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BULK VISCOUS CREATION FIELD COSMOLOGICAL MODEL WITH COSMOLOGICAL TERM IN BIANCHI TYPE I SPACE-TIME

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Abstract: We have investigated Creation Field Cosmological model in presence of bulk viscosity with cosmological term in the frame work of Bianchi Type I space-time. The assumption of scale factor $(R) = e^{H_0 t}$ leads to uniform matter density, cosmological constant (Λ), the bulk viscosity coefficient (ζ) and Hubble parameter (H). The Creation field increases with time which matches with the result as given by Hoyle and Narlikar [12] and the creation is going on. The model has no initial singularity. The deceleration parameter (q) = –1 indicates that the model leads to de-Sitter expanding universe. The state finder parameter {r, s} agrees with ΛCDM model. The model exhibits particle horizon. The model represents inflationary scenario and accelerating universe.

Key words: Bulk Viscous, Creation Field, Cosmological Term, Bianchi I.

1. Introduction

The introduction of viscosity in the cosmic fluid content has been very useful in explaining many physical aspects of dynamics of homogeneous cosmological models. It is well known that in the early stage of universe when neutrino decoupling occurred, the matter behaved like viscous fluid as given by Klimek [13]. The coefficient of viscosity is known to decrease as the universe expands. The significance of viscosity in cosmology has been studied by many authors viz. Berman [9], Beesham [8], Zimdahl [23], Gron [11], Saha [19], Bali and Singh [1].

The current accelerating expansion of the universe suggests that our universe is dominated by unknown dark energy (cosmological constant). It is the most favored candidate of dark energy representing energy density of vacuum in the context of quantum field theory. The observations of distant types Ia Supernovae (Perlmutter et al) [17], Riess et al. [18] strongly favors a positive value of cosmological constant (Λ) in order to measure the expansion rate of universe and now it is believed that universe is not only expanding but also accelerating. Many authors viz. Chen and Wu [10], Sahni and Starobinsky [20], Verma and Ram [22], Bali et al. [2], Barrow and Shaw [5] investigated cosmological models with cosmological term.

After the failure of big-bang model to describe early universe, the alternative theories of gravity have been proposed from time to time. The Steady State theory of Bondi and Gold [7] was discarded for not giving any physical justification for continuous creation of matter. To overcome this difficulty, Hoyle and Narlikar [12] adopted a field theoretic approach introducing a massless and charge less scalar field. In C- field Theory there is no big-bang type singularity as in Steady State theory of Bondi and Gold [7]. Creation is accomplished at the expense of negative energy C-field as explained by Narlikar [14]. Creation field cosmological models have been investigated by Narlikar and Padmanabhan [15], Bali and Tikekar [3], Bali and Saraf [4] in different contexts.

Friedmann-Robertson-Walker (FRW) models are unstable near the singularity and fails to describe early universe as pointed by Patridge and Wilkinson [16]. Therefore, spatially homogenous and anisotropic Bianchi Type I space-time is undertaken to study the universe in its early stage of evolution.

2. The Space-time and Field Equations

We consider the Bianchi Type I space-time as

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2 \tag{1}$$

Where A, B, C are functions of cosmic time t alone and $\sqrt{-g} = ABC$.

The modified Einstein's field equations in presence of Creation field with bulk viscosity are given by

$$R_i^j - \frac{1}{2} R g_i^j = -\left[\underset{(m)}{T}{}_i^j + \underset{(C)}{T}{}_i^j\right] - \Lambda g_i^j \tag{2}$$

(In geometrized unit where $8\pi G = 1$, c = 1)

Where

$$\underset{(m)}{T}{}_i^j = \rho v_i v^j - \zeta\theta (v_i v^j - g_i^j) \tag{3}$$

(With p=0)

and

$$\underset{(c)}{T}{}_i^j = -f \left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right) \tag{4}$$

In equation (4), the energy-momentum tensor $\underset{(c)}{T}{}_i^j$ is given by Hoyle and Narlikar [12], f > 0, a coupling constant between matter and creation field. We assume the coordinates to be co-moving so that

$v^1 = 0 = v^2 = v^3, v^4 = 1$

ρ being the matter density, ζ the coefficient of bulk viscosity and $C_i = \frac{dC}{dx^i}$.

