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CONSEQUENCE OF CHEMICAL REACTION ON AN UNSTEADY MHD RADIATIVE FLOW PAST AN ERECT POROUS PLATE WITH DIFFUSION-THERMO (DUFOUR EFFECT)

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Abstract: Aim of this paper is to investigate the influence of chemical reaction on an unsteady MHD radiative incompressible viscous electrically conducting fluid flow past a vertical porous plate with Dufour effect. The plate moves with a constant velocity in the direction of fluid flow. A uniform magnetic field is applied normal to a vertical porous plate. The governing dimensional equations are transformed into non-dimensional form. The resultant equations are then solved analytically by Laplace transform technique. The effect of different flow parameters involved in the problem on velocity, temperature, concentration, skin-friction coefficient, Nusselt number, and Sherwood number are discussed with the help of different graphs. From the graphs results of the problem are obtained. It is observed that the influence of chemical reaction reduces the fluid flow while Dufour effect increases the fluid flow.

Keywords: Chemical reaction, Dufour effect, Laplace transforms technique, MHD.

1. Introduction

MHD is concerned with the study of the interaction of magnetic field and electrically conducting fluids in motion. The magnetohydrodynamics (MHD) flow is found in meteorology, solar physics, motion of earth's core, etc. Also, it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering, and electronics. Many researchers have given attention to find out the good and useful result on MHD flow. Some of them are Seth et al.[12], Sulemana[14], Waghmode, et al.[15], Krishna et al.[7] etc.

The study about the influence of chemical reaction is very important because it is applicable in almost all branches of science and technology. The chemical reaction between fluid and a foreign mass occurs in many chemical engineering processes. Due to importance of chemical reactions, many researchers give attention to it and carry out many good works. Some of them are Chamkha et al.[2], Kar et al.[6], Bakr [1] etc.

The study of MHD in presence of radiation has attracted many researchers due to its importance. It is applicable in advanced energy conservation systems, astrophysics, geophysics, stellar and solar structures, etc. Makinde et al. [8] studied radiation and mass transfer effects on MHD flow through a porous medium. Some other researchers in this line are Jha and Samaila [5], Raju et al. [11], Chiranjeevi et al.[3] etc.

Dufour effect is the energy flux due to a mass concentration gradient occurring as a couple of effects of irreversible processes. It is a reciprocal phenomenon to the Soret effect. It is applicable in chemical engineering, nuclear reactors, geothermal energy, astrophysical studies, etc. Many authors have carried out their research works to investigate the problems related to Dufour effect. Some of them are Sharma and Bhaskar [13], Pandya, and. Shukla [9], Imtiaz et al. [4]. Prakash et al. [10] studied Dufour effects on unsteady MHD radiative fluid flow past a vertical plate through a porous medium.

Here, it is tried to study the influence of chemical reaction on an unsteady MHD radiative fluid flow past a vertical porous plate with Dufour effect. This paper is an extension of work done by Prakash et al. [10].

2. Formulation of the problem

In the present study influence of chemical reaction and Dufour effects on unsteady MHD radiative, viscous, incompressible, electrically conducting fluid flow past an infinite vertical porous plate is considered. Let a co-ordinate system be introduced, where X'-axis is along the plate in a vertically upward direction, Y'-axis remains normal to the plate and directed to fluid region and Z'-axis is width of plate as shown in **Figure 1.** Let u' and v' be the components of velocity along X'-axis and Y'-axis respectively. So u' is chosen in the upward direction along the plate and v' is chosen normal to the plate. Initially taking the plate and its surrounding fluid are at the same temperature and concentration in the flow region $Y' \geq 0$. At the time $t' \geq 0$, the plate moves with a constant velocity u= u_0. Then the temperature is also increasing with time t and concentration of the plate is raised to c'_w. A uniform magnetic field B_0 is applied normal to the fluid flow.

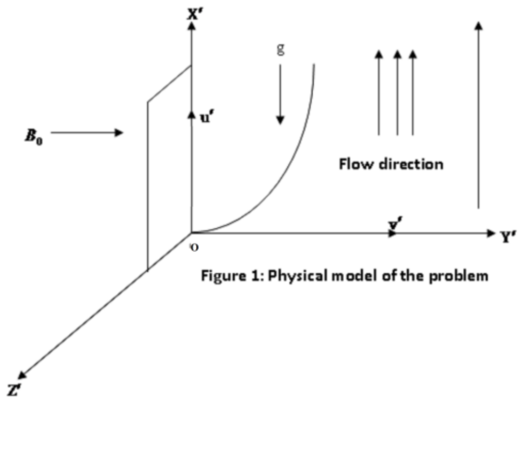

Figure 1: Physical model of the problem

The investigation is based on the following assumptions:

1. The magnetic Reynolds number is assumed very small, so induced magnetic field can be neglected.
2. Only considered variation of fluid density with temperature and concentration. Other variations of all fluid properties are completely ignored.
3. In the energy equation, viscous dissipation is neglected.
4. The flow of fluid is considered along x'-axis, the physical quantities are functions of y' and t' only.

