

MHD FLOW PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH HEAT SOURCE / SINK

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Abstract: The effect of heat source/sink past an impulsively started vertical plate under the influence of transverse magnetic field has been investigated. The governing equations of the flow are solved by Laplace transformation technique. The effect of various parameters in the problem on velocity, temperature, skin-friction and nusselt number are shown with graphs and table. The study shows that the temperature of the fluid is high without heat source/sink parameter and decreases with increasing value of heat source/sink parameter. Further, it is also shown that the velocity of the fluid flow is low without heat source/sink parameter as compared to velocity of fluid with introducing heat source/sink parameter and velocity decreases with increasing value of heat source/sink parameter.

Keywords: MHD, Skin-friction, Temperature, Heat Source/Sink.

1. Introduction

The study of MHD flow is very important because of its applications in geophysics, astrophysics, engineering and technology. The study of effect of magnetic field on temperature distribution with heat source/sink when fluid is capable of emitting and absorbing thermal radiations is of great importance in concerned with space applications and higher operating temperatures. Many researchers have investigated on effect of heat transfer and radiation on flow. Gupta et al. [8] studied free convection flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Basant et al. [1] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Elbashbeshy [3] analyzed heat and mass transfer along a vertical plate in the presence of the magnetic field under the combined buoyancy effects of thermal and species diffusion. Singh and Garg [10] have also obtained exact solution of an oscillatory free convection MHD flow in a rotating channel in the presence of heat transfer due to radiation. Recently, Garg [4] investigated magneto hydrodynamics and radiation effects on the flow due to moving vertical porous plate with variable temperature. Very Recently, Garg [5] investigated on an oscillatory hydro magnetic convective flow of viscous incompressible and electrically

conducting fluid in a vertical porous channel when the entire system consisting of channel plates and the fluid rotates about an axis perpendicular to the plates.

The study of heat generation or absorption in moving fluids is important in several physical problems such as fluids undergoing exothermic or endothermic chemical reactions. The effect of heat source/sink in thermal convection is considered where there are high temperature differences between surfaces such as space craft bodies. These effects have been studied by various researchers. Chamkha [2] studied hydro magnetic mixed convection non-darcian flow of an electrically conducting and heat-generating / absorbing fluid in a channel embedded in a uniform porous medium. Ramachandra et al. [9] studied the radiation effects on an unsteady two dimensional hydro magnetic free convective boundary layer flow of a viscous incompressible fluid past a semi-infinite vertical plate with mass transfer in the presence of heat source or sink. Suneetha et al. [11] investigated radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink. Recently, Garg et al. [6] investigated on unsteady MHD radiative and convective flow through a porous medium in a vertical channel. Very recently, Garg et al. [6,7] analyzed an unsteady MHD mixed convection flow through a porous medium filled in a vertical channel in the presence of heat radiation.

The present model has applications in geophysics, astrophysics, aero space, solar energy collection systems and also in the design of high operating temperature chemical process systems. The present study examines MHD flow past an impulsively started infinite vertical plate in presence of magnetic field and heat source/sink. The solutions are obtained in terms of exponential and complementary error functions. The results are shown with the help of graphs and tables.

2. Mathematical Analysis

Here an electrically conducting, viscous, incompressible fluid past an impulsively started infinite vertical plate is considered. The x' -axis is taken along the plate in vertically upward direction and y' -axis is taken normal to the plate. Initially, the plate and surrounding gas are at same temperature T'_∞ . At time $t' > 0$, the plate temperature is slightly raised to $T'_w - T'_\infty > 0$. The plate is given an impulsively motion with a velocity u_0 . The flow is governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) - \frac{\sigma B_0^2 u'}{\rho} \quad (1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - Q^*(T' - T'_\infty) \quad (2)$$

The initial and boundary conditions are

$$\begin{aligned} u' &= 0, T' = T'_\infty \quad \forall y', t' \leq 0 \\ u' &= u_0, T' = T'_w \quad \text{at } y'=0, t' > 0 \\ u' &\rightarrow 0, T' \rightarrow T'_\infty \quad \text{as } y' \rightarrow \infty, t' > 0 \end{aligned} \quad (3)$$

Here u' is the velocity in the x -direction, ρ the density, g the acceleration due to gravity, β the coefficient of thermal expansion, T' the temperature of the fluid near the plate, Q^* is heat source/sink, C_p the specific heat at constant pressure, K the thermal conductivity.

