tag{27}$$

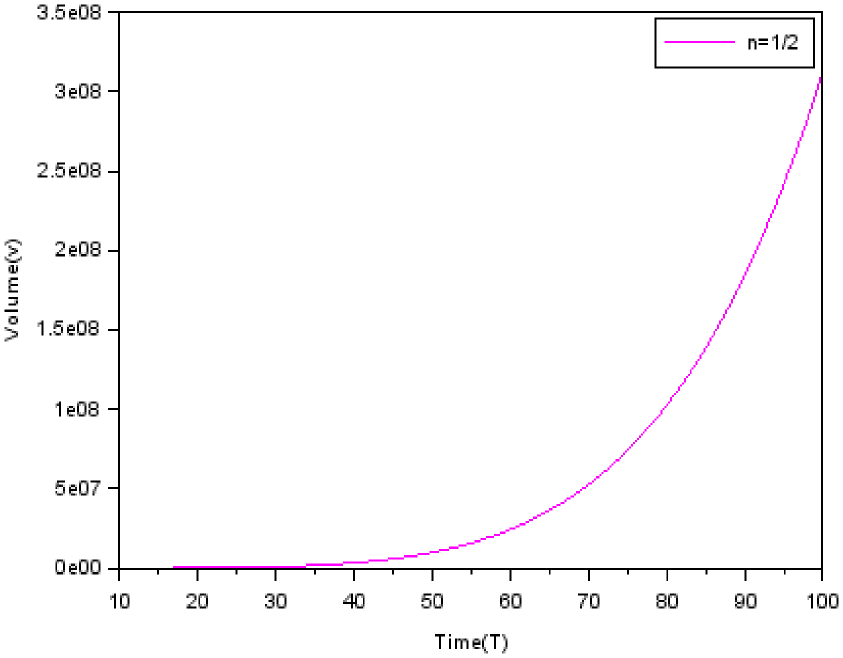

Fig.4: The plot of volume (*V*) versus Time (*T*)

Figure 4 represent the volume *V* of the universe is zero at the initial epoch $T = 0$ (*if* c = 0)

As time increases, volume of the universe increases i.e. T → ∞, V → ∞

$$\lambda=\frac{32}{3}\frac{h^2}{\beta^2}\frac{1}{T^2} \tag{28}$$

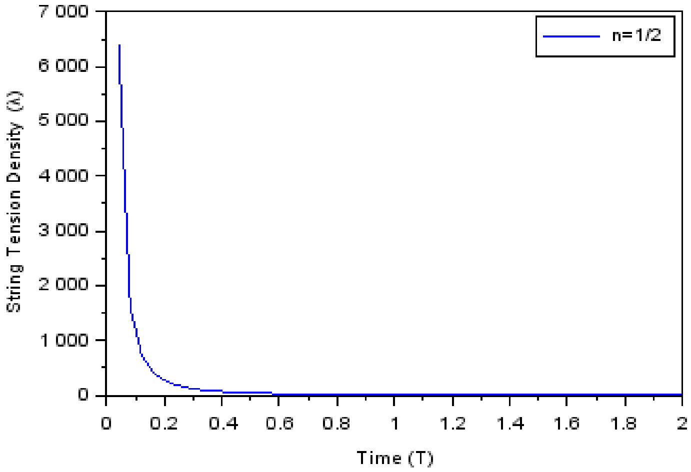

Fig.5: The plot of String Tension Density (λ) versus Time (T)

Figure 5 shows the string vanishes for $T \to \infty$ and the universe is dominated by the string for $T \to 0$.

4. Conclusion and Discussion

In this paper we have studied Bianchi type VI$_h$ string cosmological model for stiff perfect fluid distribution in general relativity. In recent observations of Type Ia supernova the present universe is accelerated and expanding. The model starts with the big bang at $T = 0$. The deceleration parameter $q < 0$ (accelerating phase) for $n = 1/2$ and the value of this deceleration parameter is in the range of observed value. The ratio of shear σ and the expansion θ tends to a finite value $\frac{\sigma}{\theta} = \frac{1}{5\sqrt{3}} \neq 0$, therefore model does not approach isotropy. The cosmic string vanishes for $T \to \infty$ and the universe is dominated by the string for $T \to 0$.

In general the model is expanding, shearing, non-rotating universe.

Acknowledgement: The authors are thankful to the Referee for valuable comments and suggestions.

References

[1] Bali R., Gupta, R. and Goyal, R. (2009). Bianchi Type I Magnetized Barotropic Perfect Fluid Cosmological Model in General Relativity. Int J Theor Phys **48**, 1186–1193.

[2] Bali R and Pareek Umesh K.(2008). Bianchi Type III Magnetized massive String Cosmological Model for Perfect Fluid Distribution in General Relativity. Astrophys Space Sci **318:** 237-242.

[3] Bali R. and Poonia Laxmi (2013). Bianchi Type VI0 Inflationary Cosmological Model in General Relativity. International Journal of Modern Physics: Conference Series **22**, 593–602

[4] Bali R. and Pradhan A. (2007) Bianchi Type-III String Cosmological Models with Time Dependent Bulk Viscosity. Chin. Phys. Lett. **24**(2), 586.