Under these assumptions, the following governing equations are considered:

Continuity Equation:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} = g\beta\left(T' - T'_\infty\right) + g\beta'\left(C' - C'_\infty\right) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma\beta_0^2 u'}{\rho} - \upsilon\frac{u'}{k'} \tag{2}$$

Energy Equation:

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \frac{D_m K_T \rho}{C_s}\frac{\partial^2 c'}{\partial y'^2} \tag{3}$$

Concentration Equation:

$$\frac{\partial c'}{\partial t'} = D\frac{\partial^2 c'}{\partial y'^2} + K'_r\left(C'_\infty - C'\right) \tag{4}$$

Initial and boundary conditions for velocity, temperature and concentration field are:

$$t' \le 0:\ u' = 0,\ T' = T'_\infty,\ C' = C'_\infty,\ \forall\ y'$$

$$t' > 0:\quad u' = u_0,\ T' = T'_\infty + \left(T'_w - T'_\infty\right)\frac{t' u_0^2}{\upsilon},\ C' = C'_w \ \text{ at } \ y' = 0$$

$$u' = 0,\quad T' \to T'_\infty,\quad C' \to C'_\infty,\ \text{ as } \ y' \to \infty \tag{5}$$

Radiative heat flux

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right) \tag{6}$$

Considered temperature differences within the flow are very small. So T'^4 can be found out by expanding T'^4 in Taylor series with T'_∞. Eliminating higher orders of the series it may be written as:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{7}$$

Applying equation (7) in equation (6), Radiative heat flux i.e. the equation (6) is transformed to following equation:

$$\frac{\partial q_r}{\partial y'} = 16a * \sigma T'^3_\infty \left(T' - T'_\infty\right) \tag{8}$$

Putting the value of equation (8) in equation (3), the equation (3) becomes:

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + 16a * \sigma T'^3_\infty \left(T'_\infty - T'\right) + \frac{D_m K_T \rho}{C_s} \frac{\partial^2 c'}{\partial y'^2} \tag{9}$$

With the help of the following non-dimensional variables,

$$u = \frac{u'}{u_0},\ t = \frac{t' u_0^2}{\upsilon},\ y = \frac{y' u_0}{\upsilon},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ \phi = \frac{C - C'_\infty}{C'_w - C'_\infty},$$

$$Gr = \frac{g\beta\upsilon\left(T'_w - T'_\infty\right)}{u_0^3},\ \Pr = \frac{\mu C_p}{k},\ Gm = \frac{g\beta\upsilon(C'_w - C'_\infty)}{u_0^3},\ \ Sc = \frac{\upsilon}{D},$$

$$M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2},\ R = \frac{16a^* \upsilon^2 \sigma T'^3_\infty}{k u_0^2},\ Du = \frac{D_m K_T (C'_w - C'_\infty)}{C_s c_p \upsilon (T'_w - T'_\infty)},\ k = \frac{u_0^2 k'}{\upsilon^2},\ Kr = \frac{Kr' \upsilon}{u_0^2} \tag{10}$$

The equations (2), (9), and (4) can be transformed to the following non-dimensional form as

$$\frac{\partial u}{\partial t} = Gr\theta + Gm\phi + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{k} \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{R}{\Pr}\theta + Du \frac{\partial^2 \phi}{\partial y^2} \tag{12}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - Kr\phi \tag{13}$$

Boundary condition equation no. (5) is also transformed to non-dimensional form as:

$$t > 0:\ \ u = 1,\ \ \theta = t,\ \ \phi = 1,\ \text{at } y = 0$$

$$u \to 0,\ \ \theta \to 0,\ \ \phi \to 0\ \ \text{as } y \to \infty \tag{14}$$

3. Solution of problem

The equations from (11)-(13) are solved using Laplace transform technique. Solutions can be written in the following way:

$$\frac{d^2\bar{u}}{dy^2}-\left(s+M+\frac{1}{k}\right)\bar{u}=-Gr\bar{\theta}-Gm\bar{\phi} \tag{15}$$

$$\frac{d^2\bar{\theta}}{dy^2}-\Pr\left(s+c\right)\bar{\theta}=-\Pr Du\frac{d^2\bar{\phi}}{dy^2},\quad \text{where } c=\frac{R}{\Pr} \tag{16}$$

$$\frac{d^2\bar{\phi}}{dy^2}-Sc\left(s+Kr\right)\bar{\phi}=0 \tag{17}$$

Boundary conditions (14) also can be changed using Laplace transform technique as:

$$\bar{u}=\frac{1}{s},\quad \bar{\theta}=\frac{1}{s^2}\left(1-e^{-s}\right),\quad \bar{\phi}=\frac{1}{s},\quad \text{at } y=0$$

$$\bar{u}\to 0,\quad \bar{\theta}\to 0,\quad \bar{\phi}\to 0\quad \text{as } y\to\infty \tag{18}$$

The equations (15)-(17) are ordinary second order differential equations. Taking boundary conditions (18) these differential equations can be solved in the following way:

$$\bar{\phi}=\frac{1}{s}e^{-y\sqrt{Sc(s+Kr)}} \tag{19}$$

$$\bar{\theta}=\frac{1}{s^2}\left(1-e^{-s}\right)e^{-y\sqrt{\Pr(s+c)}}-\frac{N_1}{s}e^{-y\sqrt{\Pr(s+c)}}+\frac{M_1}{s+L}e^{-y\sqrt{\Pr(s+c)}}+\frac{N_1}{s}e^{-y\sqrt{Sc(s+Kr)}}-\frac{M_1}{s+L}e^{-y\sqrt{Sc(s+Kr)}} \tag{20}$$