The equation (2) for the space-time (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = \zeta\theta + \frac{1}{2} f\dot{C}^2 + \Lambda \quad (5)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = \zeta\theta + \frac{1}{2} f\dot{C}^2 + \Lambda \quad (6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = \zeta\theta + \frac{1}{2} f\dot{C}^2 + \Lambda \quad (7)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} = \rho - \frac{1}{2} f\dot{C}^2 + \Lambda \quad (8)$$

Where $\dot{C} = C_4 = \frac{dC}{dt}$

3. Solution of Field Equations

To find the determinate results in terms of cosmic time t, we assume

(i) $R^3 = e^{3H_0 t}$, (9)

R being scale factor

(ii) $\rho = 3H^2$, $\zeta = \alpha\rho^{1/2}$, $\Lambda = \beta H^2$, $0 < \alpha < 1$ (10)

as given by Barrow [6]

Equations (5), (6) and (7) lead to

$$\frac{A_4}{A} - \frac{B_4}{B} = \frac{l_1}{R^3} = l_1 e^{-3H_0 t} \quad (11)$$

$$\frac{A_4}{A} - \frac{C_4}{C} = \frac{l_2}{R^3} = l_2 e^{-3H_0 t} \quad (12)$$

where l_1, l_2 are constants. Equation (9) gives

$$\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = 3H_0 \quad (13)$$

Where $R^3 = ABC$

From equations (11), (12) and (13), we have

$$\frac{A_4}{A} = H_0 + \frac{(l_1 + l_2)}{3} e^{-3H_0 t}$$

Thus, we have

$$A = e^{H_0 t}.\exp\left[-\frac{(l_1+l_2)}{9H_0}e^{-3H_0 t}\right] \tag{14}$$

Similarly

$$B = e^{H_0 t}.\exp\left[\frac{(2l_1-l_2)}{9H_0}e^{-3H_0 t}\right] \tag{15}$$

and

$$C = e^{H_0 t}.\exp\left[\frac{(2l_2-l_1)}{9H_0}e^{-3H_0 t}\right] \tag{16}$$

Therefore, the space-time (1) takes the form

$$ds^2 = dt^2 - e^{2H_0 t}.\exp\left[-\frac{2(l_1+l_2)}{9H_0}e^{-3H_0 t}\right]dx^2 - e^{2H_0 t}.\exp\left[\frac{2(2l_1-l_2)}{9H_0}e^{-3H_0 t}\right]dy^2 - e^{2H_0 t}.\exp\left[\frac{2(2l_2-l_1)}{9H_0}e^{-3H_0 t}\right]dz^2 \tag{17}$$

4. Some Physical Consequences

The Hubble parameter (H), the expansion (θ), the matter density (ρ), coefficient of bulk viscosity (ζ) and cosmological constant (Λ)) for the model (17) are given by

$$\frac{\dot{\mathrm{R}}}{\mathrm{R}} = H = H_0 \text{ (Constant)} \tag{18}$$

$$\theta = 3\mathrm{H} = 3\mathrm{H}_0 \text{ (Constant)} \tag{19}$$

$$\rho = 3\mathrm{H}^2 = 3\mathrm{H}_0^2 \text{ (Constant)} \tag{20}$$

$$\zeta = \alpha\rho^{1/2} = \alpha\sqrt{3}H_0 \text{ (Constant)} \tag{21}$$

$$\Lambda = \beta\mathrm{H}_0^2 \text{ (Constant)} \tag{22}$$

Where α, β are arbitrary constants

Conservation equation

$$[\underset{(m)}{\mathrm{T}}{}_i^j + \underset{(C)}{\mathrm{T}}{}_i^j + \Lambda]_{;j} = 0 \tag{23}$$

