Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.18, No.3&4, July-December, 2019, 201-214

FERROMAGNETIC CONVECTION IN THE PRESENCE OF DUST PARTICLES WITH MAGNETIC FIELD DEPENDENT VISCOSITY-REVISITED

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Abstract: A linear stability analysis is carried to examine the effect of magnetic field dependent (MFD) viscosity on the thermal convection in a ferrofluid in the presence of dust particles and a uniform vertical magnetic field has been investigated. An essential correction is applied to the paper by Sunil et al. (J. Appl.Math. Comput.,27,(2008), 7-22), so as to predict the correct behavior of MFD viscosity.The critical wave number and the critical Rayleigh number for the onset of instability, for the case of stationary modes using free boundary conditions, are computed numerically for sufficiently large values of the magnetic parameter M_1. Numerical results are obtained and are illustrated graphically. It is shown that magnetic field dependent viscosity has a stabilizing effect whereas dust particles always have a destabilizing effect on the system. Further, magnetizationM_3 may have a stabilizing or destabilizing effect.

Mathematics Subject Classification: 76E06, 76E20, 76E25.

Keywords and Phrases: Ferrofluid, Convection, Magnetic field dependent viscosity, Dust particles, Magnetization.

Introduction

Ferrofluids are the colloidal suspensions of magnetic submicron sized particles, covered by a surfactant (such as oleic acid) to prevent their agglomeration and dispersed in an organic carrier (e.g. water, ester, kerosene or heptane). In recent decades, the studies on ferromagnetic fluids have attracted the attention of many researchers because of the diverse applications of magnetic fluids. These applications include the contrast enhancement of magnetic resonance imaging (MRI), sealing of rotating shafts, magnetic drug targeting hyperthermia, cooling of loudspeakers, pressure seals of compressors and blowers etc.(Rosensweig [20], Odenbach [5-6]).

Ferroconvection is a concept which is very much similar to Bénard convection and several researchers have given their contribution in the development of the same. A fundamental introduction to ferrohydrodynamics is provided by Rosensweig [20]. Finlayson [2] has studied the convective instability of a ferrofluid layer heated from below. Lalas and Carmi [4] have studied the thermal instability of ferrofluids without considering the buoyancy effects. For a broad view of the subject, one may refer to Shliomis [29], Gupta and Gupta [3], Schwab et al. [24], Stiles and Kagan [30], Rudraiah and Shekar [21], Qin and Kaloni [18], Aniss et al. [1], Sunil and Mahajan [32] and Prakash [8-10].

In nature, there exist many fluid systems in which the fluid contains dust (or suspended) particles. The study of these systems is very useful from the physical and mathematical point of view. Saffman [22] has studied the stability of a laminar flow of a dusty gas. Scanlon and Segel [23] have investigated some effects of suspended particles on the onset of Bénard convection. For a comprehensive view of the subject, one may refer to Sharma et al. [25], Palaniswamy and Purushotam [7], Sharma and Sunil [26], Sharma et al. [27] and Sunil et al. [33-34].

Ferroconvection has gained vital importance because of the astonishing physical properties of ferrofluids. Amongst this viscosity of ferrofluid is one such property. Several research papers have been published in the recent past considering the influence of magnetic field dependent (MFD) viscosity on ferromagnetic convection. For details of such investigations, one may refer to Shliomis [28], Vaidyanathan et al. [37, 39, 40], Ramanathan and Suresh [19], Sunil et al. [31, 35], Prakash and Gupta [13] and Prakash [11].Sunil et al. [36] have theoretically investigated the effect of magnetic field dependent (MFD) viscosity on the thermal convection in a ferrofluid in the presence of dust particles.

In the research papers cited in the preceding paragraph, these researchers have carried out their analysis by considering the MFD viscosity in the form$\mu = \mu_1(1 + \boldsymbol{\delta}.\boldsymbol{B})$, where μ_1 is fluid viscosity in the absence of magnetic field $\boldsymbol{B}$ and $\boldsymbol{\delta}$ is the variation coefficient of viscosity. They have resolvedμ into componentsμ_x,μ_y and μ_z which is not justified since μ, being a scalar quantity, cannot be decomposed in such a manner. Though theyhave studied a very important problem of ferrohydrodynamics, but their results cannot be relied upon due to this wrong assumption. Recently, Prakash and Bala [12] and Prakash et al. [14-17] has pointed out this mistake and made the corrections in some problems.In the present communication, particular attention has been given to the above cited paper by Sunil et al. [36]. Keeping in view this fact, now we have reformulated the basic equations considered by Sunil et al. [36] and then carried out mathematical and numerical analysis to correct the existing findings.