Where,

$$k_1=\frac{-G'}{I},\quad G'=\Pr DuSc,\quad I=Sc-\Pr,\quad J=ScKr-\Pr c,\quad c=\frac{R}{\Pr},$$

$$L=\frac{J}{I},\quad M_1=\frac{k_1\left(Kr-L\right)}{L},\quad N_1=k_1+M_1$$

$$\bar{u}=\frac{1}{s}e^{-y\sqrt{s+N}}+\frac{p_1}{s+Q_1}\frac{1}{s^2}\left(1-e^{-s}\right)e^{-y\sqrt{s+N}}-\frac{p_2}{s+Q_1}\frac{1}{s}e^{-y\sqrt{s+N}}+\frac{p_3}{s+Q_1}\frac{1}{s+L}e^{-y\sqrt{s+N}}+\frac{p_4}{s+Q_2}\frac{1}{s}e^{-y\sqrt{s+N}}-$$
$$\frac{p_5}{s+Q_2}\frac{1}{s+L}e^{-y\sqrt{s+N}}+\frac{p_6}{s+Q_2}\frac{1}{s}e^{-y\sqrt{s+N}}-\frac{p_1}{s+Q_1}\frac{1}{s^2}\left(1-e^{-s}\right)e^{-y\sqrt{\Pr(s+c)}}+\frac{p_2}{s+Q_1}\frac{1}{s}e^{-y\sqrt{\Pr(s+c)}}$$
$$-\frac{p_3}{s+Q_1}\frac{1}{s+L}e^{-y\sqrt{\Pr(s+c)}}-\frac{p_4}{s+Q_2}\frac{1}{s}e^{-y\sqrt{Sc(s+Kr)}}+\frac{p_5}{s+Q_2}\frac{1}{s+L}e^{-y\sqrt{Sc(s+Kr)}}-\frac{p_6}{s+Q_2}\frac{1}{s}e^{-y\sqrt{Sc(s+Kr)}} \tag{21}$$

Where,

$$p_1 = \frac{Gr}{A},\ p_2 = \frac{E}{A},\ p_3 = \frac{F}{A},\ p_4 = \frac{E}{G},\ p_5 = \frac{F}{G},\ p_6 = \frac{Gm}{G},\ Q_1 = \frac{B}{A}, Q_2 = \frac{H}{G},$$

$$A = Pr - 1,\ B = Prc - N,\ c = \frac{R}{Pr},\ E = GrN_1,\ F = GrM_1,\ H_1 = ScKr - N,\ G = Sc - 1$$

Applying inverse Laplace transform technique in equations (19)-(21), solutions are written using exponential (erf) and complementary error(erfc) functions as:

$$\phi(y,t) = \psi_1 = f(Sc, Kr, y, t) \quad (22)$$

$$\psi_1 = \frac{1}{2}\left[e^{y\sqrt{Sc}\sqrt{Kr}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Krt} \right) + e^{-y\sqrt{Sc}\sqrt{Kr}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Krt} \right) \right]$$

$$\theta(y,t) = \psi_7 + N_1\psi_8 + M_1 e^{-Lt}\psi_9 \quad (23)$$

Where, $\psi_7 = \psi_2 - \psi_3,\ \psi_8 = \psi_1 - \psi_4,\ \psi_9 = \psi_5 - \psi_6$

$$\psi_2 = \beta(Pr, c, y, t),\ c = \frac{R}{Pr}$$

$$= \left(\frac{t}{2} + \frac{y\sqrt{Pr}}{4\sqrt{c}} \right) e^{y\sqrt{Prc}} erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{ct} \right) + \left(\frac{t}{2} - \frac{y\sqrt{Pr}}{4\sqrt{c}} \right) e^{-y\sqrt{Prc}} erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{ct} \right),$$

$$\psi_3 = \beta(Pr, c, y, t-1) H(t-1),$$

$$N_1 = k_1 + M_1,\ k_1 = \frac{-G'}{I},\ G' = Pr\, DuSc,\ M_1 = \frac{k_1(Kr - L)}{L}$$

$$\psi_4 = f(Pr, c, y, t),\ c = \frac{R}{Pr}$$

$$= \frac{1}{2}\left[e^{y\sqrt{Pr}\sqrt{c}} erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{ct} \right) + e^{-y\sqrt{Pr}\sqrt{c}} erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{ct} \right) \right]$$

$$\psi_5 = f(Pr, c - L, y, t),\ L = \frac{J}{I},\ J = ScKr - Prc,\ I = Sc - Pr,\ \psi_6 = f(Sc, Kr - L, y, t),$$

$$u(y,t) = \eta_1 + p_1\eta_9 - p_2\eta_{12} + p_3\eta_{15} + p_4\eta_{18} - p_5\eta_{20} + p_6\eta_{18} - p_1\eta_{26} + p_2\eta_{27} - p_3\eta_{29} - p_4\eta_{32} + p_5\eta_{34} - p_6\eta_{32} \quad (24)$$

$$p_1 = \frac{Gr}{A},\ p_2 = \frac{E}{A},\ p_3 = \frac{F}{A},\ p_4 = \frac{E}{G},\ p_5 = \frac{F}{G},\ p_6 = \frac{Gm}{G},\ Q_1 = \frac{B}{A}, Q_2 = \frac{H}{G},$$

$$A = Pr - 1,\ B = Prc - N,\ c = \frac{R}{Pr},\ E = GrN_1,\ F = GrM_1,\ H_1 = ScKr - N,\ G = Sc - 1$$

$$\eta_1 = \frac{1}{2}\left[e^{y\sqrt{N}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Nt} \right) + e^{-y\sqrt{N}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Nt} \right) \right] = f(N, y, t),$$

$$\eta_9 = \eta_7 - \eta_8,\ \eta_7 = \frac{1}{Q_1^2}\eta_6,\ \eta_6 = \eta_4 - \eta_2 + \eta_5,\ \eta_4 = e^{-Q_1 t}\eta_2,\ \eta_2 = f\left(N - Q_1, y, t\right),$$

$$\eta_8 = \frac{1}{Q_1^2}\left[e^{-Q_1 t} f\left(N - Q_1, y, t-1\right)H\left(t-1\right) - f\left(N, y, t-1\right)H\left(t-1\right) + Q_1\beta\left(N, y, t-1\right)H\left(t-1\right)\right]$$