leads to

$$\frac{\mathrm{d}}{\mathrm{dt}}\dot{\mathrm{C}}^2 + 2\left(\frac{\mathrm{A}_4}{\mathrm{A}} + \frac{\mathrm{B}_4}{\mathrm{B}} + \frac{\mathrm{C}_4}{\mathrm{C}}\right)\dot{\mathrm{C}}^2 = \frac{2\dot{\rho}}{\mathrm{f}} + \frac{2(\rho - \zeta\theta)}{\mathrm{f}}\left(\frac{\mathrm{A}_4}{\mathrm{A}} + \frac{\mathrm{B}_4}{\mathrm{B}} + \frac{\mathrm{C}_4}{\mathrm{C}}\right) + \dot{\Lambda} = 0 \tag{24}$$

Thus, we have

$$\frac{\mathrm{d}}{\mathrm{dt}}\dot{\mathrm{C}}^2 + 6H_0\dot{\mathrm{C}}^2 = \frac{2}{f}3H_0(3H_0^2 - 3\alpha\sqrt{3}H_0^2) \text{ Using (10)} \tag{25}$$

Or

$$\frac{d}{dt}\dot{C}^2 + 6H_0\dot{C}^2 = 18\frac{H_0^3}{f}(1-\alpha\sqrt{3}) \text{ Where } 0 < \alpha < 1 \tag{26}$$

Which leads to

$$\dot{C}^2\, e^{6H_0 t} = \gamma^2\, \frac{e^{6H_0 t}}{6H_0} \tag{27}$$

Where $\gamma^2 = 18\frac{H_0^3}{f}(1-\alpha\sqrt{3})$

Thus, we have

$$\dot{C}^2 = \frac{3H_0^2}{f}(1-\alpha\sqrt{3}) = \text{Constant.}$$

which leads to $C \propto t$

The deceleration parameter (q) for the model (17) is given by

$$q = -\frac{\frac{\ddot{R}}{R}}{\frac{\dot{R}^2}{R^2}} = -1 \tag{28}$$

State Finder Parameter{r, s}

The state finder parameters effectively differentiate between forms of dark energy and provide simple diagnosis whether the given model fits into the basic observational data. The state finder diagnostic pair {r,s} is given by Sahni et al. [21] as

$$\gamma = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} = 1 \tag{29}$$

$$\boldsymbol{s} = \frac{\gamma-1}{3(q-\frac{1}{2})} = 0 \tag{30}$$

Which agrees with ΛCDM model.

Particle Horizon / Event Horizon

The co-ordinate distance to the horizon $\gamma_H(t)$ is the maximum distance a null ray could have traveled at time t starting from infinite past. Thus particle horizon for the model (17) is given by

$$\gamma_H(t) = \int_{-\infty}^{\infty} \frac{dt}{R^3} \tag{31}$$

We would extend the proper time t to (–∞) in the past because of non-singular nature of the space-time. Now, we have

$$\gamma_{\mathrm{H}}(\mathrm{t}) = \int_0^\infty e^{-3H_0 t} dt = \mathit{finite} \tag{32}$$

Thus, the model (17) has particle horizon.

5. Conclusion

For the model (17), the matter density (ρ), the coefficient of bulk viscosity (ζ), the expansion (θ), Hubble parameter (H) and the cosmological Constant (Λ) are constants. The conservation equation in presence of C- field is satisfied. The Creational field (C) is proportional to time which agrees with the result as given by Hoyle and Narlikar [12]. The model (17) has no initial singularity. The deceleration parameter q= -1 leads to de-Sitter model of expanding universe. The state finder parameter {r, s} agrees with the result of ΛCDM model. The model (17) has particle horizon i.e. the model permits communicable region. The spatial volume increases exponentially with time indicating inflationary scenario. The model also represents accelerating universe.

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