Mathematical Formulation

Consider a ferromagnetic Boussinesq fluid layer of infinite horizontal extension and finite vertical thickness d in the presence of dust particles heated from below which is kept under the action of a uniform vertical magnetic field $\boldsymbol{H}$ (see Fig.1).

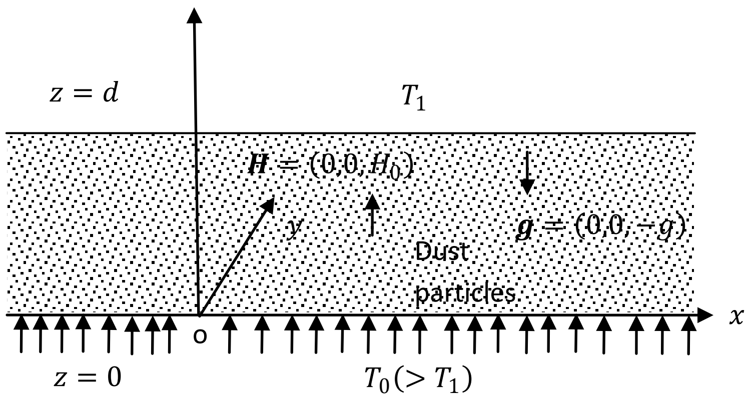

Fig.1. Geometrical Configuration.

The fluid is assumed to be incompressible having a variable viscosity, given by $\mu = \mu_1(1 + \boldsymbol{\delta}.\boldsymbol{B})$, where μ_1 is the viscosity of the fluid when there is no magnetic field applied, μ is the magnetic field dependent viscosityand $\boldsymbol{B}$ is the magnetic induction. The variation coefficient of viscosity $\boldsymbol{\delta}$ has been taken to be isotropic, i.e. $\delta_1 = \delta_2 = \delta_3 = \delta$.As a first approximation for small field variation, linear variation of magneto viscosity has been used (Sunil et al. [36]).

The basic governing equations for the above model are given by (Sunil et al. [36]).

$$\nabla.\boldsymbol{q} = 0, \tag{1}$$

$$\rho_0\left[\frac{\partial}{\partial t} + \boldsymbol{q}.\nabla\right]\boldsymbol{q} = -\nabla p + \rho\boldsymbol{g} + KN(\boldsymbol{q}_d - \boldsymbol{q}) + \nabla.(\boldsymbol{HB}) + \mu\nabla^2\boldsymbol{q}, \tag{2}$$

$$\left[\rho_0 C_{V,H} - \mu_0\boldsymbol{H}.\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt} + \mu_0 T\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{V,H}.\frac{D\boldsymbol{H}}{Dt} + mNC_{pt}\left(\frac{\partial}{\partial t} + \boldsymbol{q}_d.\nabla\right)T = K_1\nabla^2 T, \tag{3}$$

where $\boldsymbol{q}, \boldsymbol{q}_d, p, \rho, N, \boldsymbol{H}, \boldsymbol{M}, \mu, \boldsymbol{g} = (0,0,-g)$, $C_{V,H}$,C_{pt}, μ_o, T and K_1 denote respectively the velocity of ferrofluid, velocity of dust particles, pressure, density, number density of the dust particles, magnetic field,magnetization, variable viscosity, acceleration due to gravity, specific heat at constant volume and magnetic field, specific heat of dust particles, magnetic permeability, temperature and thermal conductivity.$K = 6\pi\mu r$, r being the particle radius, is the Stokes drag coefficient.