$$\eta_5 = Q_1\eta_3,\ \eta_3 = \beta\left(N, y, t\right), \eta_{12} = \eta_{10} - \eta_{11}, \eta_{10} = \frac{1}{Q_1}\eta_1 \eta_{11} = \frac{1}{Q_1}\eta_4, \eta_{15} = \frac{1}{L - Q_1}\left(\eta_4 - \eta_{14}\right),$$

$$\eta_{14} = e^{Lt}\eta_{13}, \eta_{13} = f\left(N - L, y, t\right), \eta_{18} = \frac{1}{Q_2}\eta_{17}, \eta_{17} = \eta_1 - \eta_{16},$$

$$\eta_{19} = e^{-Lt} f\left(N - L, y, t\right), \eta_{16} = e^{-Q_2 t} f\left(N - Q_2, y, t\right), \eta_{20} = \frac{1}{L - Q_2}\left(\eta_{16} - \eta_{19}\right),$$

$$\eta_{26} = \eta_{24} - \eta_{25}, \eta_{24} = \frac{1}{Q_1^2}\left(\eta_{21} - \eta_{22} + \eta_{23}\right), \eta_{21} = e^{-Q_1 t} f\left(Pr, c - Q_1, y, t\right), \eta_{22} = f\left(Pr, c, y, t\right),$$

$$\eta_{23} = Q_1\beta\left(Pr, c, y, t\right), \eta_{27} = \frac{1}{Q_1}\left(\eta_{22} - \eta_{21}\right), \eta_{29} = \frac{1}{L - Q_1}\left(\eta_{21} - \eta_{28}\right),\ \eta_{28} = e^{-Lt} f\left(Pr, c - L, y, t\right),$$

$$\beta\left(N, y, t\right) = \frac{1}{2}\left[\left(t + \frac{y}{2\sqrt{N}}\right)e^{y\sqrt{N}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Nt}\right) + \left(t - \frac{y}{2\sqrt{N}}\right)e^{-y\sqrt{N}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Nt}\right)\right]$$

$$\eta_{32} = \frac{1}{Q_2}\left(\eta_{30} - \eta_{31}\right), \eta_{30} = f\left(Sc, Kr, y, t\right),\ \eta_{31} = e^{-Q_2 t} f\left(Sc, Kr - Q_2, y, t\right)$$

$$\eta_{34} = \frac{1}{L - Q_2}\left(\eta_{31} - \eta_{33}\right),\ \ \eta_{33} = e^{-Lt} f\left(Sc, Kr - L, y, t\right)$$

$$f\left(Pr, c, y, t\right) = \frac{1}{2}\left[e^{y\sqrt{Pr\, c}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Pr\, ct}\right) + e^{-y\sqrt{Pr\, c}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Pr\, ct}\right)\right]$$

$$\beta\left(Pr, c, y, t\right) = \frac{1}{2}\left[\left(t + \frac{y}{2\sqrt{Pr\, c}}\right)e^{y\sqrt{Pr\, c}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Pr\, ct}\right) + \left(t - \frac{y}{2\sqrt{Pr\, c}}\right)e^{-y\sqrt{Pr\, c}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Pr\, ct}\right)\right]$$

4. Sherwood Number

From the concentration field, rate of mass transfer coefficient in terms of Sherwood number (Sh) in non-dimensional form can be written as:

$$Sh=-\left[\frac{\partial\phi}{\partial y}\right]_{y=0}=\frac{\sqrt{Sc}}{\sqrt{\pi t}}e^{-Kr\,t}+\sqrt{ScKr}\,erf\left(\sqrt{Krt}\right)==\in\left(Sc,Kr,t\right)=\in_1 \tag{25}$$

5. Nusselt Number

From the temperature field, rate of heat transfer coefficient in terms of Nusselt number (Nu) can be calculated in .non-dimensional form as given by:

$$Nu=-\left[\frac{\partial\theta}{\partial y}\right]_{y=0}=(\in_3-\in_2)+N_1\left(\in_4-\in_1\right)+M_1e^{-Lt}\left(\in_6-\in_5\right) \tag{26}$$

$$\in_2=\in\left(Pr,c,t\right)=-\frac{\sqrt{Pr}}{\sqrt{\pi t}}e^{-ct}-\sqrt{Pr\,c}\,erf\left(\sqrt{ct}\right),$$

$$\in_3=\in\left(Pr,c,t-1\right),\in_4=q\left(Pr,c,t\right),$$

$$\in_5=\in\left(Pr,c-L,t\right)$$
$$=-\frac{\sqrt{Pr}}{\sqrt{\pi t}}e^{-t(c-L)}-\sqrt{Pr\left(c-L\right)}\,erf\left(\sqrt{t\left(c-L\right)}\right)$$

$$\in_6=\in\left(Sc,Kr-L,t\right)$$
$$=-\frac{\sqrt{Sc}}{\sqrt{\pi t}}e^{-t(Kr-L)}-\sqrt{Sc\left(Kr-L\right)}\,erf\left(\sqrt{t\left(Kr-L\right)}\right)$$

6. Skin-friction co-efficient

From the velocity field, the non-dimensional coefficient of skin-friction at the plate can be found out and it is denoted by τ.