Introducing the following non-dimensional quantities,

$$u = \frac{u'}{u_0}, y = \frac{\sqrt{G}y'u_0}{v}, t = \frac{Gt'u_0^2}{v}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \quad (4)$$

$$M = \frac{\sigma B_0^2 v}{\rho G u_0^2}, H = \frac{Q^* v^2}{k u_0^2 G}, Pr = \frac{\mu c_p}{k}, G = \frac{g \beta v (T'_w - T'_\infty)}{u_0^3}$$

where Pr the Prandtl number, H the heat source/sink parameter, M the magnetic field parameter, G the Grashof number and θ the dimensionless temperature.

Then in view of (4), equations (1) and (2) reduce to

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - H\theta \quad (5)$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta - Mu \quad (6)$$

with following boundary conditions

$$u = 0, \theta = 0 \quad \forall y', t \leq 0$$

$$u = 1, \theta = 1 \quad \text{at } y = 0, t > 0 \quad (7)$$

$$u \rightarrow 0, \theta \rightarrow 0 \quad \text{as } y \rightarrow \infty, t > 0$$

3. Solution of the Problem

Solving equations (5) subject to the initial and boundary conditions (7) by the usual Laplace transformation technique,

$$L \left(Pr \frac{\partial \theta}{\partial t} \right) = L \left(\frac{\partial^2 \theta}{\partial y^2} - H\theta \right)$$

$$Pr [sL(\theta(y, t)) - (\theta(y, 0))] = \frac{\partial^2 L(\theta(y, t))}{\partial y^2} - HL(\theta(y, t))$$

$$[D^2 - (H + s Pr)] L(\theta(y, t)) = 0$$

$$L(\theta(y, t)) = A_1 e^{y\sqrt{H+sPr}} + A_2 e^{-y\sqrt{H+sPr}} \quad (8)$$

Using boundary conditions (7), $A_1 = 0, A_2 = \frac{1}{s}$

$$\text{Therefore, } L(\theta(y, t)) = \frac{1}{s} e^{-y\sqrt{H+sPr}} \quad (9)$$

Now, Solving equations (6) subject to the initial and boundary conditions (7) by the usual Laplace transformation technique,

$$L\left(\frac{\partial u}{\partial t}\right) = L\left(\frac{\partial^2 u}{\partial y^2} + \theta - Mu\right)$$

$$sL(u(y, t)) - u(y, 0) = \frac{\partial^2 L(u(y, t))}{\partial y^2} + L(\theta(y, t)) - ML(u(y, t))$$

$$[D^2 - (M + s)]L(u(y, t)) = -L(\theta(y, t))$$

$$A.E = A_3 e^{y\sqrt{s+M}} + A_4 e^{-y\sqrt{s+M}}$$

$$P.I = \frac{1}{1 - P_r} \left[\frac{1}{s(s-a)} \right] e^{-y\sqrt{H+sp_r}}$$

Therefore, $L(u(y, t)) = A.E. + P.I.$

$$L(u(y, t)) = A_3 e^{y\sqrt{s+M}} + A_4 e^{-y\sqrt{s+M}} + \frac{1}{1 - P_r} \left[\frac{1}{s(s-a)} \right] e^{-y\sqrt{H+sp_r}} \quad (10)$$

$$\text{Using boundary conditions, } A_3 = 0, A_4 = \frac{1}{s} - \frac{1}{1 - P_r} \left[\frac{1}{s(s-a)} \right]$$

Therefore,

$$L(u(y, t)) = \left[\left(1 + \frac{1}{b}\right) \frac{1}{s} + \left(\frac{1}{b}\right) \frac{1}{s-a} \right] e^{-y\sqrt{s+M}} + \left(\frac{1}{b}\right) \left[\frac{1}{s-a} - \frac{1}{s} \right] e^{-y\sqrt{H+sp_r}} \quad (11)$$

Where 's' is the Laplace transformation parameter. By taking inverse Laplace transformation of equation (9) and (11), we get general solution of problem.

