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C-FIELD COSMOLOGICAL MODEL WITH VARIABLE G AND BULK VISCOSITY IN FRW SPACE-TIME

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Abstract: Cosmological models with variable bulk viscosity and gravitational constant in C-field cosmology for FRW space-time are investigated. To find the deterministic model of the universe, we assume that $G = R^n$ and discussed for n = -1, -2, R being scalar factor. In both the cases, creation field (C) increases with time, G (gravitational constant) decreases with time. For n = -1, ρ (matter density) decreases with time and for n = -2, ρ (matter density) is constant. These results match with the astronomical observations and explained

Key words: C-field cosmology; Variable G and bulk viscosity, FRW space-time.

1. Introduction

In the early stage of cosmic expansion, it has been argued for a long time that the different picture of the universe at the initial stage of cosmological evolution may appear due to dissipative process caused by viscosity as viscosity counteracts the cosmological collapse. The effect of bulk viscosity on the evolution of cosmological models has been studied by number of authors viz. Misner [20,21], Banerjee and Santos [8], Banerjee et al. [7], Bali and Jain [3], Sahni and Starobinski [26], Bali and Pradhan [5], Bali and Kumawat [4], Bali et al. [6], Brevik and Gron [12]. On the basis of large number of hypothesis, Dirac [15] suggested the variation of G with time. Therefore, in an evolving universe, it is natural to consider G = G(t) as G couples geometry to matter. Brans and Dicke [11], Canuto et al. [13] proposed an alternative theory of gravitation for the possible extension of general relativity with time variation of G. Pochoda and Schwarzschild [25], Gamow [16] studied the solar evolution in the presence of time varying gravitational constant. Demarque et al. [14] and Barrow [9] assumed the ansatz $G \propto t^{-n}$ where n is a constant.

The big-bang model based on Einstein’s field equations successfully explains the three important observations in Astronomy: (i) the phenomena of expanding universe, (ii) primordial nucleosynthesis, (iii) the observed isotropy of the cosmic back ground radiation. However, the big-bang model is known to have the short coming in the following aspects: (i) the model has singularity in the past and possible one in future, (ii) the conservation of energy is violated, (iii) it leads to a very small particle horizon, (iv) no consistent scenario exists that explains the origin, evolution and characteristic of structures in the universe at small scale, (v) flatness problem.

Therefore, it is natural to replace the simple 'big-bang' model and consideration of relevant improvement which may take care of the difficulties mentioned above. The attempts to improve upon the big-bang model may be classified into two models (i) quantum cosmological models (ii) inflationary models.

It is unfortunate fact that we do not have a complete quantum theory of gravity. The inflationary models (Linde[19]) are also not without serious drawbacks of their own and does not solve the problem of singularity. Therefore, it is interesting to consider the fact that if a model successfully explains creation of positive energy then it is necessary to have some degree of freedom which acts as a negative energy mode. All quantum gravitational models which describe the creation consistently (Atkatz and Pagels [1], Padmanabhan [24]) use such a energy mode arising from the scale degree of freedom of gravity. Thus introducing a negative-energy field may provide a natural way for creating the matter. Hoyle and Narlikar [18] adopted a field theoretic approach introducing a massless and chargeless scalar field in the Einstein-Hilbert action to account for creation of matter. In C-field (creation field), there is no big-bang type singularity as in the steady state theory of Bondi and Gold [10]. Narlikar [22] has pointed out that the proper consideration of matter creation can resolve the problem of singularity. Narlikar and Padmanabhan [23] have discussed the importance of creation field cosmology and have explained that it is the possible solution to singularity, horizon and flatness problems. Vishwakarma and Narilikar [27] emphasized that creation of matter plays a very crucial role in cosmology and provides a natural explanation to the various explosive phenomena occurring in local and extra galactic universe. Bali [2] investigated barotropic field model in creation field cosmology with variable G using FRW space-time.

