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INFLATIONARY UNIVERSE IN BIANCHI TYPE-II SPACE-TIME WITH HIGGS FIELD AND FLAT POTENTIAL IN GENERAL RELATIVITY

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Abstract: In this paper, we have investigated inflationary scenario in Bianchi Type II space-time with flat potential and Higgs field in general relativity. We find that Higgs field evolves slowly but universe expands. The model represents inflationary scenario and accelerating universe. The anisotropy is small & the model isotropizes at late time.

Keywords: Inflationary, Bianchi II, Flat Potential, General Relativity.

1. Introduction

Bianchi type II space-time play a significant role in description of early stages of evolution of universe. The importance of Bianchi type-II space time is given by Asseo and Sol [1]. Bianchi type-II models have been studied by Hajj-Boutros [12], Dunn and Tupper [10], Banerjee et al. [7], Bali and Singh [3] to name a few. Astronomical observations support the existence of anisotropic universe that approaches to isotropic phase at large as mentioned by Land and Magueijo [13]. The stage of accelerated expansion of the universe is known as inflation. The notion of inflation is to explain the flat, homogeneous and isotropic nature of the present day universe. In the beginning of 1980, Guth [11] proposed inflationary model of the universe in the context of grand unified theory (GUT) which was accepted soon as model of early universe due to appearance of a flat potential as in GUT phase transition. In inflationary models, the universe undergoes a phase transition due to the evolution of Higgs field ϕ. Inflation occurs if potential $V(\phi)$ has flat region and in this region ϕ evolves slowly but the universe expands in an exponential way due to the vacuum field energy as mentioned by Stein-Schabes [16]. The flat part of the universe is associated with vacuum energy which is identified as cosmological constant (Λ). Barrow and Turner [8] pointed out that the large anisotropy prevents transition into an inflationary era as per Guth original inflationary scenario. Rothman and Ellis [14] have investigated that we can have solution of isotropy problem if we work with anisotropic metrics and these can be isotropized and inflated under very general circumstances. Inflationary cosmological models are also investigated by Bali [4], Chakravorty [9], Bali and Jain [5], Bali and Singh [6] to name a few.

2. The metric and field equations

We consider Bianchi type II space-time in the form

$$ds^2 = -dt^2 + R^2(dx^2 + dz^2) + S^2(dy + xdz)^2 \quad (1)$$

Where R and S are metric potentials and are functions of cosmic time t alone. The Einstein field equations are given by as

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \quad (2)$$

$$T_{ij} = \partial_i \varphi \partial_j \varphi - \left[\frac{1}{2} \partial_\alpha \partial_\varphi^d 1 V(\varphi)\right] g_{ij} \quad (3)$$

The energy conservation law coincides with equation of motion for ϕ and we have

$$\frac{1}{\sqrt{-g}} \partial_i\left(\sqrt{-g}\partial^i \varphi\right) = -\frac{dV}{d\varphi} \quad (4)$$

Which leads to

$$\varphi_{44} + \left(2\frac{R_4}{R} + \frac{S_4}{S}\right)\varphi_4 = -\frac{dV(\varphi)}{d\varphi} \quad (5)$$

The Einstein field equation (2) with (3) for the space-time (1) leads to

$$2\frac{R_{44}}{R} + \frac{R_4^2}{R^2} + \frac{3}{4}\frac{S^2}{R^4} = -\frac{1}{2}\varphi_4^2 + V(\varphi) \quad (6)$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{R_3} + \frac{1}{4}\frac{S^2}{R^4} = 1\frac{1}{2}\varphi_4 + V(\varphi) \quad (7)$$

$$2\frac{R_4 S_4}{RS} + \frac{R_4^2}{R^2} - \frac{1}{4}\frac{S^2}{R^4} = \frac{1}{2}\varphi_4^2 + V(\varphi) \quad (8)$$

3. Solution of field equations

Equations (6) and (7) lead to

$$\frac{R_{44}}{R} + \frac{R_4^2}{R^2} - \frac{S_{44}}{S} - \frac{R_4 S_4}{RS} - \frac{S^2}{R^4} = 0 \quad (9)$$

For deterministic solution of equation (9), we assume that shear (σ) is proportional to the expansion (θ) i.e. $\frac{\sigma}{\theta}$ = constant. This leads to

$$R = S^n \quad (10)$$

where Q and S we metric potentials and, n is a constant. The condition $\frac{\sigma}{\theta}$ = constant is assumed as per Astronomical observations mentioned by Thorne [ ]. Using equation (10) in (9), we have

$$2S_{44} + 4n\frac{S_4^2}{S} = \frac{1}{(n-1)S^{4n-3}} \quad (11)$$

which leads to

$$\left(\frac{ds}{dt}\right)^2 = f^2 = \frac{1}{2(n-1)} S^{4-4n} + \alpha^2 S^{-4n} \quad (12)$$

where $S_4 = f(S)$ and α^2 is constant.