The solutions are given by

$$\theta(y, t) = \frac{1}{2} \left[e^{y\sqrt{H}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} \sqrt{P_r} + \sqrt{\frac{Ht}{P_r}} \right) + e^{-y\sqrt{H}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} \sqrt{P_r} - \sqrt{\frac{Ht}{P_r}} \right) \right] \quad (12)$$

$$\begin{aligned} u(y, t) &= \frac{1}{2} \left(1 + \frac{1}{b} \right) \left[e^{y\sqrt{M}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{Mt} \right) + e^{-y\sqrt{M}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{Mt} \right) \right] \\ &\quad - \frac{1}{2b} e^{at} \left[e^{y\sqrt{c}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{ct} \right) + e^{-y\sqrt{c}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{ct} \right) \right] \\ &\quad + \frac{1}{2b} e^{at} \left[e^{y\sqrt{c}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} \sqrt{P_r} + \sqrt{\frac{c}{P_r} t} \right) + e^{-y\sqrt{c}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} \sqrt{P_r} - \sqrt{\frac{c}{P_r} t} \right) \right] \\ &\quad - \frac{1}{2b} \left[e^{y\sqrt{H}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} \sqrt{P_r} + \sqrt{\frac{H}{P_r} t} \right) + e^{-y\sqrt{H}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} \sqrt{P_r} - \sqrt{\frac{H}{P_r} t} \right) \right] \quad (13) \end{aligned}$$

Where $a = \frac{H-M}{1-P_r}$, $b = H - M$, $c = \frac{H-MP_r}{1-P_r}$ and $\operatorname{erfc}(x)$ is complementary error function defined as $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$, $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$.

4. Skin-Friction Coefficient

Skin friction is derived from velocity field. The effect of t , H , M and Pr on Skin Friction coefficient in the dimensionless form is given by

$$\tau = - \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

$$\tau = \left(1 + \frac{1}{b}\right) \left[\sqrt{M} \operatorname{erf} \sqrt{Mt} + \frac{1}{\sqrt{\pi t}} e^{-Mt} \right] - \frac{1}{b} e^{at} \left[\frac{1}{\sqrt{\pi t}} e^{-ct} + \sqrt{c} \operatorname{erf} \sqrt{ct} - \sqrt{c} \operatorname{erf} \sqrt{\frac{c}{Pr} t} - \sqrt{\frac{Pr}{\pi t}} \frac{1}{b} \left[\sqrt{\frac{Pr}{\pi t}} e^{-\frac{Ht}{Pr}} + \sqrt{H} \operatorname{erf} \sqrt{\frac{H}{Pr} t} \right] \right]$$

5. Nusselt Number

The Nusselt number is the measure of rate of heat transfer which is derived from temperature field. The effect of t , H and Pr on Nusselt number in the dimensionless form is given by

$$N_u = - \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = \sqrt{\frac{Pr}{\pi t}} e^{-\frac{Ht}{Pr}} + \sqrt{H} \operatorname{erf} \sqrt{\frac{H}{Pr} t}$$

6. Results and Discussions

The problem of flow past an impulsively started infinite vertical plate with heat source/sink has been formulated to understand the physical meaning of the problem. We have computed the expression for u and θ for different values of magnetic field parameter M , Prandlt number Pr , heat source/sink parameter H and time t . The purpose of the calculations given here is to assess the effects of the parameters H , M and t upon the nature of the flow.

