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STIFF FLUID COSMOLOGICAL MODEL WITH C-FIELD IN LOCALLY ROTATIONALLY SYMMETRIC BIANCHI TYPE-I SPACE TIME

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Abstract: We have investigated stiff fluid cosmological model with C-field in LRS Bianchi type-I space time. To get the deterministic solution, we assume that σ (shear tensor) is proportional to θ(expansion) which leads to $A = B^n$, where n is constant. The physical and geometrical aspects of the model are also discussed.

Keywords : C-field cosmology, stiff fluid, LRS Bianchi type-I

1. Introduction

In modern physical cosmology, the cosmological principle is the notion that the spatial distribution of matter in the universe is homogeneous and isotropic where homogeneous is defined as "The universe is same in all locations" while isotropic means "The universe is same in all directions". These concepts are important because most of the modern cosmology is based on the "Cosmological principle" the assumption that on large scale, the universe is both homogeneous and isotropic. The cosmological observations obtained by Type I_a Supernova Tonry et al. [17] and large scale structure Tegmark et al. [16], Percival et al. [13]. Sharif and Siddiya [15] have studied of Homogeneous and Isotropic universe in f(R, T) gravity. Katanaev [9] also investigated cosmological model with homogeneous and isotropic spatial sections. Shamir [14] investigated the exact vacuum solutions of Bianchi type I, III space times. Chandel and Ram [7] have studied spatially homogeneous cosmological models in f(R, T) theory of gravity.

The distribution of matter can be satisfactorily describe by perfect fluid due to the large scale distribution of galaxies in our universe. Stiff fluid cosmological models create interest in the study because for these models, the speed of light is equal to speed of sound and its governing equation have the same characteristic as those of gravitational field. Zel'dovich [21] and Barrow [5] have discussed the relevance of stiff fluid equation of state (p = ρ) to the matter content of the universe in the early state of evolution of universe. Wesson [20] has investigated an exact solution of Einstein's field equation with Stiff fluid equation of state. Mohanty et al. [10] have investigated cylindrically symmetric Zel'dovich fluid distribution in general relativity.

Bali and Sharma [4] have investigated Bianchi type-I stiff fluid magnetized cosmological model in General Relativity.

In the early universe, all the investigations dealing with physical process use a model of the universe usually called a big-bang model. The big-band model based on Einstein field equation successfully explains the three important observation in Astronomy : (i) The phenomena of expanding universe, (ii) Primordial nucleo synthesis, (iii) The observed isotropy of the cosmic background radiation.

The astronomical observation in the late eighties has revealed that the predications of FRW type models do not always meet with our requirements as we believe earlier, the big bang model is known to have the following short comings :

(i) The model has singularity in the past and possibly one in future.

(ii) The conservation of energy is violated in the big-bang model.

(iii) The big-bang models based on reasonable equations of state lead to a very small particle horizon in the early epochs of the universe. This fact gives rise to the 'Horizon problem'.

(iv) No consistent scenario exists within the frame work of big-bang model that explains the origin, evaluation and characteristic of structure in the universe of small scales.

(v) Flatness problem

Thus alternative theories were proposed from time to time. The most well known theory is the 'Steady State Theory' by Bondi and Gold. In this theory the universe does not have any singular beginning nor an end on the cosmic time scale to maintain constancy of matter density. They envisaged a very slow but continuous creation of matter in contrast to the irrespective certain at t = 0 of the standard FRW model. However, it suffers the serious disqualification for not giving any physical justification in the form of any dynamical theory for continuous creation of matter, also the principle energy conservation is scarifies in this formulism.

To remove this problem, Hoyle and Narlikar [8] adopted a field theoretic approach introducing a massless and chargeless scalar field in Einstein Hilbert action to account for creation of matter. In C-field theory, there is no big-bang type singularity as in the steady state theory of Bondi and Gold [6], Narlikar and Padmanabhan [11] have investigated the solution of Einstein's field equation which admit radiation and negative energy massless scalar c-field as source. Bali and Tikekar [3] have investigated C-field cosmological model for dust distribution in Flat FRW model with variable gravitation constant.