The density equation of state is

$$\rho = \rho_0[1 + \alpha(T_0 - T)], \tag{4}$$

Where$\alpha$is coefficient of volume expansion and $\rho_0$is the density at some properly chosen mean temperature T_0.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force on the particles is neglected. Inter-particle reactions are also ignored since it is assumed that the distances between particles are quite large as compared to their diameters. The effects due to pressure, gravity, viscous force on the particles are negligibly small, and therefore ignored (Sunil et al. [36]). If mN is the mass of particles per unit volume, then the equations of motion and continuity of the dust particles, under the above assumptions are

$$mN\left[\frac{\partial}{\partial t}+ \boldsymbol{q}_d.\nabla\right]\boldsymbol{q}_d = KN(\boldsymbol{q}-\boldsymbol{q}_d), \tag{5}$$

$$\frac{\partial N}{\partial t}+\nabla.(N\boldsymbol{q}_d)=0. \tag{6}$$

For a non-conducting fluid with no displacement current, the Maxwell's equations are given by

$$\nabla.\boldsymbol{B}=0,\nabla\times\boldsymbol{H}=0, \tag{7a,b}$$

where magnetic induction is given by

$$\boldsymbol{B}=\mu_0(\boldsymbol{H}+\boldsymbol{M}). \tag{8}$$

Eqs. (7a) and (8), thus gives

$$\nabla.(\boldsymbol{H}+\boldsymbol{M})=0. \tag{9}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field as well as the temperature as (Finlayson [2])

$$\boldsymbol{M}=\left(\frac{\boldsymbol{H}}{H}\right)M(H,T). \tag{10}$$

The linearized magnetic equation of state is

$$M=M_0+ \chi(H-H_0)-K_2(T-T_0) \tag{11}$$

where M_0 is the magnetization when magnetic field is H_0 and temperature T_0,

$\chi=\left(\frac{\partial \boldsymbol{M}}{\partial H}\right)_{H_0,T_0}$ is magnetic susceptibility, $K_2=-\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{H_0,T_0}$ is the pyromagnetic coefficient.

The basic state is assumed to be quiescent state and is given by

$$\boldsymbol{q}=\boldsymbol{q}_b=0,\boldsymbol{q}_d=(\boldsymbol{q}_d)_{\boldsymbol{b}}=0,\rho=\rho_b(z),p=p_b(z),T=T_b(z)=-\beta z+T_0,\beta=\frac{T_0-T_1}{d},\boldsymbol{H}_b=\left(H_0-\frac{K_2\beta z}{1+\chi}\right)\hat{\boldsymbol{k}},\boldsymbol{M}_b=\left(M_0+\frac{K_2\beta z}{1+\chi}\right)\hat{\boldsymbol{k}},N=N_b=N_0,\ H_0+M_0=H_0{}^{ext}. \tag{12}$$

Only the spatially varying parts of H_o and M_o contributes to the analysis, so that the direction of the external magnetic field is unimportant and the convection is the same whether the external magnetic field is parallel or antiparallel to the gravitational force (Finlayson [2]).

Now following Finlayson [2], we analyze the stability of basic state by introducing the following perturbations

$$\boldsymbol{q} = \boldsymbol{q}_b + \boldsymbol{q}', \boldsymbol{q}_d = (\boldsymbol{q}_d)_{\boldsymbol{b}} + \boldsymbol{q}_d{}', \rho = \rho_b(z) + \rho', p = p_b(z) + p', T = T_b(z) + \theta',$$
$$\boldsymbol{H} = \boldsymbol{H}_b(z) + \boldsymbol{H}', \boldsymbol{M} = \boldsymbol{M}_b(z) + \boldsymbol{M}', N = N_b + N'. \quad (13)$$

where $\boldsymbol{q}' = (u', v', w')$, $\boldsymbol{q}_d{}' = (l', r', s')$, ρ', p', θ', $\boldsymbol{H}'$ and $\boldsymbol{M}'$are infinitesimal perturbations in velocity of ferrofluid, velocity of dust particles, density, pressure, temperature, magnetic field intensity and magnetization. Further, using Eq. (13) into Eqs. (1) – (11) and utilizing the basic state solutions, we obtain the linearized perturbation equations in the form

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0, \quad (14)$$

$$L_0\rho_0\frac{\partial u'}{\partial t} = L_0\left[-\frac{\partial p'}{\partial x} + \mu_0(H_0 + M_0)\frac{\partial H_1'}{\partial z} + \mu_1[1 + \delta\mu_0(H_0 + M_0)]\nabla^2 u'\right] - mN_0\frac{\partial u'}{\partial t}, \quad (15)$$

$$L_0\rho_0\frac{\partial v'}{\partial t} = L_0\left[-\frac{\partial p'}{\partial y} + \mu_0(H_0 + M_0)\frac{\partial H_2'}{\partial z} + \mu_1[1 + \delta\mu_0(H_0 + M_0)]\nabla^2 v'\right] - mN_0\frac{\partial v'}{\partial t}, \quad (16)$$