In this paper, we have investigated C-field cosmological model with variable bulk viscosity and gravitational constant in the frame work of FRW model for positive and negative curvature. To get the deterministic solution in terms of cosmic time t,, we have assumed $G = R^n$ where n = -1, -2 and R is the scale factor and n is a constant. The physical aspects of the model have been discussed and results match with the astronomical observations.

2. Metric and Field Equations

We consider the FRW space-time as

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right] \qquad \ldots(1)$$

Where $K \neq 0$.

Einstein's modified field equations by the introduction of C-field are given by

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi G\,[{}^m T_i^j + {}^C T_i^j] \qquad \ldots(2)$$

where

$${}^m T_j^i = \rho v_i v^j - \xi\theta\,(v_i v^j - g_i^j) \qquad \ldots(3)$$

$${}^C T_j^i = -f\left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right) \qquad \ldots(4)$$

where $f > 0$ and $C_i = \frac{dC}{dx^i}$. We assume that the coefficient of bulk viscosity (ξ) is inversely proportional to expansion (θ) i.e. ξθ = β (constant). Now the field equations (2) for metric (1) lead to

$$\frac{3\dot{R}^2}{R^2} + \frac{3K}{R^2} = 8\pi G\left[\rho - \frac{1}{2} f\,\dot{C}^2\right] \qquad \ldots(5)$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{K}{R^2} = 8\pi G\left[\frac{1}{2} f\dot{C}^2 + \beta\right] \qquad \ldots(6)$$

3. Solution of Field Equations

The conservation equation

$$[8\pi G\, T_i^j]_{;j} = 0 \qquad \ldots(7)$$

leads to

$$8\pi\dot{G}\left(\rho - \frac{1}{2} f\,\dot{C}^2\right) + 8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + 3\rho\frac{\dot{R}}{R} - 3f\dot{C}^2\frac{\dot{R}}{R} - 3\beta\frac{\dot{R}}{R}\right] = 0 \qquad \ldots(8)$$

which yields $\dot{C} = 1$ when used in source equation.

Equation (5) and (6) lead to

$$\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{2K}{R^2} = 4\pi G\,[\rho + \beta] \qquad \ldots(9)$$

Using $\dot{C} = 1$ in equation (6), we have

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{K}{R^2} = 4\pi G[f + 2\beta] \qquad \ldots(10)$$

To get the deterministic solution in terms of cosmic time t, we assume

$$G = R^{n} \qquad \ldots(11)$$

where n is a constant and R the scale factor.

Equations (10) and (11) lead to

$$2\ddot{R} + \frac{\dot{R}^{2}}{R} + \frac{K}{R} = A R^{n+1} \qquad \ldots(12)$$

Where $A = 4\pi[f + 2\beta]$ …(13)

To get the solution of equation (12), we assume $\dot{R} = F(R)$. This leads to $\ddot{R} = F F'$ with $F' = \frac{dF}{dR}$. Thus equation (12) leads to

$$\frac{dF^{2}}{dR} + \frac{F^{2}}{R} = A R^{n+1} - \frac{K}{R} \qquad \ldots(14)$$

which leads to

$$F^{2} = \frac{A R^{n+2}}{n+3} - K \qquad \ldots(15)$$

where $A = 4\pi[f + 2\beta)]$ and constant of integration has been taken zero for simplicity.

From equation (15), we have

$$\frac{dR}{\sqrt{R^{n+2} - \frac{K(n+3)}{A}}} = \sqrt{\frac{A}{n+3}}\, dt \qquad \ldots(16)$$

To obtain the deterministic value of R in terms of cosmic time t, we consider two cases

Case I: n = -1

Equation (16) for n = -1leads to

$$\frac{dR}{\sqrt{R - \frac{2K}{A}}} = \sqrt{\frac{A}{2}}\, dt \qquad \ldots(17)$$