Bali and Kumawat [1] have investigated C-field cosmological model with variable G in FRW space-time. Bali and Saraf [2] studied C-field cosmological model for dust distribution with varying Λ in FRW space-time. To get deterministic model satisfying conservation equation $T^{j}_{i;j} = 0$ they have assumed $\Lambda = \frac{1}{R^2}$ where R is the scale factor.

Tyagi and Singh [19] have studied a cosmological model for barotropic fluid distribution in C-field cosmology with varying cosmological constant (Λ) in Bianchi type-III space time. LRS Bianchi type V perfect fluid cosmological model in C-field theory with variable Λ have been studied by Tyagi and Singh [18]. Parikh and Tyagi [12] have studied time-dependent Λ in Bianchi type IX space time with barotropic perfect fluid in C-field.

In this paper, we have investigated stiff fluid cosmological model with C-field in LRS Bianchi type-I space time. The stiff fluid condition leads to p = ρ, p-being isotropic pressure. To get deterministic solution, we have assume σ (shear tensor) proportional to θ(expansion) which leads to $A = B^n$, where n is constant. We find that creation field increases with time. The other physical and geometrical aspects of the model are also discussed.

2. The Metric and Field Equations

We consider the LRS Bianchi type-I space time as

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2) \tag{1}$$

where the metric potential A and B are function of t alone and $\sqrt{-g} = AB^2$

Einstein modified field equation by introduction of C-field are given by Hoyle and Narlikar as

$$R^j_i - \frac{1}{2} R g^j_i = -8\pi G \left[\underset{(m)}{T^j_i} + \underset{(c)}{T^j_i} \right] \tag{2}$$

The energy momentum tensor $\underset{(m)}{T^j_i}$ for perfect fluid and $\underset{(c)}{T^j_i}$ for Creation field are given by Hoyle and Narlikar [8] as

$$\underset{(m)}{T^j_i} = (p+\rho) v_i v^j - p g^j_i \tag{3}$$

$$\underset{(c)}{T^j_i} = -f\left(C_i C^j - \frac{1}{2} g^j_i C^\alpha C_\alpha \right) \tag{4}$$

where f > 0 is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$.

The modified field equation (2) for the metric (1) leads to

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}=8\pi G\left(-p+\frac{1}{2}f\,\dot{C}^2\right) \tag{5}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=8\pi G\left(-p+\frac{1}{2}f\,\dot{C}^2\right) \tag{6}$$

$$\frac{2A_4B_4}{AB}+\frac{B_4^2}{B^2}=8\pi G\left(\rho-\frac{1}{2}f\,\dot{C}^2\right) \tag{7}$$

where the suffix 4 denoted the differentiation with respect to time t.

3. Solution of Field Equations

The conservation equation

$$[8\pi G T_i^j]_{;j}=0 \tag{8}$$

leads to

$$\frac{d}{dt}\dot{C}^2+2\left(\frac{A_4}{A}+\frac{2B_4}{B}\right)\dot{C}^2=\frac{2\dot{\rho}}{f}+\frac{4\rho}{f}\left(\frac{A_4}{A}+\frac{2B_4}{B}\right) \tag{9}$$

The source equation of C-field $C^i_{,i}=\frac{n}{f}$ leads to C = t for large r

Thus $\dot{C}=1$

Equation (5), (6) and (7) are three equations in 4 unknowns A, B, P and ρ, to get deterministic solution, a supplementary condition between metric potential

$$A = B^n \tag{10}$$

Using $\dot{C}=1$ in equation (5), (6) and (7) which leads to

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}=8\pi G\left(-p+\frac{1}{2}f\right) \tag{11}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=8\pi G\left(-p+\frac{1}{2}f\right) \tag{12}$$

$$\frac{2A_4B_4}{AB}+\frac{B_4^2}{B^2}=8\pi G\left(\rho-\frac{1}{2}f\right) \tag{13}$$