$$L_0\rho_0\frac{\partial w'}{\partial t} = L_0\left[-\frac{\partial p'}{\partial z} + \mu_0(H_0 + M_0)\frac{\partial H_3'}{\partial z} + \mu_1[1 + \delta\mu_0(H_0 + M_0)]\nabla^2 w' + \rho_0 g\alpha\theta' - \mu_0 K_2\beta H_3' + \frac{\mu_0 K_2^2\beta\theta'}{(1+\chi)}\right] - mN_0\frac{\partial w'}{\partial t}, \quad (17)$$

$$L_0\left[(\rho C_1 + mN_0C_{pt})\frac{\partial\theta'}{\partial t} - \mu_0 T_0 K_2\frac{\partial}{\partial t}\left(\frac{\partial\phi'}{\partial z}\right)\right] = L_0K_1\nabla^2\theta' + L_0\left(\rho C_1\beta - \frac{\mu_0 T_0 K_2^2\beta}{1+\chi}\right)w' + mN_0\beta C_{pt}w', \quad (18)$$

where $\rho C_1 = \rho_0 C_{V,H} + \mu_0 K_2 H_0$, $L_0 = \left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)$,(19)

$$\frac{\partial}{\partial x}(H_1' + M_1') + \frac{\partial}{\partial y}(H_2' + M_2') + \frac{\partial}{\partial z}(H_3' + M_3') = 0, \vec{H}' = \nabla\phi', \quad (20)$$

where ϕ' is the perturbed magnetic potential,

and $H_3' + M_3' = (1 + \chi)H_3' - K_2\theta'$, $H_i' + M_i' = \left(1 + \frac{M_o}{H_o}\right)H_i'(i = 1, 2)$, (21)

where we have assumed $K_2\beta d \ll (1 + \chi)H_0$. (Finlayson [2])

Eliminating u', v', p' between Eqs. (15)-(17) using Eq. (14), we have

$$\left[L_0\rho_0\frac{\partial}{\partial t} + mN_0\frac{\partial}{\partial t}\right](\nabla^2 w') = L_0\left[\rho_0 g\alpha\nabla_1^2\theta' - \mu_0 K_2\beta\frac{\partial}{\partial z}(\nabla_1^2\phi') + \frac{\mu_0 K_2^2\beta\nabla_1^2\theta'}{(1+\chi)} + \mu_1[1 + \delta\mu_0(H_0 + M_0)]\nabla^4 w'\right], \quad (22)$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Further, Eqs. (20) and (21) yields

$$(1 + \chi)\frac{\partial^2\phi'}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right)\nabla_1^2\phi' - K_2\frac{\partial\theta'}{\partial z} = 0. \quad (23)$$

Now following normal mode technique, we analyze the perturbations w', θ' and ϕ' into two dimensional periodic waves and considering disturbances characterized by a particular wave number k. Thus we assume to all quantities describing the perturbation a dependence on x, y and t of the form

$$\left(w', \theta', \phi'\right)(x, y, z, t) = \left[w''(z), \theta''(z), \phi''(z)\right]\exp\left[i\left(k_x x + k_y y\right) + nt\right], \quad (24)$$

where k_x and k_y are the wave numbers along the x– and y– directions respectively and $k = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number.

Using (24) in Eqs. (18), (22) and (23) and non-dimensionalizing the variables by setting

$$z_* = \frac{z}{d}, w_* = \frac{d}{\nu}w'', a = kd, t_* = \frac{\nu}{d^2}t, \omega = \frac{nd^2}{\nu}, D_* = \frac{\partial}{\partial z_*}, \theta_* = \frac{K_1 a R^{1/2}}{\rho C_1 \beta \nu d}\theta'', \phi_* = \frac{(1+\chi)K_1 a R^{1/2}}{K_2 \rho C_1 \beta \nu d^2}\phi'', \ k_{1*} = \frac{k_1}{d^2}, \ \nu = \frac{\mu_1}{\rho_0}, P_r = \frac{\nu \rho C_1}{K_1}, \ \delta_* = \mu_0 \delta H_0(1+\chi), \ R = \frac{g\alpha\beta\, d^4 \rho C_1}{K_1 \nu}, M_1 = \frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha \rho_0 g}, M_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_1}, M_3 = \frac{1+\frac{M_0}{H_0}}{(1+\chi)}, \tau = \frac{m\nu}{Kd^2}, L_{0*} = \left(\tau\frac{\partial}{\partial t_*} + 1\right), f = \frac{mN_0}{\rho_0}, \ h = \frac{mN_0 C_{pt}}{\rho C_1} \quad (25)$$

