Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.17, No.3&4, July-December, 2018, 111-126

A TWO UNIT STANDBY SYSTEM WITH SKILLED AND REGULAR REPAIRMEN TO REPAIR PRIORITY AND ORDINARY UNITS

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Abstract: The paper deals with a model of two non-identical unit (ordinary and priority) cold standby systemassuming that one of the unit gets priority unit in operation. Each unit has two possible modes-normal and total failure. The repair of each unit is performed by separate repairman namely regular and skilled. The regular repairman is always available with the system and attends the failed ordinary unit instantaneously whereas the skilled repairman is called from the outside to come at the system whenever the priority unit fails. The skilled repairman leaves the system as soon as the priority unit is repaired. A repaired unit by either of the tworepairmen always works as good as new. The distributions of waiting time for skilled repairman and repair time of each unitare assumed to be exponential. The failure time distributions of each unit are taken to be general with different C.d.f's. The various measures of the system effectiveness are obtained by using regenerative point technique.

Keywords: Coldstandby, MTSF, regenerative point, availability, transition probability and mean sojourn time.

1. Introduction

Repair maintenance is one of the important device to increase the reliability, availability and expected life of the system.In a single unit system, there is no improvement in reliability and expected life of the system whereas point-wise as well as steady-state availabilities of the system increase. But in case of a redundant system all the measuresof system effectiveness such as reliability, availability and expected profit earned by the system increase due to the repair maintenance. Generally, for analyzing a system model, a single repairman is considered to repair a failed unit. The repairman may always be available with the system if the cost of repairman is not too much high, otherwise he may be called to come at the system from outside whenever required. When repairman is called from outside he may take some significant time to reach at the system and during this time the failed unit waits for repairman. This time is known as arrival time of repairman or waiting time of repairman.

Various authors including, Gopalan and D'Souza [3], Gopalan and Radhakrishnan [4, 5], Gupta et.al. [6], Agnihotri and Satsangi [1], Mahmoud and Esmail [9] etc. analyzed two and more unit redundant system models with a single repairman who is always available with the system to repair a failed unit. Some of the authors including Gupta and Coworkers [2, 8] analyzed the two-unit redundant system models considering that the repairman is not always available with the system. They have assumed that a single repairman is called to come at the system whenever required and the repairman takes some significant time to reach at the system and this time is known as waiting time or arrival time of repairman. Gupta and Bhardwaj [7] further analyzed a two identical unit cold standby system model assuming tworepairmen. One is regular who is always available with the system and other is skilled who is arranged from the outside and he takes some significant time to reach at the system. As soon as a unit fails, it is attended immediately by regular repairman. If regular repairman is unable to repair a failed unit within a specified time limit, whose probability is 'q' then, the skilled repairman is intimated to reach at the system and the failed unit waits for skilled repairman till he arrives at the system. The skilled repairman completes the repair of failed unit with certainty.

The purpose of the present paper is to analyze a two non-identical unit cold standby system assuming one of the units gets priority in operation. The repair of each unit is performed by separate repairman named as regular and Skilled. Regular repairman is less costly and remains always available with the system to attend the repair of a failed ordinary unit. The skilled repairman who is more costly is not always available with the system and is called from the outside for attending a failed priority unit for its repair and he takes some significant time to reach at the system and this time is known as arrival time of skilled repairman.Using regenerative point technique, the following measures of system effectiveness are obtained-

i. Transition probabilities and mean sojourn times in various states of the system.

ii. Reliability and mean time to system failure (MTSF).

iii. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).

iv. The expected busy period of skilled andregular repairmen during time interval (0, t).

v. Net expected profit earned by the system in time interval (0, t) and in steady-state.

2. Model Description and Assumptions

i. The system consists of two non-identical units (unit-1 and unit-2). Unit-1 gets priority in operation. Therefore, initially unit-1 is operative and unit-2 is kept into cold standby.

ii. Each unit has two possible modes: Normal (N) and Total failure (F).

iii. Two repairmen skilled and regular are considered in the system to repair a failed unit. The regular repairman is always available with the system and attends the failed ordinary unit.The skilled repairman is called from the outside to come at the system whenever the priority unit fails. The skilled repairman takes some significant time to reach at the system and this time is known as arrival timeof the skilled repairman.

iv. A repaired unit by either of the two repairmen always works as good as new.

v. The skilledrepairman leaves the system as soon as the priority unit is repaired.

vi. The distribution of arrival time for skilled repairman and the distribution of repair timeof each unit are taken to be exponentials with different parameters.

vii. The failure time distributions of each unit are taken to be general with different C.d.f's.