Table 1: Temperature Results of Fluid without Heat Source/Sink and with Heat Source/Sink Parameter (taking $Pr=0.71$ and $M=1$ as constants)

y ($t=0.2$)	$\theta(y, t)$ ($H=0$)	$\theta(y, t)$ ($H=2$)	t ($y=0.2$)	$\theta(y, t)$ ($H=0$)	$\theta(y, t)$ ($H=2$)
0	1.0000	1.0000	0	0	0
0.1	0.8940	0.8485	0.1	0.7063	0.6588
0.2	0.7899	0.7151	0.2	0.7899	0.6551
0.3	0.6894	0.5981	0.3	0.8278	0.6648
0.4	0.5941	0.4960	0.4	0.8506	0.6681
0.5	0.5053	0.4076	0.5	0.8662	0.6693
0.6	0.4241	0.3316	0.6	0.8777	0.6699
0.7	0.3510	0.2669	0.7	0.8867	0.6701
0.8	0.2865	0.2125	0.8	0.8940	0.6702
0.9	0.2305	0.1672	0.9	0.9000	0.6703
1	0.1828	0.1299	1	0.9051	0.6703

The temperature profiles are shown in Figs. 1-3 and Table-1. The effect of Heat source/sink parameter on temperature of the fluid is shown in Table-1 and it is shown graphically by Fig.1. From Table-1, it is observed that temperature of fluid is high without heat source/sink parameter as compared to introducing heat source/sink parameter in the flow. This effect is shown in fig.1. We observed from graph that temperature decreases with increasing value of H.

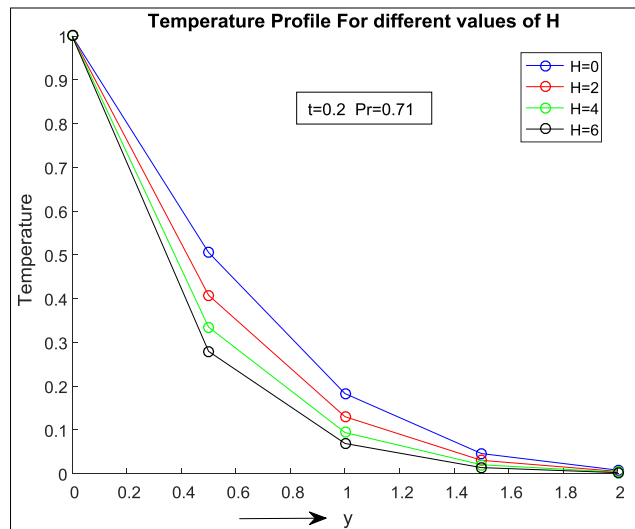


Fig.1: Effect of Heat Source/Sink Parameter on Temperature Profile

Fig.2 shows temperature profile at H=2 and Pr=0.71 (for air) at increasing values of t. We observed that temperature of the fluid increases as the time is increased.

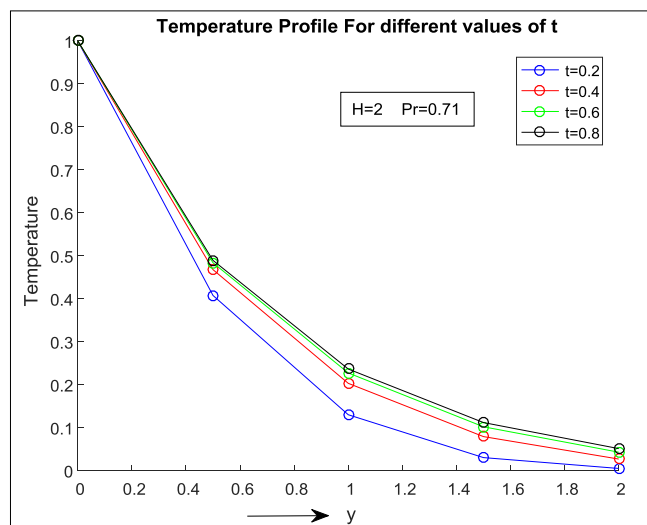


Fig.2: Temperature Profile at different time

In fig.3, the effect of Prandlt number on temperature of fluid has been presented. It is observed that the increasing the value of Prandlt number results in the decreasing the temperature of fluid. This is because the smaller values of Prandlt number are equivalent to increasing thermal conductivity. So heat is able to diffuse away from the heated surface more rapidly than for higher values of Prandlt number.