Therefore the metric (1) can be written as

$$ds^2 = -\left(\frac{dt}{ds}\right)^2 ds^2 + S^{2n}(dx^2 + dz^2) + S^2(dy + xdz)^2 \tag{13}$$

After suitable transformation of coordinates, the space-time (13) leads to

$$ds^2 = -\frac{dT^2}{\frac{T^{4-4n}}{2(n-1)} + \alpha^2 T^{-4n}} + T^{2n}(dx^2 + dz^2) + T^2(dy + xdz)^2 \tag{14}$$

Special model

To get the deterministic result in terms of cosmic time t, we assume n = 3/2. Thus equation (12) leads to

$$\frac{S^3 dS}{\sqrt{S^4 + \alpha^2}} = dt \tag{15}$$

From equation (15), we have

$$S^4 = 4\,(t+\gamma)^2 - \alpha^2 \tag{16}$$

Where γ is a constant and

$$R^2 = S^3 = [4(t+\gamma)^2 - \alpha^2]^{3/4} \tag{17}$$

$$\xi = 2(t+\gamma) \Rightarrow \xi^2 = S^4 + \alpha^2 = 4(t+\gamma)^2$$

$$S^4 = 4\tau^2 - \alpha^2$$

$$R^2 = S^3 = (4\tau^2 - \alpha^2)^{3/4}$$

The metric (1) in terms of cosmic time t leads to

$$ds^2 = -d\tau^2 + (4\tau^2 - \alpha^2)^{\frac{3}{4}}(dx^2 - dz^2) + (4\tau^2 - \alpha^2)^{\frac{1}{2}}(dy - xdz)^2 \tag{18}$$

where $t + \gamma = \tau$

4. Physical and geometrical aspects

The spatial volume (V) the expansion (θ), the shear (σ), the deceleration parameter (V) for the model (18) are given by

$$V = R^2 S = 4\tau^2 - \alpha^2 \tag{19}$$

$$\theta = 2\frac{R_4}{R} + \frac{S_4}{S} = \frac{8\tau}{4\tau^2 - \alpha^2} \tag{20}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{R_4}{R} - \frac{S_4}{S}\right) = \frac{\tau}{\sqrt{3}\,4\tau^2 - \alpha^2} \tag{21}$$

$$\frac{\sigma}{\theta} = \frac{1}{8\sqrt{3}} = \text{a very small quantity} \tag{22}$$

$$q = -\frac{\frac{\ddot{V}}{V}}{\frac{\dot{V}^2}{V^2}} = -\frac{(4\tau^2 - \alpha^2)}{8\tau^2} \tag{23}$$

The spatial volume (V), expansion (θ), shear (σ) for the model (13) are given by :

$$V = R^2 S = S^{2n+1} = T^{2n+1} \tag{24}$$

where 2n + 1 > 0.

$$\theta = 2\frac{R_4}{R} + \frac{S_4}{S}$$

$$= (2n+1)\sqrt{\frac{1}{2(n-1)}T^{2-4n} + \alpha^2 T^{-4n-2}} \tag{25}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{R_4}{R} - \frac{S_4}{S}\right)$$

$$= \frac{n-1}{\sqrt{3}}\sqrt{\frac{1}{2(n-1)}T^{2-4n} + \alpha^2 T^{-4n-2}} \tag{26}$$

$$\frac{\sigma}{\theta} = \frac{n-1}{(2n+1)\sqrt{3}} = Constant \tag{27}$$

where n > 1. The rate of Higgs field (ϕ) for the model (14) is given by

$$\phi_4 = \frac{l}{T^{2n+1}} \tag{28}$$

5. Determination of Higgs fields (ϕ)

From equation (5), we have

$$\varphi_{44} + \left(2\frac{R_4}{R} + \frac{S_4}{S}\right)\varphi_4 = 0 \tag{29}$$

as potential V(ϕ) is constant.

Equation (24) leads to

$$\phi = \frac{\beta}{4\alpha} log\left(\frac{\tau - \alpha/2}{\tau + \alpha/2}\right) \tag{30}$$

where β is constant.

6. Conclusion

The spatial volume for the model (IF) increases with time showing inflations scenario. The shear (σ) leads to zero for large time. This indicates that the model isotropizes at late time. Since $\frac{\sigma}{\theta}$ is very small. It indicates that expansion dominates over shear. The deceleration parameter (q) is negative i.e. q < 0, which shows that the model represents accelerating universe. The Hubble parameter decreases with time. The Higgs fields (ϕ) evolves slowly for τ > α/2 but universe expands. In the model (14), the spatial volume increases with time showing inflationary scenario. The deceleration parameter q < 0 which shows accelerating phase of universe. The spatial volume (V) for the model (14) increases with time indicating inflationary scenario. The Higgs field (ϕ) evolves slowly but universe expands. The ratio of σ and θ is very small and the model isotropizes at late time.

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