$$\tau=\left[\frac{\partial u}{\partial y}\right]_{y=0}=\in_7+p_1\left(\in_8-\in_9\right)-p_2\left(\in_{10}-\in_{11}\right)+p_3\in_{14}+p_7\in_{17}- \tag{27}$$
$$p_5\in_{20}-p_1\left(\in_{21}-\in_{22}\right)-p_2\in_{25}-p_3\in_{27}-p_7\in_{30}+p_5\in_{33}$$

$$\in_7=f\left(N,t\right)=-\frac{1}{\sqrt{\pi t}}e^{-Nt}-\sqrt{Nt}$$

$$\in_8=\frac{1}{Q_1^2}\left[e^{Q_1t}f\left(N-Q_1,t\right)-f\left(N,t\right)+Q_1\beta\left(N,t\right)\right]$$

$$\in_9=\in_8\,H\left(t-1\right),\ \in_{10}=\frac{1}{Q_1}f\left(N,t\right),\ \in_{11}=\frac{1}{Q_1}e^{-Q_1t}f\left(N-Q_1,t\right),$$

$$\in_{12}=\frac{1}{Q_1^2}e^{-Q_1t}f\left(N-Q_1,t\right),\beta\left(N,t\right)=-\frac{t}{\sqrt{\pi}}e^{-Nt}-\left(tN+\frac{1}{2\sqrt{N}}\right)\sqrt{Nt},$$

$$\in_{13}=e^{-Lt}f(N-L,t),\ \in_{14}=\frac{1}{L-Q_1}(\in_{12}-\in_{13}),\in_{15}=f(N,t),\ \in_{16}=e^{-Q_2t}f(N-Q_2,t)$$

$$\in_{17}=\frac{1}{Q_2}(\in_{15}-\in_{16}),\ \in_{18}=e^{-Q_2t}f(N-Q_2,t),\in_{19}=e^{Lt}f(N-L,t),\in_{20}=\frac{1}{L-Q_2}(\in_{18}-\in_{19})$$

$$\in_{21}=\frac{1}{Q_1^2}\left[e^{-Q_1t}f(Pr,c-Q_1,t)-f(Pr,c,t)+Q_1\beta(Pr,c,t)\right]$$

$$f(Pr,c-Q_1,t)=-\frac{\sqrt{Pr}}{\sqrt{\pi t}}e^{-(c-Q_1)t}-\sqrt{Pr}\sqrt{c-Q_1}\,erfc\left(\sqrt{(c-Q_1)t}\right)$$

$$\in_{23}=f(Pr,c,t),\in_{24}=e^{-Q_1t}f(Pr,c-Q_1,t),\in_{25}=\frac{1}{Q}(\in_{23}-\in_{24}),$$

$$\in_{26}=e^{-Lt}f(Pr,c-L,t),\ \in_{27}=\frac{1}{L-Q_1}(\in_{24}-\in_{26}),$$

$$\in_{22}=\frac{1}{Q_1^2}\left[e^{-Q_1t}f(Pr,c-Q_1,t-1)H(t-1)-f(Pr,c,t-1)H(t-1)+Q_1\beta(Pr,c,t-1)H(t-1)\right]$$

$$\in_{28}=f(Sc,Kr,t)=\frac{\sqrt{Sc}}{\sqrt{\pi t}}e^{-\sqrt{Kr\,t}}-\sqrt{Sc}\sqrt{Kr}\,erf\left(\sqrt{Kr\,t}\right)$$

$$\in_{30}=\frac{1}{Q_2}(\in_{28}-\in_{29}),\in_{31}=e^{-Q_2t}f(Sc,Kr-Q_2,t),\in_{32}=e^{-Lt}f(Sc,Kr-L,t)$$

$$\in_{29}=e^{-Q_1t}f(Sc,Kr-Q_2,t)=e^{-Q_1t}\left[-\frac{\sqrt{Sc}}{\sqrt{\pi t}}e^{-\sqrt{(Kr-Q_2)t}}-\sqrt{Sc}\sqrt{Kr-Q_2}\,erf\left(\sqrt{(Kr-Q_2)t}\right)\right]$$

$$\in_{33}=\frac{1}{L-Q_1}\left[e^{-Q_2t}f(Sc,Kr-Q_2,t)-e^{-Lt}f(Sc,Kr-L,t)\right]$$

$$\beta(Pr,c,t)=\frac{-\sqrt{t}\sqrt{Pr}}{\sqrt{\pi}}e^{-ct}-\left(t\sqrt{c}\sqrt{Pr}+\frac{\sqrt{Pr}}{2\sqrt{c}}\right)erfc\sqrt{ct}$$

7. Results and Discussions

In order to understand the physical situations of the problem, we have carried out numerical calculations of non-dimensional velocity, temperature, concentration, Skin-friction co-efficient, the rate of heat transfer in terms of Nusselt number Nu and rate of mass transfer in terms of Sherwood number Sh by taking some arbitrary values of different parameters involved in the problem, viz. magnetic parameter M. chemical reaction parameter Kr, Dufour number Du, Schmidt number Sc and radiation parameter R.

The effects of these values are discussed through different graphs and results are interpreted physically.

Figures 2-5 depict the variations of velocity field u against y under the influence of magnetic parameter M, chemical reaction parameter Kr, Dufour number Du and Schmidt number Sc. From **Figures 2 and 3**, it is evident that the increasing value of M and Kr affects the fluid velocity. The fluid velocity is decelerated under the action of M and Kr. This is due to the fact that the presence of transverse **magnetic field** set in Lorentz force and consumption of chemical species results in retarding force on the **velocity field**. In **Figure 5** it is seen the same trend of performance, i.e. obligation of Schmidt number Sc tends to slow down the fluid flow because as we know that as **velocity rises**, **mass** transfer coefficient **rises** because of fresh fluid elements coming into the **mass** transfer interface. But in **Figure 4,** opposite behavior with respect to the above mentioned figures is observed, i.e. fluid motion is accelerated due to the increasing value of Dufour number Du.