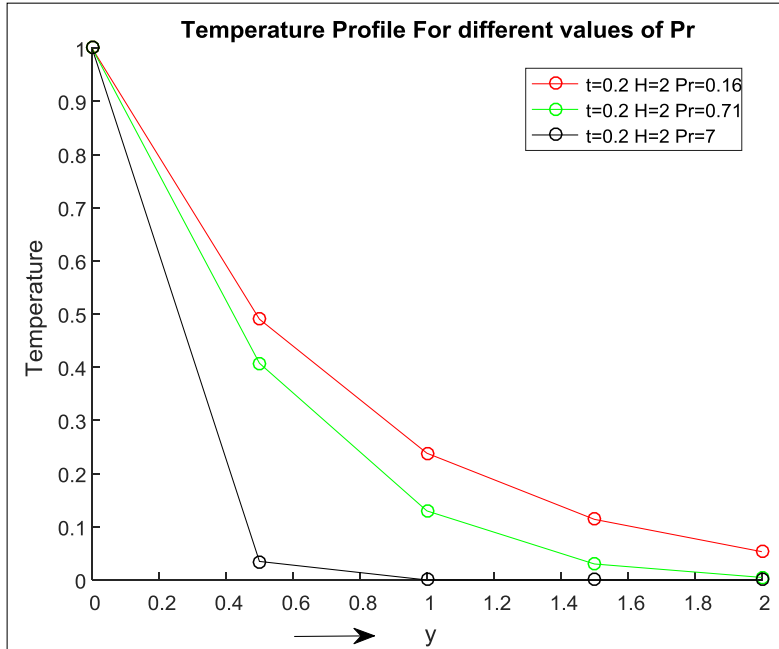


Fig.3: Temperature Profile at different Values of Prandlt Number

Table 2: Velocity Results of Fluid without Heat Source/Sink and with Heat Source/Sink Parameter (taking Pr=7 and M=1 as constants)

y (t=0.2)	u(y, t) (H=0)	u(y, t) (H=2)	t (y=0.2)	u(y, t) (H=0)	u(y, t) (H=2)
0	1.0000	1.0000	0	-0.7461	0.6933
0.1	0.6803	1.0100	0.1	0.1522	1.0726
0.2	0.4132	0.9886	0.2	0.4132	0.9886
0.3	0.2224	0.9157	0.3	0.5471	0.9325
0.4	0.1062	0.7972	0.4	0.6314	0.8941
0.5	0.0462	0.6554	0.5	0.6905	0.8662
0.6	0.0198	0.5137	0.6	0.7349	0.8451
0.7	0.0100	0.3877	0.7	0.7697	0.8287
0.8	0.0068	0.2839	0.8	0.7980	0.8157
0.9	0.0058	0.2029	0.9	0.8216	0.8051
1	0.0052	0.1418	1	0.8417	0.7965

The comparative study of velocity of fluid without heat source/sink parameter and with heat source parameter is shown by Table-2 and Fig.4. It is observed that velocity of fluid is low in absence of heat source/sink parameter. This effect is clearly depicted in Fig. 4. The graph also depicts that velocity of fluid decreases with increasing values of heat source/sink parameter.