From equation (17), we have

$$R = (at+b)^2 + \frac{2K}{A} \qquad \ldots(18)$$

where

$$a = \frac{1}{2}\sqrt{\frac{A}{2}} \qquad \ldots(19)$$

$$b = \frac{N}{2} \qquad \ldots(20)$$

where N is a constant of integration. Thus, we have

$$G = R^{-1} = \left[(at+b)^2 + \frac{2K}{A}\right]^{-1} \qquad \ldots(21)$$

From equations (9), (18) and (21), we have

$$8\pi\rho = \frac{20a^2(at+b)^2 + \frac{8ka^2}{A} + 4K}{\left[(at+b)^2 + \frac{2K}{A}\right]} - 8\pi\beta \qquad \ldots(22)$$

After using the value of R given by (18), the metric (1) lead to

$$ds^2 = dt^2 - \left[(at+b)^2 + \frac{2K}{A}\right]^2 \left[\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \qquad \ldots(23)$$

Now equation (8) leads to

$$8\pi\left[G\dot{\rho} + \rho\dot{G}\right] - 4\pi Gf\dot{C}^2 - 8\pi fG\dot{C}\ddot{C} + 24\pi G\rho\frac{\dot{R}}{R} - 24\pi Gf\dot{C}^2\frac{\dot{R}}{R} - 24\pi\beta G\frac{\dot{R}}{R} = 0 \qquad \ldots(24)$$

Using equations (18), (21) and (22) into equation (24), we get

$$\dot{C}^2\left[(at+b)^2 + \frac{2K}{A}\right]^5 = \frac{1}{4\pi f}\int\left\{\frac{-80\pi\beta a(at+b)}{\left[(at+b)^2 + \frac{2K}{A}\right]} + \frac{80a^3(at+b)^3 + \frac{96Ka^3(at+b)}{A} + 8Ka(at+b)}{\left[(at+b)^2 + \frac{2K}{A}\right]^2}\right\} \times \left[(at+b)^2 + \frac{2K}{A}\right]^5 dt \qquad \ldots(25)$$

To find deterministic value of $\dot{C}$, we assume a = 1, b = 0. Thus (25) leads to

$$\dot{C}^2\left[t+\frac{2K}{A}\right]^5=\frac{1}{4\pi f}\int\left\{-80\pi\beta t\left[t^2+\frac{2K}{A}\right]^4+\left[80t^3+\frac{96Kt}{A}+8Kt\right]\left[t^2+\frac{2K}{A}\right]^3\right\}dt \quad \ldots(26)$$

Equation (26) leads to

$$\dot{C}^2\left[t+\frac{2K}{A}\right]^5=\frac{1}{4\pi f}\left\{(8-8\pi\beta)\left[t^2+\frac{2K}{A}\right]^5+\left(\frac{-8K}{A}+K\right)\left[t^2+\frac{2K}{A}\right]^4\right\} \ldots(27)$$

using $a=1, a=\frac{1}{2}\sqrt{\frac{A}{2}}$ and $A=4\pi[f+2\beta]$ in equation (27), we have

$$\dot{C}^2=1 \quad \ldots(28)$$

which leads to

$$C = t \quad \ldots(29)$$

We find $\dot{C}=1$ which agree with the value used in the source equation.

The metric (1) for a = 1, b = 0 leads to

$$ds^2=dt^2-\left[t^2+\frac{2K}{A}\right]^2\left[\frac{dr^2}{1-Kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right] \quad \ldots(30)$$

The homogeneous mass density ρ, gravitational constant G, and deceleration parameter (q) for the model (23) are given by

$$8\pi\rho=\frac{20a^2(at+b)^2+\frac{8Ka^2}{A}+4K}{\left[(at+b)^2+\frac{2K}{A}\right]}-8\pi\beta \quad \ldots(31)$$

$$G=\left[(at+b)^2+\frac{2K}{A}\right]^{-1} \quad \ldots(32)$$

$$q=-\left[\frac{2a^2(at+b)^2+\frac{4a^2K}{A}}{4a^2(at+b)^2}\right] \quad \ldots(33)$$