we get (dropping the asterisks for convenience)

$$(D^2 - a^2)[L_0\{(1+\delta M_3)(D^2 - a^2) - \omega\} - \omega f]w = aR^{1/2}L_0\left\{(1+M_1)\theta - M_1 D\phi\right\}, \quad (26)$$

$$L_0[D^2 - a^2 - \omega(1+h)P_r]\theta + L_0 P_r M_2 \omega D\phi = -[h + L_0(1 - M_2)]aR^{1/2}w, \quad (27)$$

$$(D^2 - a^2 M_3)\phi = D\theta, \quad (28)$$

where z is the real independent variable such that $0 \le z \le 1$, D is differentiation with respect to z, a^2 is square of the wave number, $P_r > 0$ is the Prandtl number, ω is the complex growth rate, $R > 0$ is the Rayleigh number, h is the dust parameter, $M_1 > 0$ is the magnetic number which defines ratio of magnetic forces due to temperature fluctuation to buoyant forces, $M_2 > 0$ is a non-dimensional parameter which defines the ratio of thermal flux due to magnetization to magnetic flux, $M_3 > 0$ is the measure of the nonlinearity of magnetization, $\omega = \omega_r + i\omega_i$ is a complex constant in general such that ω_r and ω_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$ and $\phi(z) = \phi_r(z) + i\phi_i(z)$ are complex valued functions of the real variable z such that $w_r(z)$, $w_i(z)$, $\theta_r(z)$, $\theta_i(z)$, $\phi_r(z)$ and $\phi_i(z)$ are real valued functions of the real variable z.

Since, M_2 is of very small order (Finlayson[2]), it is neglected in the subsequent analysis and thus Eq. (27) takes the form

$$L_0[D^2 - a^2 - \omega(1+h)P_r]\theta = -[h + L_0]aR^{1/2}w, \quad (29)$$

The horizontal boundaries are considered to be free. Hence the boundary conditions are

$$w = 0 = \theta = D^2 w = D\phi \text{ at } z = 0 \text{ and } z = 1 \text{(Both the boundaries are free)} \quad (30)$$

It may further be noted that Eqs.(26) and (28)-(30) describe an eigenvalue problem for ω and govern ferromagnetic convection, with MFD viscosity, in the presence of dust particles heated from below.

Mathematical Analysis

Exact solutions for free boundaries:

Following the analysis of Finlayson [2], the exact solutions satisfying the boundary conditions (30) aregiven by

$$w = Asin\pi z, \theta = Bsin\pi z, \phi = -\frac{C}{\pi} cos\pi z, D\phi = Csin\pi z,$$

where$A$, B and $C$are constants. Substitution of above solutions in Eqs. (26) and (28)-(29) yield a system of three linear homogeneous algebraic equations in the unknowns A, B and$C$.For the existence of non-trivial solutions of this system, the determinant of the coefficients of$A$, B and$C$must vanish. This determinant on simplification yields

$$U\omega^2 + V\omega + W = 0, \tag{31}$$

where,$U = L_0 P_r k^2 (1+h)(\pi^2 + a^2 M_3)(f + L_0)$, (32)

$$V = L_0 k^4 (\pi^2 + a^2 M_3)(f + L_0) + L_0{}^2 P_r k^4 (1+h)(\pi^2 + a^2 M_3)(1 + \delta M_3), \tag{33}$$

$$W = L_0{}^2 k^6 (\pi^2 + a^2 M_3)(1 + \delta M_3) - L_0 R a^2 \pi^2 (L_0 + h) - L_0 R a^4 M_3 (L_0 + h)(1 + M_1), \tag{34}$$

and $k^2 = (\pi^2 + a^2)$.

By substituting $\omega = i\omega_i$ in Eq. (31), we obtain marginal state of convection. Further, when $\omega_i = 0$, condition for stationary convection is determined which in turn yields the Rayleigh number for stationary convection as

$$R = \frac{\pi^4 (1+x_1)^3 (1+x_1 M_3)(1+\delta M_3)}{x_1 h_1 [1 + x_1 (1+M_1) M_3]}, \tag{35}$$

where, $x_1 = \frac{a^2}{\pi^2}$ and $h_1 = (1+h)$.