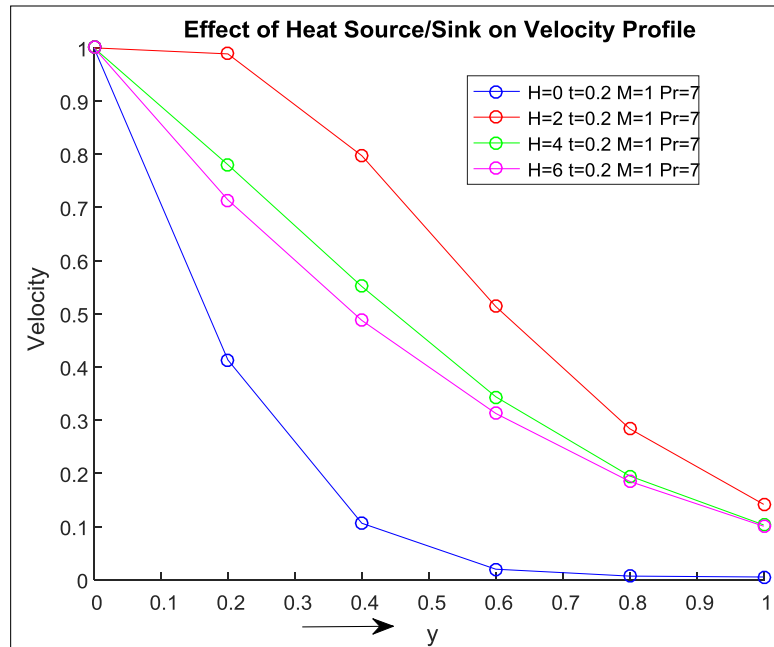


Fig.4: Velocity Profile for different values of Heat Source/Sink Parameter

Table 3: Velocity Results of Fluid without Magnetic Field Parameter and with Magnetic Field Parameter (taking $Pr=0.71$ and $H=2$ as constants)

y (t=0.2)	u(y, t) (M=0)	u(y, t) (M=1)	t (y=0.2)	u(y, t) (M=0)	u(y, t) (M=1)
0	1.0000	1.0000	0	0.5499	1.0412
0.1	0.8747	0.8973	0.1	0.7178	0.8195
0.2	0.7543	0.8062	0.2	0.7543	0.8062
0.3	0.6435	0.7284	0.3	0.7729	0.8007
0.4	0.5462	0.6651	0.4	0.7866	0.7990
0.5	0.4661	0.6175	0.5	0.7980	0.7992
0.6	0.4059	0.5864	0.6	0.8081	0.8004
0.7	0.3677	0.5723	0.7	0.8171	0.8020
0.8	0.3531	0.5754	0.8	0.8252	0.8038
0.9	0.3631	0.5959	0.9	0.8325	0.8055
1	0.3987	0.6337	1	0.8392	0.8071

Table-3 shows the effect of Magnetic field parameter on velocity Profile. It is observed that velocity of fluid is slow without magnetic field. Magnetic field parameter accelerates the motion of the fluid. This effect is shown graphically by Fig.5.