Adding equation (11) and (13), using stiff fluid condition (p = ρ), we have

$$\frac{B_{44}}{B}+\frac{B_4^2}{B^2}+\frac{A_4B_4}{AB}=0 \tag{14}$$

Equation (10) and (14) leads to

$$B_{44}+(n+1)\frac{B_4^2}{B}=0 \tag{15}$$

To get the deterministic value of A and B, we assume

$$B_4=f(B) \tag{16}$$

Using equation (15) and (16) leads to

$$\frac{d}{dt}(f^2)+2(n+1)\frac{f^2}{B}=0 \tag{17}$$

Equation (17) leads to

$$f=\frac{M}{B^{(n+1)}} \quad \text{where M is a integration constant} \tag{18}$$

Equation (18) leads to

$$B=(Lt+K)^{\frac{1}{n+2}} \tag{19}$$

where $L=M(n+2)$ and $K=P(n+2)$ constant

Equation (10) and (19) leads to

$$A=(Lt+K)^{\frac{n}{n+2}} \tag{20}$$

Using equation for (7), (19) and (20) which leads to

$$\rho=\frac{1}{8\pi G}\left[\frac{\alpha L^2}{(Lt+k)^2}\right]+\frac{f}{2} \tag{21}$$

Thus the metric (1) after using equation (19) and (20) leads to

$$ds^2=dt^2-(Lt+K)^{\frac{2n}{(n+2)}}dx^2-(Lt+K)^{\frac{2}{n+2}}(dy^2+dz^2) \tag{22}$$

Using equation (19), (20) into equation (9) we have

$$\frac{d}{dt}(\dot{C}^2)+\left(\frac{2L}{Lt+K}\right)\dot{C}^2=\left(\frac{2L}{Lt+K}\right) \tag{23}$$

Equation (23) leads to

$$(\dot{C}^2)(Lt+K)^2 = (Lt+K)^2 + Q \tag{24}$$

$$\dot{C}^2 = 1 \tag{25}$$

Thus we have $\dot{C} = 1$ or C = t (26)

For simplicity, we assume that integral constant Q = 0 which leads to

We find C = t, which agrees with the value used in the source equation. Thus Creation field C is proportional to time t.

4. Some Physical and Geometrical Features

The homogeneous mass density (ρ), the creation field (C), spatial volume (R^3), the deceleration parameter (q), shear tensor (σ) and expansion (θ) of the model (22) are given by

$$\rho = \frac{1}{8\pi G}\left[\frac{\alpha L^2}{(Lt+k)^2}\right] + \frac{f}{2} \tag{27}$$

$$C = t \tag{28}$$

$$R^3 = (Lt+K) \tag{29}$$

$$q = 2 \tag{30}$$

$$\theta = \frac{L}{Lt+K} \tag{31}$$

$$\sigma^2 = \frac{L^2}{(Lt+K)^2}\left[\frac{2n^2 - 4n + 2}{6(n+2)^2}\right] \tag{32}$$

$$\frac{\sigma}{\theta} = \text{Constant} \tag{33}$$

5. Conclusion and Discussion

The scale factor R increases with time. The deceleration parameter q > 0, hence the model (22) represents decelerating universe. The density and pressure decrease as time increases. The model (22) passes through a singular state at $t = -K/L$, this is explained as Creation exists all the time , so there is a big crunch between $t = -K/L$ to ∞ and Creation is going on from $t = -K/L$ to ∞. During this period the model exists. Since at $t \to \infty$, $\frac{\sigma}{\theta}$ is constant, therefore model (22) does not approach to isotropy at late time. Since $\theta \neq \infty$ at t = 0, hence model (22) is free from initial singularity.

The coordinate distance γ_H to the horizon is the maximum distance a null ray could have travelled at time t starting from infinite past i.e.

$$\gamma_H = \int_{-\infty}^{t} \frac{dt}{R^3(t)}$$

We could extent the proper time t to in the past because of non-singular nature of space time, thus

$$\gamma_H(t) = \int_0^t \frac{dt}{R^3(t)} = \int_0^t \frac{dt}{(Lt+K)}$$

The integral of diverge at lower limit shows the model is free from event horizon.

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