3. Notations and States of the System

a) Notations:

E	:	Set of regenerative states $=\{S_0 \text{ to } S_5\}$.
θ	:	Constant arrival rate of the skilled repairman.
$\beta_1,\ \beta_2$	:	Constant repair rate of unit-1and unit-2.
$F_1(\bullet), F_2(\bullet)$	:	C.d.f. of failure time of unit-1 and unit-2 and $f_1(\bullet), f_2(\bullet)$ denote the corresponding p.d.fs.
n_1	:	Mean failure time of unit-1 $= \int \overline{F_1}(t)dt$
qij(•)	:	p.d.f. of transition time from regenerative state S_i to S_j.
$q_{ij}^{(k)}(\bullet)$	:	p.d.f. of transition time from regenerative state S_i to S_j via non-regenerative state S_k.
~	:	Symbol for Laplace-stieltjes transforms i.e. $\tilde{A}(s) = \int e^{-st} dA(t)$
*	:	Symbol for Laplace-transform i.e. $A^*(s) = \int e^{-st} A(t) dt$
©	:	Symbol for ordinary convolution i.e. $A(t)©B(t) = \int_0^t A(u)B(t-u)du$

The limits of the integration are not mentioned whenever they are 0 to ∞.

b) Symbols for the states of the systems:

N_o^1 / N_o^2 : Unit-1/unit-2in Normal (N) mode andoperative.

N_s^2 : Unit-2in Normal (N) mode and standby.

F_{WR}^1 / F_R^1 : Unit-1 in total failure (F) mode and under waiting/repairfor/by skilledrepairman.

F_r^2 : Unit-2 in total failure (F) mode and under repair by regular repairman.

Considering the above symbols and keeping in view the assumptions stated in section-2, the possible states of the system model are shown in transition diagram (Fig.1). The epochs of transitions into the states S_3 from S_1 and S_0 from S_5, are non-regenerative while all the other entrance epochs into the states are regenerative. It is to be noted that the behavior of states S_0 and S_3 is regenerative as well as non-regenerative as these states contain both regenerative and non-regenerative points.

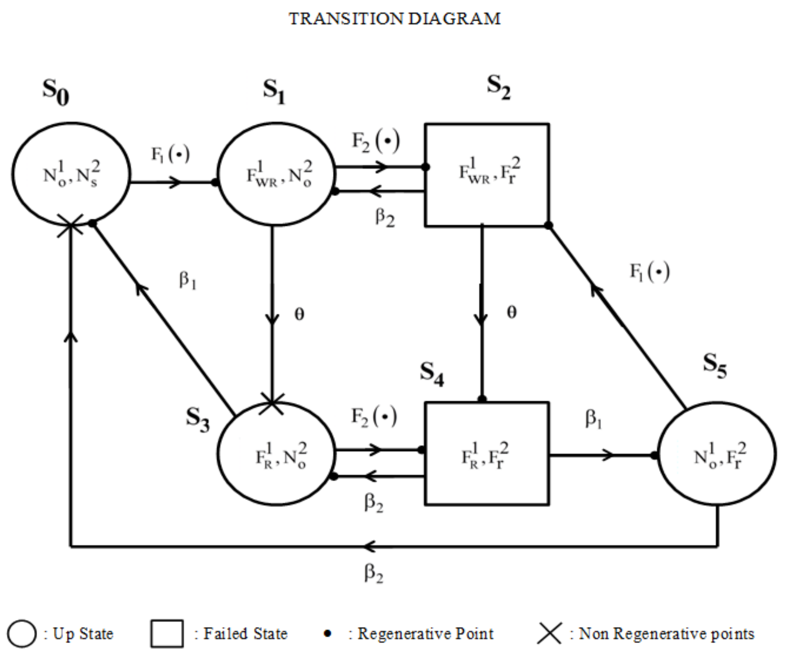

Fig.1

4. Transition Probabilities

1. The direct or one-step steady state transition probabilities can be obtained as follows:

$$p_{01} = \int dF_1(t) = 1$$

Similarly,

$$p_{12} = \int e^{-\theta t} dF_2(t) = \tilde{F}_2(\theta),$$

$$p_{21} = \int \beta_2 e^{-(\theta+\beta_2)t} dt = \frac{\beta_2}{\theta+\beta_2}$$

$$p_{24} = \int \theta e^{-(\theta+\beta_2)t} dt = \frac{\theta}{\theta+\beta_2},$$

$$p_{30} = \int \beta_1 e^{-\beta_1 t} \overline{F_2}(t) = 1 - \tilde{F}_2(\beta_1)$$

$$p_{34} = \int e^{-\beta_1 t} dF_2(t) = \tilde{F}_2(\beta_1),$$

$$p_{43} = \int \beta_2 e^{-(\beta_1+\beta_2)t} dt = \frac{\beta_2}{\beta_1+\beta_2}$$

$$p_{45} = \int \beta_1 e^{-(\beta_1+\beta_2)t} dt = \frac{\beta_1}{\beta_1+\beta_2},$$