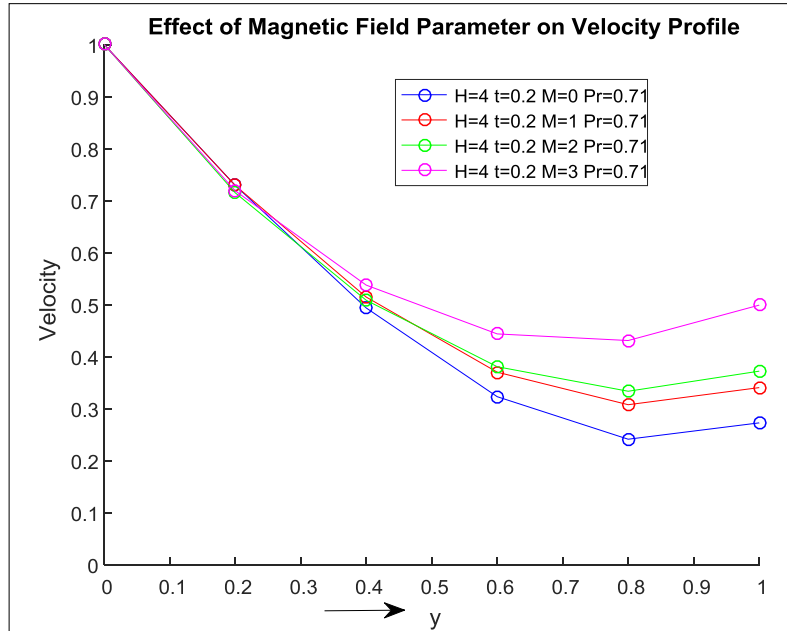


Fig.5: Velocity Profile at different values of Magnetic field parameter

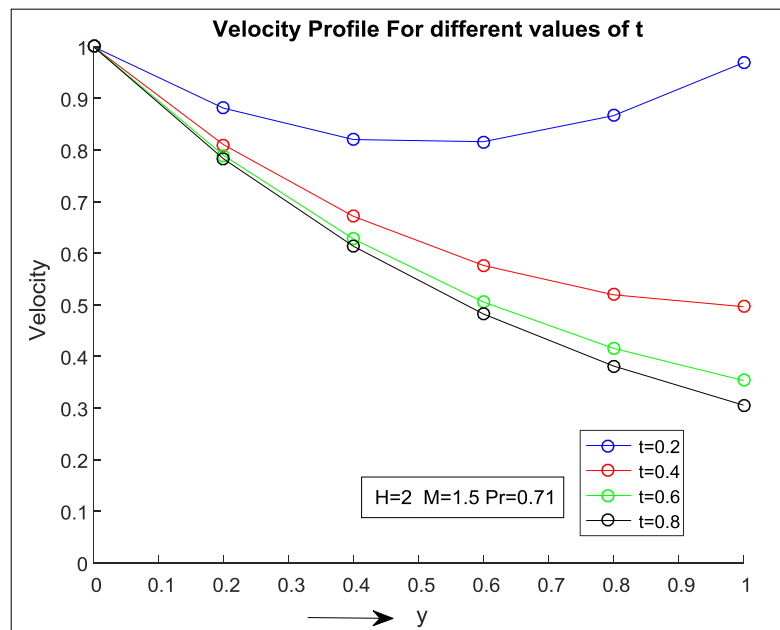


Fig.6: Effect of time on Velocity Profile

The variations in velocity with time t are presented in Fig.6. For $H=2$ and $Pr=0.71$, it is observed that velocity decreases with increased in time

The Skin-Friction coefficient is presented for different values of flow parameters t ; H , M and Pr are shown in Table-4. Skin friction coefficient determines the shear stress. From this table, we conclude that shear stress increases with increasing values of t , H , M and Pr . The effect of these parameters on skin friction is shown graphically by Fig.7.

Table-4: Values of Skin-Friction Coefficient at different values of flow parameters

Time (t)	Heat Source/ Sink (H)	Magnetic field (M)	Prandlt Number(Pr)	Skin Friction (τ)
0.2	2	1	0.71	2.7200
0.2	4	1	0.71	3.8388
0.2	6	1	0.71	7.7592
0.2	2	1.5	0.71	3.0628
0.2	2	1.8	0.71	4.2519
0.4	2	1	0.71	3.3821
0.6	2	1	0.71	5.0064
0.2	2	1	0.16	0.9612
0.2	2	1	0.025	-2.7902