With M_1very large, Eq. (35) yields the magnetic thermal Rayleigh number $N = RM_1$ for stationary mode as

$$N = \frac{\pi^4 (1+x_1)^3 (1+x_1 M_3)(1+\delta M_3)}{x_1{}^2 h_1 M_3}. \tag{36}$$

To find the minimum value N_c of N with respect to wave number x_1, Eq. (36) is differentiated with respect to x_1 and equated to zero and the following polynomial in x_1 is obtained.

$$(1 + \delta M_3)[2M_3 x_1{}^5 + (1 + 3M_3) x_1{}^4 - (3 + M_3) x_1{}^2 - 2x_1] = 0. \tag{37}$$

The above equation is solved numerically for various values of M_3 (see Table 1) using the software Scientific Workplace and the minimum value$x_c$of $x_1$is obtained each time, hence the critical wave number is obtained. On using this value in Eq. (36), the critical magnetic Rayleigh numberN_c is obtained, above which the instability sets in as stationary convection.

Table 1: Critical magnetic thermal Rayleigh numbers and wave numbers of the unstable modes at marginal stability for the onset of stationary convection for different values of magnetic field dependent viscosity, non-buoyancy magnetization and dust parameters.

δ	M_3	x_c	$h_1 = 1$ N_c	$h_1 = 3$ N_c	$h_1 = 5$ N_c	$h_1 = 7$ N_c	$h_1 = 9$ N_c
0.01	1	1	1574.1	524.71	314.83	224.88	174.9
	5	0.6899	922.87	307.62	184.57	131.84	102.54
	10	0.6131	853.23	284.41	170.65	121.89	94.803
	15	0.58134	849.36	283.12	169.87	121.34	94.373
	20	0.56370	863.18	287.73	172.64	123.31	95.909
0.03	1	1	1605.3	535.1	321.06	229.33	178.37
	5	0.6899	1010.8	336.92	202.15	144.39	112.31
	10	0.6131	1008.4	336.12	201.67	144.05	112.04
	15	0.58134	1070.9	356.98	214.19	152.99	118.99
	20	0.56370	1150.9	383.64	230.18	164.42	127.88
0.05	1	1	1636.5	545.49	327.29	233.78	181.83
	5	0.6899	1098.7	366.22	219.73	156.95	122.07
	10	0.6131	1169.1	389.72	232.70	166.21	129.28
	15	0.58134	1292.5	430.84	258.50	184.64	143.61
	20	0.56370	1438.6	479.54	287.73	205.52	159.85
0.07	1	1	1667.6	555.88	333.53	238.23	185.29
	5	0.6899	1186.5	395.52	237.31	169.51	131.84
	10	0.6131	1318.6	439.54	263.72	188.37	146.51
	15	0.58134	1514.1	504.69	302.82	216.30	168.23
	20	0.56370	1726.4	575.45	345.27	246.62	191.82
0.09	1	1	1698.8	566.27	339.76	242.69	188.76
	5	0.6899	1274.4	424.81	254.89	182.06	141.60
	10	0.6131	1473.8	491.25	294.75	210.54	163.75
	15	0.58134	1735.7	578.55	347.13	247.95	192.85
	20	0.56370	2014.1	671.36	402.82	287.73	223.79