Figures 6-9 demonstrate temperature distribution θ against y under the effects of chemical reaction parameter Kr, Prandtl number Pr, Dufour number Du and radiation parameter R. In Figure 6; it has been observed that the increasing value of Kr does not affect fluid temperature. The fluid temperature remains same. In Figures 7 and 9, it is seen that the rising of Pr and R leads to a decrease in fluid temperature. Physically, the increase of Pr means the decrease of thermal conductivity of the fluid. Also, the fluid temperature can be controlled by using radiation. But Figure 8 shows reverse behavior. The fluid temperature quickly rises for the increasing values of Dufour number Du. The variation of θ with Dufour parameter Du shows that the actual temperature experiences an enhancement in the flow region with Du. It is further noticed from these figures that fluid temperature asymptotically falls from its maximum value y=0 to its minimum value $y \to \infty$.

Figures 10-11 display the fluid concentration level ϕ against y under the effects of Kr and Sc. From both the figures, it is found that the concentration level of fluid is getting reduced with the increasing values of Kr and Sc. This means that the concentration level of fluid drops due to increase of Kr and high mass diffusivity Sc. From **Figures 10-11** it is also observed that the concentration level of fluid asymptotically falls from its maximum value at y=0 to its minimum value at $y \to \infty$.

Figures 12-13 show variations of Skin-friction co-efficient τ against t under the action of magnetic parameter M and chemical reaction parameter Kr. In both the figures, it is evident that an increase in M and Kr tends to minimize the co-efficient of Skin-friction.

The effects of chemical reaction parameter Kr and radiation parameter R on the coefficient of rate of heat transfer in terms of Nusselt number Nu have been observed in **Figures 14-15.** It is seen from **Figure 14** that an acceleration in the reaction Kr leads to reduce the heat transfer rate, i.e. Nusselt number Nu. But **Figure 15** displays opposite behavior. It can be seen from this figure that Nu is reduced due to the increasing values of radiation parameter R.

Figures 16-17 depict the influence of Kr and Sc on the coefficient of rate of mass transfer in terms of Sherwood number Sh. In both the figures, it is seen that Sh rises with higher values of both Kr and Sc, i.e. the mass flux of the fluid gets enlarged under the effects of chemical reaction parameter Kr and mass diffusivity Sc.

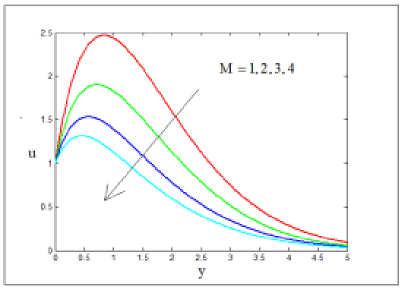

Figure 2: Velocity versus y under Kr=1, Gr=10, Gm=5, R=2, Sc=2.01, Pr=0.71, K=5, Du=0.03, t=0.4

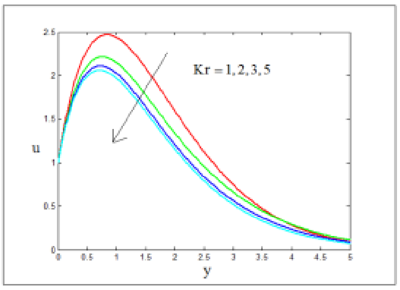

Figure 3: Velocity versus y under M=3, Gr=10, Gm=5, R=2, Sc=2.01, Pr=0.71, K=5, Du=0.03, t=0.4

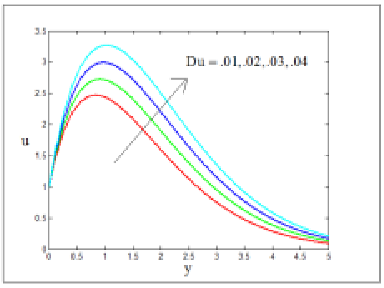

Figure 4: Velocity versus y under Kr=1, Gr=10, Gm=5, R=2, Sc=2.01, Pr=0.71, K=5, M=3, t=0.4

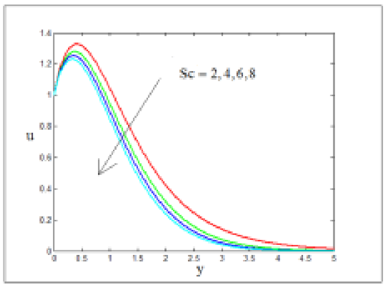

Figure 5: Velocity versus y under Kr=1, Gr=10, Gm=5, R=2, M=3, Pr=0.71, K=5, Du=0.03, t=0.4