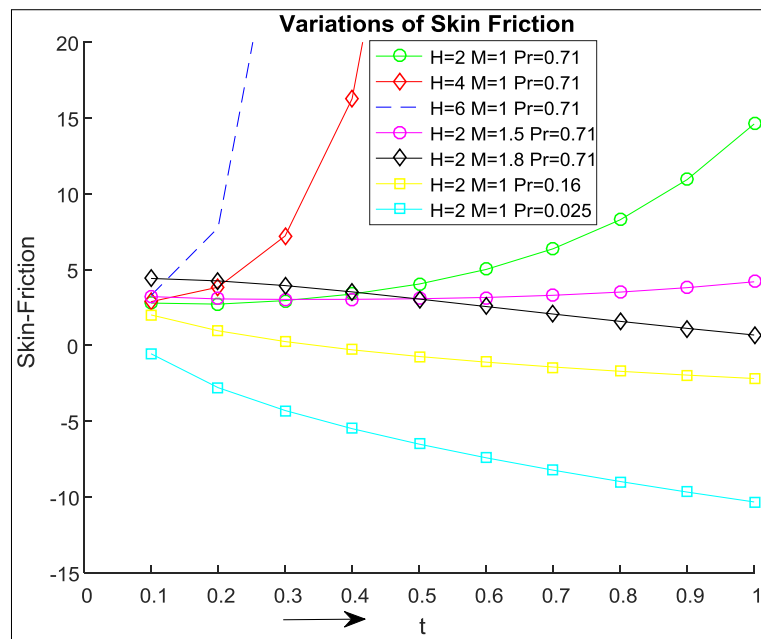


Fig.7: Effect of Parameters H , M and Pr on Skin Friction Coefficient

Table-5 shows the variations of parameters t , H and Pr on the nusselt number which determines the rate of heat transfer. It is observed that rate of heat transfer increases with increasing values of H and Pr . It is also observed that rate of heat transfer decreases when time is increased.

Table-5: Values of Nusselt Number at different values of flow parameters

Time (t)	Heat Source/Sink (H)	Prandlt Number(Pr)	Nusselt Number (Nu)
0.2	2	0.71	1.61147
0.2	4	0.71	2.077969
0.2	6	0.71	2.48404
0.4	2	0.71	1.469346
0.6	2	0.71	1.434161
0.2	2	0.16	1.4198
0.2	2	0.025	1.4142

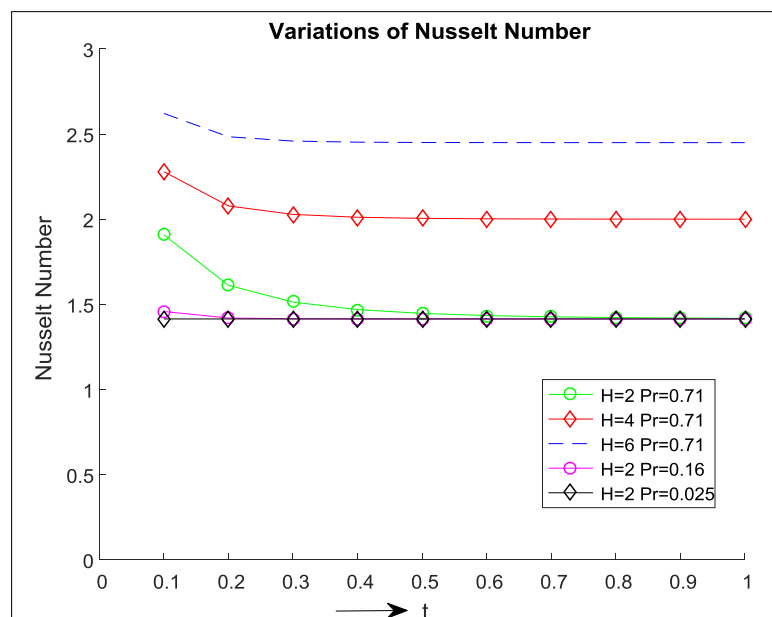


Fig.8: Effect of Parameters H and Pr on Nusselt Number

8. Conclusions

The study of MHD flow past an impulsively started infinite vertical plate with heat source/sink and magnetic field can be concluded as

- (i) The temperature of the fluid is high without heat source/sink parameter and decreases with increasing value of heat source/sink parameter. Also temperature of fluid increases with progression in time. Further, increasing the value of Prandlt number results in the decreasing the temperature of fluid.

- (ii) The velocity of the fluid flow is low without heat source/sink parameter and velocity decreases with increasing value of heat source/sink parameter and also when time is increased. It is also observed that magnetic field parameter accelerate the motion of the fluid.
- (iii) The shear stress increases with increasing value of time, heat source/sink parameter, magnetic field parameter and Prandlt number.
- (iv) The rate of heat transfer increases with increasing heat source/sink parameter and Prandlt number but decreases with increasing the time.

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