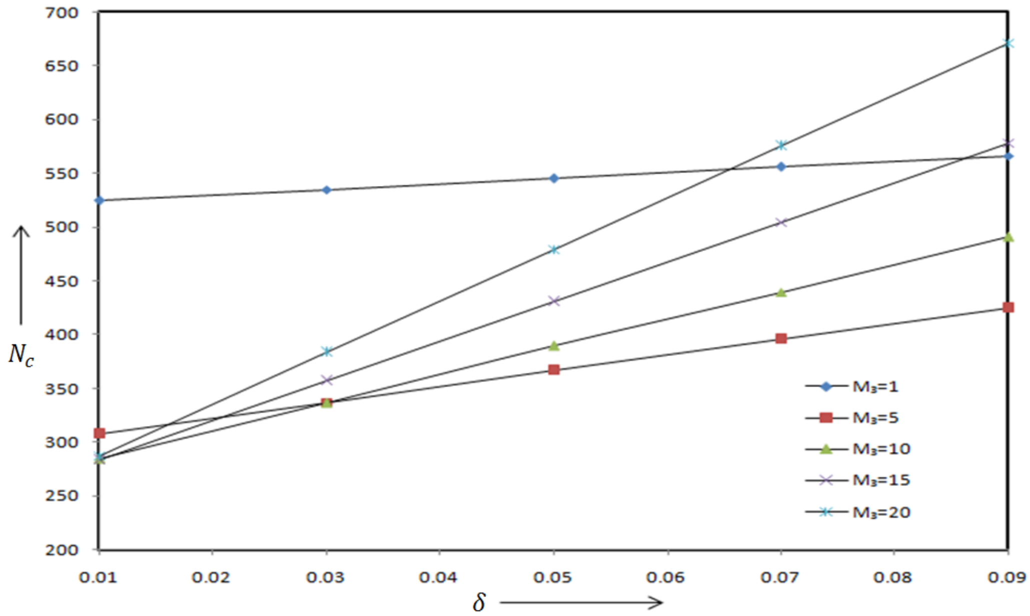

Fig. 2. The variation of critical magnetic Rayleigh number (N_c) with magnetic field dependent viscosity (δ) for stationary convection for $h_1 = 3$.

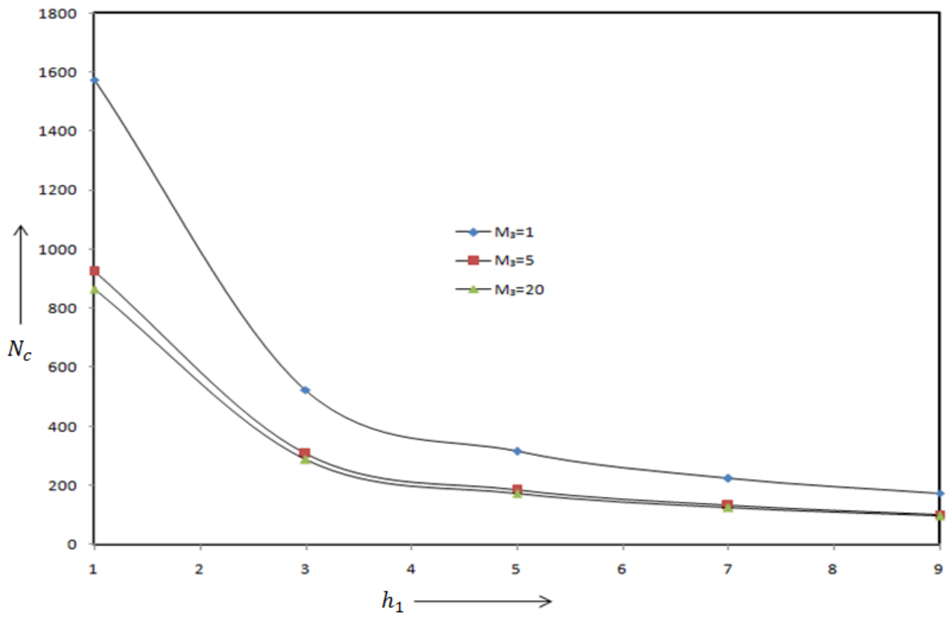

Fig. 3. The variation of critical magnetic Rayleigh number (N_c) with dust particle parameter (h_1) for stationary convection for $\delta = 0.01$.

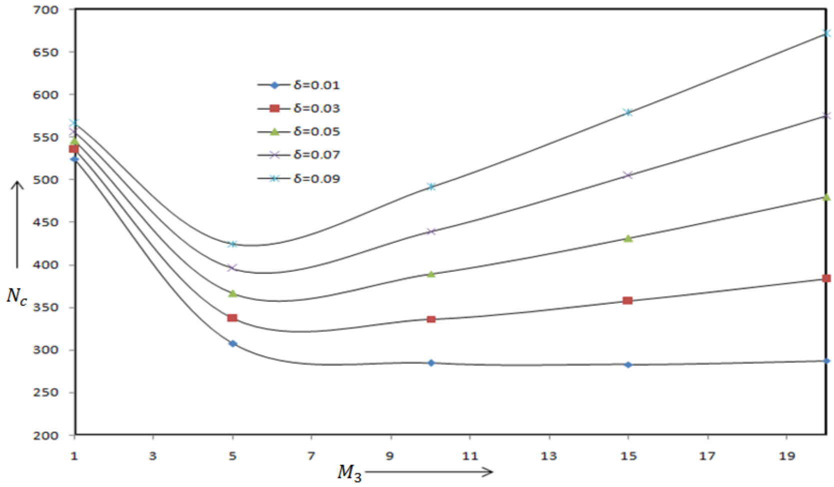

Fig. 4. The variation of critical magnetic Rayleigh number (N_c) with magnetization parameter (M_3) for stationary convection for $h_1 = 3$.