[5] Bali R. and Sharma K. (2003). Bianchi type-I Stiff Fluid Magnetized Cosmological Model in General Relativity, Astrophys Space Sci, **283**, 11-22.

[6] Bali R and Singh Deo Karan (2005) Bianchi Type-V Bulk Viscous Fluid String Dust Cosmological Model in General Relativity, Astrophys Space Sci. **300**, 387–394.

[7] Bhattacharjee R., Baruah K.K., (2001), String Cosmologies with Scalar Field, Indian J. Pure Appl. Math. **32**, 47-53.

[8] Bijan Saha, Mihai Visinescu. (2010), Bianchi type-VI model with cosmic strings in the presence of a magnetic field, Rom. J. Phys. **55**, 1064-1074.

[9] Clocchiatti A., et al. (2006) Hubble Space Telescope and Ground-Based Observations of Type Ia Supernovae At Redshift 0.5: Cosmological Implications, The Astrophysical Journal, **642**, 1-21.

[10] Cunha, C.E., Lima, M., Ogaizu, H., Frieman, J., Lin, H. (2009), Estimating the redshift distribution of photometric galaxy samples-II. Applications and tests of a new method, Mon. Not. R.Astron. Soc.**396**, 2379-2398.

[11] Das, A., Banerjee, A., Chakraborty, S. et al. (2018). Perfect fluid cosmological Universes: One equation of state and the most general solution. Pramana - J Phys **90**, 19.

[12] Gore V, Chouhan D.S., Sharma A., (2020). Homogeneous Bianchi Type String VI0 Cosmological Model for Anti-Stiff Perfect Fluid Distribution in General Relativity, Solid State Technology, **63**(1), 927-935.

[13] Hindmarsh M.B., Kibble T.W.B. (1995). Cosmic Strings, Rept. Prog. Phys, **58**, 477-562.

[14] Hinshaw G. et al., (2003). First-Year Wilkinson Microwave Anisotrop Probe (WMAP) Observations: The angular Power Spectrum, ApJS **148**, 135-159.

[15] Jain K., Chhajed D., Tyagi A., (2019). Magnetized LRS Bianchi Type-I Massive String Cosmological Model for Perfect Fluid Distribution with Cosmological Term, International Journal of Scientific Research in Physics and Applied Sciences, **7**, 167-172.

[16] Kibble, T. W. B. (1976). Topology of cosmic domains and strings, J. Phys. A: Math. Gen. **9**(8), 1387-1398.

[17] Krori, K.D., Chaudhury, T., Mahanta, C.R. (1990). Some exact solutions in string Cosmology, Gen Relat Gravit **22**,123–130.

[18] Ladke L.S., Tripade V.P., Mishra R. D. (2021). Bianchi Type-VIh Bulk Viscous String and Fluid Cosmological Models in Gravity, IJARESM, **9**, 1540-1552.

[19] Mishra B., Sahoo P.K., (2014). Bianchi type VI h perfect fluid cosmological model in f(R,T) Theory, Astrophys Space Sci **352**, 331-336.

[20] Perlmutter S., Turner M. S., White M., (1999). Constraining Dark Energy with Type Ia Supernovae and Large-Scale Structure, Phys. Rev. Lett., **83,670**-673.

[21] Riess, A.G., et al. (Supernova Search Team Collaboration), (1998), The Astronomical Journal, **116**, 1009-1038.

[22] Riess, A.G., et al. (2004), Type Ia Supernova Discoveries At Z > 1 from the Hubble Space Telescope: Evidence For Past Deceleration And Constraints On Dark Energy Evolution Astrophysical Journal, **607**, 665-687.

[23] Roy, S.R., Prasad, A. (1991). Inhomogeneous generalizations of Bianchi type VIh models with perfect fluid. Astrophys Space Sci **181**, 61–71.

[24] Schuecker, R., et al. (1998). The Muenster Redshift Project. III Observational Constraints on the Deceleration Parameter, The Astrophysical Journal, **496**, 635-647.

[25] Sharma A., Tyagi, A. & Chhajed, D. (2016). Inhomogeneous Bianchi type-VI$_0$ String Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity, Prespacetime Journal.**7**, 615-622.

[26] Tikekar R. and Patel L.K. (1994). Some exact solutions in Bianchi VI$_0$ string cosmology, PARMANA-Journal of Physics, **42**, 483-489.

[27] Tikekar R., Patel L.K. (1992). Some Exact Solutions of String Cosmology in Bianchi III Space-Time, General Relativity and Gravitation, **24**, 397-404.

[28] Tonry, J.L., et al. (2003). Cosmological Results From High-Z Supernovae, The Astrophysical Journal, **594**, 1–24.

[29] Tripathi, S.K., Behera, D. (2010). Bianchi type-VIh String Cloud Cosmological Models with Bulk viscosity. Astrophys Space Sci **330**, 191-201.

[30] Santhi, Vijaya M.et al (2019). J. Phys. Conf. Ser. 1344 012038.

[31] Vilenkin A., (1981).Cosmic Strings, Phys. Rev. D **24**, 2082-2089.

[32] Ya. B. Zeldovich, (1962). Sov. Phys. JETP 14, 11437.