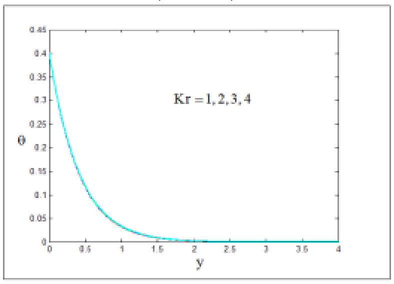

Figure 6: Temperature versus y under R=4, Du=0.03, Sc=2.01, Pr=0.71, t=0.4

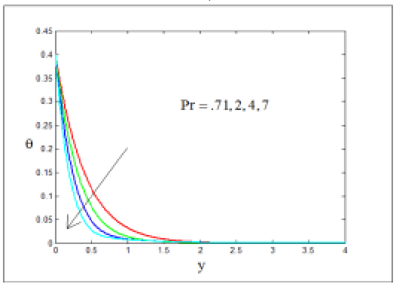

Figure 7: Temperature versus y under R=4, Kr=1, Du=0.03, Sc=2.01, t=0.4

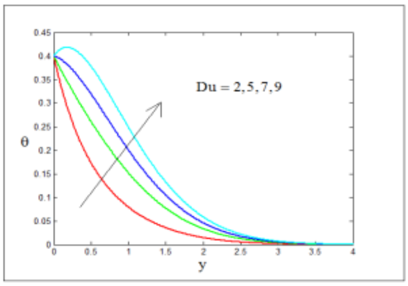

Figure 8: Temperature versus y under R=4, Kr=1, Pr=0.71, Sc=2.01, t=0.4

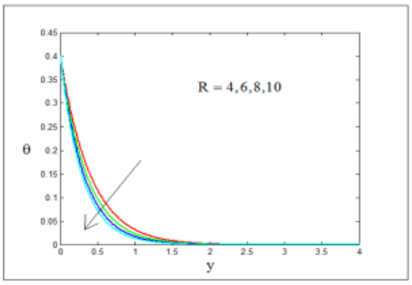

Figure 9: Temperature versus y under Pr=0.71, Kr=1, Du=0.03, Sc=2.01, t=0.4

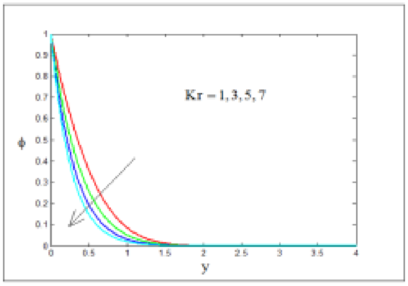

Figure 10: Concentration versus y under Sc=0.60, t=0.4

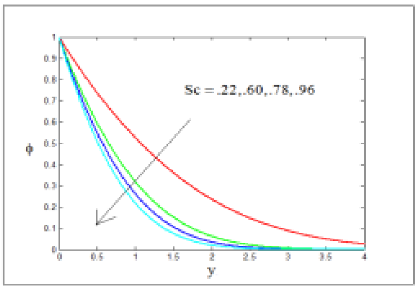

Figure 11: Concentration versus y under Kr=1, t=0.4

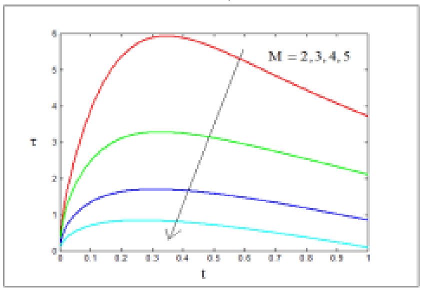

Figure 12: Skin friction versus t under Kr=1, Gr=10, Gm=5, R=2, Sc=2.01, Pr=0.71, K=5, Du=0.03

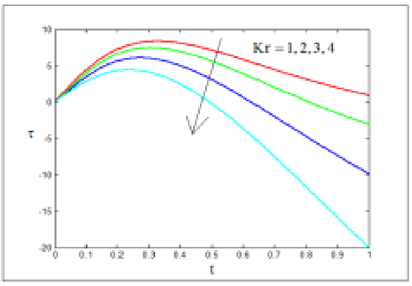

Figure 13: Skin friction versus t under M=3, Gr=10, Gm=5, R=2, Sc=2.01, Pr=0.71, K=5, Du=0.03