Case II: n = -2

For n = -2 equation (16) leads to

$$\frac{dR}{\sqrt{1-\frac{K}{A}}} = \sqrt{A}\,dt \qquad \ldots(34)$$

From equation (34), we have

$$R = \left[\sqrt{A-K}\,t + N\right] \qquad \ldots(35)$$

where N is a constant of integration. Thus, we have

$$G = R^{-2} = \left[\sqrt{A-K}\,t + N\right]^{2} \qquad \ldots(36)$$

From equations (9), (35) and (36), we have

$$8\pi\rho = 4A - 8\pi\beta \qquad \ldots(37)$$

After using the value of R given by (35), the metric (1) led to

$$ds^2 = dt^2 - \left[\sqrt{A-K}\,t + N\right]^2\left[\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right] \qquad \ldots(38)$$

Using equations (35), (36) and (37) into equation (24), we get

$$\dot{C}^2\left[\sqrt{A-K}\,t + N\right]^4 = \frac{1}{4\pi f}\int\left\{(4A - 32\pi\beta)\sqrt{A-K}\left[\sqrt{A-K}\,t + N\right]^3\right\}dt \qquad \ldots(39)$$

Equation (39) leads to

$$\dot{C}^2 = \frac{4A - 32\pi\beta}{16\pi f} \qquad \ldots(40)$$

using $A = 4\pi[f + 2\beta]$ in equation (40), we have

$$\dot{C}^2 = 1 \qquad \ldots(41)$$

which leads to

$$\dot{C} = 1 \qquad \ldots(42)$$

which leads to

$$C = t \qquad \ldots(43)$$

Hence we find $\dot{C} = 1$ which agree with the value used in the source equation. Thus creation field C is proportional to time t and the metric (1) for the constraints mentioned above, leading to

$$ds^2 = dt^2 - \left[\sqrt{A - Kt} + N\right]^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \quad \text{...(44)}$$

The homogeneous mass density ρ, gravitational constant G, and deceleration parameter (q) for the model (44) are given by

$$8\pi\rho = 4A - 8\pi\beta \quad \text{...(45)}$$

$$G = \left[\sqrt{A - Kt} + N\right]^2 \quad \text{...(46)}$$

$$q = 0 \quad \text{...(47)}$$

4. Conclusion

For model (23), the spatial volume increases as time increases. Thus inflationary scenario exists. The matter density decreases as time increases. The deceleration parameter $q < 0$ indicating accelerating universe. The creation field (C) increases with time and $\dot{C} = 1$. The gravitational constant decreases with time. Setting a = 1, b = 0 we find that matter density (ρ) is constant, creation field increases with time and gravitation constant decreases with time. The spatial volume increases with time. The deceleration parameter $q < 0$ indicates accelerating universe.

For model (38), the spatial volume increases as time increases. Thus inflationary scenario exists. The creation field (C) increases with time .The matter density ρ is constant. This can be interpreted as the matter is supposed to move along the geodesic normal to the surface (t = constant). When matter moves further apart, it is assumed that more matter is created continuously to maintain the density at constant value (Hoyle and Narlikar[18], Hawking and Ellis[17]). The gravitational constant decreases with time. The deceleration parameter $q = 0$ gives Milne universe as mentioned in Narlikar's book Introduction to Cosmology.

References

[1] Atkatz, D., and Pagels, H. (1982). Origin of the universe as a quantum tunneling event, Phys. Lett. B **108**, 389-393.

[2] Bali, R. (2013). C-field cosmological model for barotropic fluid distribution with variable gravitational constant, Intech Book “Aspects of Todays Cosmology”.

[3] Bali, R., and Jain, D.R. (1991). Viscous fluid universe filled with stiff fluid in general relativity, Astrophys. and Space-Science **185**, 211-212.

[4] Balil, R., and Kumawat, P. (2008). Bulk viscous L.R.S. Bianchi type V tilted stiff fluid cosmological model in general relativity, Phys. Lett. B **665**, 332-337.