Discussion of results

In the present paper the influence of magnetic field dependent viscosity on the thermal convection in aferrofluid layer heated from below in the presence of dust particles andsubjected to a uniform vertical magnetic field has been investigated. The magnetization parameter M_1 is considered to be 1000 (Vaidyanathan et al. [38]). The value of M_2 being negligible (Finlayson [2]), has been taken to be zero. The values of the parameter M_3 are varied from 1 to 20. The values of the magnetic field dependent viscosity parameterδ, is varied from 0.01 to 0.09.

Specific attention has been paid to a paper published by Sunil et al. [36].These researchers have carried out their analysis by taking MFD viscosity as$\mu = \mu_1(1 + \boldsymbol{\delta}.\boldsymbol{B})$. Further they decomposedμ into components μ_x, μ_y and $\mu_z$along the coordinate axes which is not justified. This is because μ, being a scalar quantity, cannot be decomposed into components. Thus a correction to their work is very much neededso that a correct interpretation may be given to the problem under investigation. Thus the basic equations have been restructured and thereafter mathematical and numerical analysis has been carried out. Results thus derived have significant variations from the previous results which were otherwise obtained by using wrong assumption. The explanation of the results obtained in table 1 and various figures is as follows:

From table 1 and from figure 2, it is evident that the critical value of magnetic Rayleigh number $N_c = (RM_1)_c$ increasesdirectly with the MFD viscosity parameter δ, and hence delays the onset of convection.Thus the MFD viscosity always has a stabilizing effect on the system.

It is also found from table 1 and from figure 3, thatthe critical value of magnetic Rayleigh number $N_c = (RM_1)_c$ decreases with the increase in the dust particle parameter $h_1$and thereby prepones the onset of convection. Hence dust particles have a destabilizing effect on the system. It is also evident from figure 3 that in the absence of dust particles $(h_1 = 1)$, the critical magnetic Rayleigh number N_c is very high however in the presence of dust particles $(h_1 > 1)$, the critical magnetic Rayleigh number N_c is reduced significantly because heat capacity of fluid increases due to dust particles.

Further, we may note from table 1 and from figure 4, that the variation of the critical magnetic Rayleigh number $N_c = (RM_1)_c$ is studied with respect tothe magnetization parameter M_3 for different valuesof the MFD viscosity parameterδ. It is clear thatN_c decreases for lower values of M_3 and then increases for higher values of M_3. It is also evident that the lower values of N_c are needed for onset of convection with an increase in M_3 for lower values of δ, whereas higher values of N_c are needed for onset of convection with an increase in M_3 for higher values of δ hence justifying the competition between the stabilizing effect of MFD viscosity δ and the destabilizing effect of non-buoyancy magnetization M_3.It is worth mentioning here that the result obtained here and the result obtained by Sunil et al. [39] (certainly using the wrong assumption regarding MFD viscosityμ) predicts almost the same behavior (except in figure 4 for $\delta = 0.01$) but the values of the critical magnetic Rayleigh number N_c obtained herein are definitely higher than those of Sunil et al. [36].

Conclusion

A linear stability analysis using normal mode technique has been carried out to study the effect of MFD viscosity on a ferromagnetic fluid layer heated from below in the presence of dust particles. A correction is applied to Sunil et al. [36] whereinmagnetic field dependent viscosity $\mu = \mu_1(1 + \boldsymbol{\delta}.\boldsymbol{B})$ is decomposed into components which is otherwise not permissible as μ is a scalar quantity. Thus the results obtained in the present analysis are on correct footing and show the correct behavior of the problem. Further, the values of the critical magnetic Rayleigh number N_c are definitely higher than those of Sunil et al. [36].

Acknowledgment: The authors are also thankful to the Referee for valuable comments and suggestions.

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