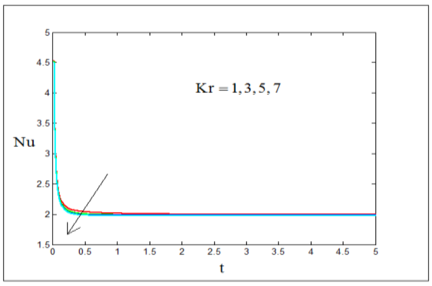

Figure 14: Nusselt number versus t under R=4, Du=0.03, Sc=2.01, Pr=0.71

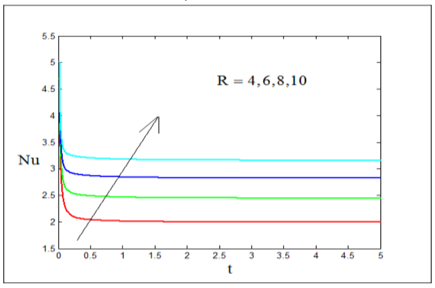

Figure 15: Nusselt number versus t under Kr=1, Du=0.03, Sc=2.01, Pr=0.71

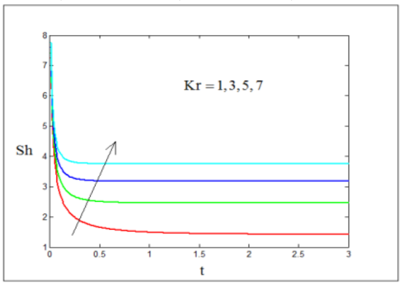

Figure 16: Sherwood number versus t under Sc=2.01

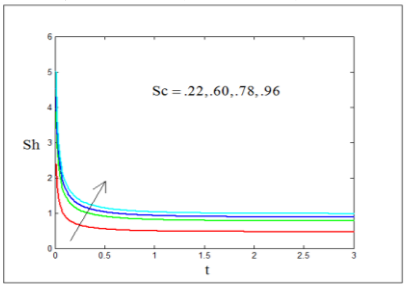

Figure 17: Sherwood number versus t under Kr=1

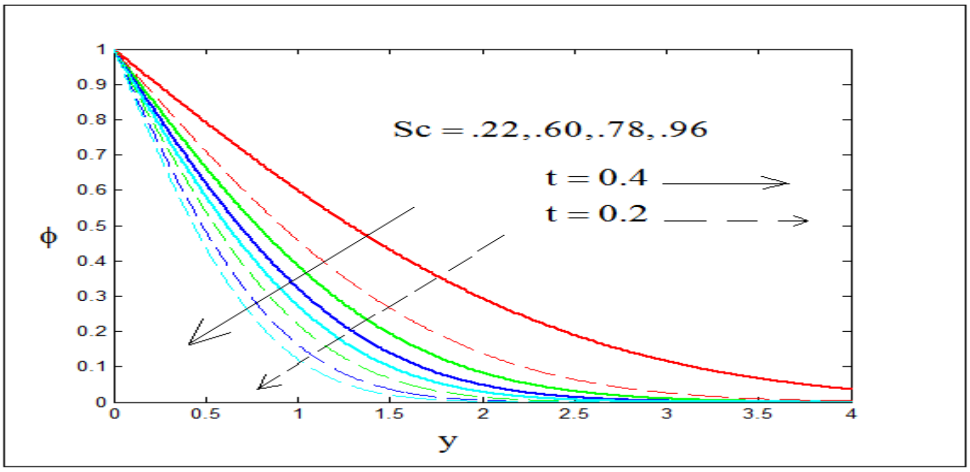

Figure 18: Concentration versus y under Kr=0

8. Comparison of Results

To compare the results of the present paper, the work of Prakash et al. [10] is considered. Comparing figure 18 with figure 19 (the figure 11 of the original work done by Prakash et al. [10]), it is observed that the two figures are almost identical in nature as the behavior of the primary fluid velocity versus normal coordinate y is concerned. That is, there is an excellent agreement between the results obtained by Prakash et al. [10] and the present authors.

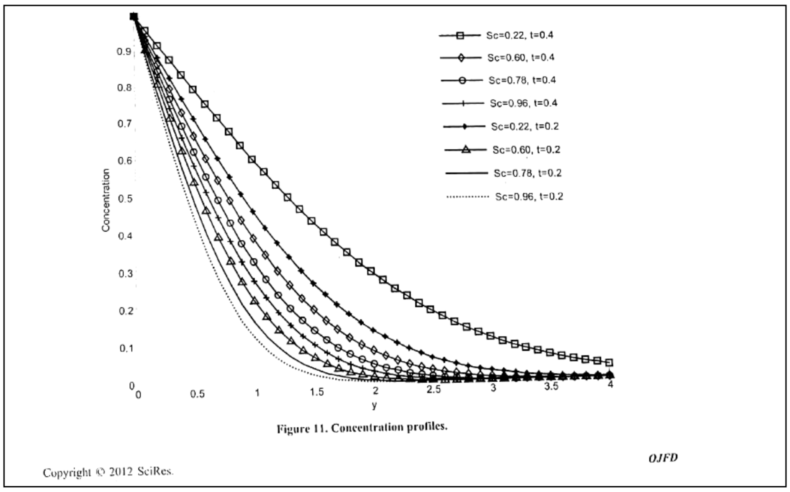

Figure 19 [Figure 11 of Prakash et al.(10)]: Concentration versus y under, t=0.2 and t=0.4

9. Conclusion

The present investigation of the problem gives the following conclusions:

1. The fluid velocity is retarded under the influence of magnetic parameter M, chemical reaction Kr and Schmidt number Sc, but fluid motion gets accelerated under the action of Dufour number Du.
2. Influence of chemical reaction Kr cannot affect fluid temperature, but the influence of Pr and R steadily falls down the fluid temperature. On the other hand, the fluid temperature gets increased by virtue of Dufour number Du.
3. The concentration level of fluid is decelerated under the action of the chemical reaction parameter Kr and Sc.
4. An increase in magnetic intensity M and chemical reaction parameter Kr tends to minimize the co-efficient of Skin friction.
5. Nusselt number Nu rises for increasing values of R but decreases as Kr increases.
6. Sherwood number Sh gets accelerated under the action of Kr and Sc.

10. Utility of the present study:

As regards the utility of the results obtained in this study, it can be said that the fluid flow under the influence of magnetic intensity, chemical reaction, mass flux, and Dufour effect has its practical applications in many chemical engineering processes. Applying high magnetic intensity, chemical reaction, and mass flux, the fluid velocity can be controlled. Such a result may be applicable in various chemical process industries, petroleum industries, power industries, polymer productions, manufacturing of ceramics or glassware, etc. Another result indicates that with the use of high radiation, the fluid temperature can be reduced, which may be useful in many branches of engineering processes, energy conservation systems, astrophysics, aeronautics, etc. On the other hand, it is found that any increase in Dufour effect also raises fluid motion and fluid temperature. This result may be applicable in chemical engineering, isotope separation processes, and geophysical applications, etc. It is also obtained that under the influence of chemical reaction and mass diffusivity the concentration level of fluid gets reduced. This result may also have application potentials in many chemical engineering processes, petroleum technology, gas production, polymer production, etc.

11. Nomenclature

B_0=Strength of applied magnetic field, T'=Dimensional temperature of fluid, T'_∞ Dimensional temperature far away from the plate, T'_W=Dimensional temperature at the plate, C'=Dimensional concentration, C'_W=Dimensional concentration at the plate. C'_∞ =Dimensional concentration far away from the plate, C_P=Specific heat at constant pressure, D_m=Co-efficient of mass diffusivity, D=Molecular mass diffusivity, K_T =Thermal conductivity, $a^\bullet$=Absorption co-efficient, K=Non-dimensional porosity number, Pr=Prandtl number, Sc=Schmdit number, R=Radiation parameter, Du=Dufour number, G_r , G_m =Grashof number for heat and mass transfer respectively, g=Acceleration due to gravity, erf=Error function, erfc=Complementory error function, H(t-1)=Unit step function. **Greek symbols:** ρ =Density of fluid, σ =Electrical conductivity, υ =Kinematic viscosity, μ =co-efficient of viscosity, β_0 =Volumetric expansion co-efficient with concentration, θ, ϕ =Non-dimensional temperature and concentration respectively, κ = Thermal conductivity of fluid. **Subscripts:** W=Wall condition, ∞=Free stream condition.

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