[5] Bali, R., and Pradhan, A. (2007). Bianchi type-III string cosmological models with time dependent bulk viscosity, Chin. Phys. Lett. **24**, 585-588.

[6] Bali, R., Singh, P., and Singh, J.P. (2012). Bianchi Type V viscous fluid cosmological models in presence of decaying vacuum energy, Astrophys. Space Sci. **34**, 701-706.

[7] Banerjee, A., Duttachoudhary, S.B., and Sanyal, A.K. (1986). Bianchi type-II cosmological model with viscous fluid, Gen. Relativ. Grav. **18**, 461-477.

[8] Banerjee, A., and Santos, N.O. (1986). Homogeneous anisotropic cosmological models with viscous fluid and magnetic field, Gen. Relativ. Grav. **18**, 1251-1262.

[9] Barrow, J.D. (1978). A cosmological limit on the possible variation of G, Mon. Not. R. Astron. Soc. **184**, 677-682.

[10] Bondi, H. and Gold, T.(1948). The steady state theory of the expanding universe, Mon. Not. Roy. Astron. Soc. **108**, 252-270.

[11] Brans, C., and Dicke, R.H. (1961). Mach's Principle and a Relativistic Theory of Gravitation, Phys. Rev. **24**, 925-935.

[12] Brevik, I., and Gron, O. (2013). Relativistic viscous universe models. In Recent Advances in Cosmology; Travena, A., Soren, B., Eds.; Nova Scientific Publishers: New York, NY, USA, 97-127.

[13] Canuto, V., Hsich, S.H., and Adms., P. J. (1977b). Scale-Covariant Theory of Gravitation and Astrophysical Applications, Phys. Rev. Lett. A **39**, 429-432.

[14] Demarque, P., Krauss, I.M., Guenther, D.B., and Nydam, D. (1994). The Sun as a Probe of Varying G, Astrophys. J. **437**, 870-878.

[15] Dirac, P.A.M. (1937). The Cosmological Constants, Nature **139**, 323-333.

[16] Gamow, G. (1967c). Does gravity change with time ? Proc. Natl. Acad. Sci. U.S.A. **57**, 187-193.

[17] Hawking, S.W. and Ellis, G.F.R. (1973). The Large Scale Structure of Space – Time, Cambridge University Press, Cambridge 126.

[18] Hoyle, F. and Narlikar, J.V. (1964). A new theory of gravitation, Proc. Roy. Soc. A **282**, 191-207.

[19] Linde, A.D. (1982). A new inflationary universe scenario : A possible solution of the Horizon, flatness, homogeneity, isotropy and primordial monopole problem, Phys. Rev. D **25**, 2065-2073.

[20] Misner, C.W. (1967), Transport Processes in the Primordial Fireball, Nature **214**, 40-41.

[21] Misner, C.W. (1968). The Isotropy of the Universe, Astrophys. J. 151, 431 -458.

[22] Narlikar, J.V. (1973). Singularity and matter creation in cosmological models, Nature **242**, 135-136.

[23] Narlikar, J.V. and Padmanabhan, T. (1985). Creation field cosmology : A possible solution to singularity, horizon and flatness problems, Phys. Rev. D **32**, 1928-1934.

[24] Padmanabhan, T. (1983). Instability of the flat space and the origin of conformal fluctuations, Phys. Lett. A **93**, 116-118.

[25] Pochoda, P., and Schwarzschild, M. (1963). Variation of the Gravitational Constant and the Evolution of the Sun, Astrophys. J. **139**, 587-593.

[26] Sahni, V., and Starobinsky, A. (2000). The case for a positive cosmological Λ-term, Int. J. Mod. Phys. D **9**, 373-443.

[27] Vishwakarma, R.G. and Narlikar, J.V. (2007). Modeling repulsive gravity with creation, J. Astrophys. and Astronomy **28**, 17-27.