$$p_{52} = \int e^{-\beta_2 t} dF_1(t) = \tilde{F}_1(\beta_2)$$

2. The two-steps or indirectsteady state transition probabilities are as follows:

$$p_{10}^{(3)} = \frac{\theta\beta_1}{\theta-\beta_1} \int \left(e^{-\beta_1 t} - e^{-\theta t}\right) \overline{F_2}(t) dt = \frac{\theta\beta_1}{\theta-\beta_1} \left[\left(\frac{1-\tilde{F}_2(\beta_1)}{\beta_1} \right) - \left(\frac{1-\tilde{F}_2(\theta)}{\theta} \right) \right]$$

$$p_{14}^{(3)} = \frac{\theta}{\theta-\beta_1} \int \left(e^{-\beta_1 t} - e^{-\theta t}\right) dF_2(t) = \frac{\theta}{\theta-\beta_1} \left[\tilde{F}_2(\beta_1) - \tilde{F}_2(\theta) \right]$$

$$p_{51}^{(0)} = \int \left(1 - e^{-\beta_2 t}\right) dF_1(t) = 1 - \tilde{F}_1(\beta_2)$$

It can be easily verified that,

$$p_{01} = 1, \qquad p_{10}^{(3)} + p_{12} + p_{14}^{(3)} = 1, \qquad p_{21} + p_{24} = 1$$

$$p_{30} + p_{34} = 1, \qquad p_{43} + p_{45} = 1, \qquad p_{51}^{(0)} + p_{52} = 1 \qquad (1\text{-}6)$$

5. Mean Sojourn Times

Themean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before making the transition into any other state. If T_0 denotes the sojourn time in state S_0, then mean sojourn time in state S_0 is;

$$\psi_0 = \int P(T_0 > t)\,dt = \int \bar{F}_1(t)\,dt = n_1 \tag{7}$$

Similarly,

$$\psi_1 = \int e^{-\theta t}\bar{F}_2\,dt = \frac{1-\tilde{F}_2(\theta)}{\theta}, \qquad \psi_2 = \int e^{-(\theta+\beta_2)t}dt = \frac{1}{(\theta+\beta_2)}$$

$$\psi_3 = \int e^{-\beta_1 t}\bar{F}_2(t)\,dt = \frac{1-\tilde{F}_2(\beta_1)}{\beta_1}, \qquad \psi_4 = \int e^{-(\beta_1+\beta_2)t}dt = \frac{1}{\beta_1+\beta_2}$$

$$\psi_5 = \int e^{-\beta_2 t}\tilde{F}_1(t)\,dt = \frac{1-\tilde{F}_1(\beta_2)}{\beta_2} \tag{8-12}$$

6. Analysis of Characteristics

a) Reliability of the System and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0,t) given that at t=0 it starts from state $S_i \in E.$ To obtain it we assume the failed states S_2 and S_4 as absorbing. Now using simple probabilistic arguments we have the following recurrence relations in $R_i(t)$; i=0, 1.

$$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t)$$

$$R_1(t) = W_1(t) + q_{10}^{(3)}(t) © R_0(t) \tag{13-14}$$

Where,

$$Z_0(t) = \bar{F}_1(t),$$

and,

$$W_1(t) = e^{-\theta t}\bar{F}_2(t) + \int_0^t \theta e^{-\theta u}du\, e^{-\beta_1(t-u)}\bar{F}_2(t) = \left[\frac{\theta e^{-\beta_1 t} - \beta_1 e^{-\theta t}}{\theta-\beta_1}\right]\bar{F}_2(t)$$

Taking Laplace transforms of relations (13-14) and simplifying the resulting set of algebraic equationsfor $R_0^*(s)$, we get;

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* W_1^*}{1 - q_{01}^* q_{10}^{(3)*}} \tag{15}$$

Here also for brevity we have omitted the argument‘s’ from $q_{ij}^*(s), Z_0^*(s)$ and $W_1^*(s)$.

The expression of mean time to system failure is given by,

$$E(T_0) = \lim_{s\to 0} R_0^*(s)$$

Observing $q_{ij}^*(0) = p_{ij}, Z_0^*(0) = \psi_0$ and $W_1^*(0) = \int W_1(t)\,dt = \dfrac{\theta\psi_3 - \beta_1\psi_1}{\theta - \beta_1}$

We have,

$$E[T_0] = \frac{\left[\psi_0 + \left(\dfrac{\theta\psi_3 - \beta_1\psi_1}{\theta - \beta_1}\right)\right]}{\left(1 - p_{10}^{(3)}\right)} \tag{16}$$

b) Availability Analysis

Let $A_i^j(t)$ be the probability that the system is up (operative) at epoch t, due to the working of unit-1 and 2 respectively for j=1, 2 when system initially starts from state $S_i \in E$. Using the basic probabilistic argumentsinregenerative point technique as in case of reliability, one can obtain the recurrence relations for $A_i^j(t)$; i=0, 1, ---, 5 as follows;

$$A_0^j(t) = (1-\delta)Z_0(t) + q_{01}(t) © A_1^j(t)$$

$$A_1^j(t) = \delta W_1(t) + q_{12}(t) © A_2^j(t) + q_{10}^{(3)}(t) © A_0^j(t) + q_{14}^{(3)}(t) © A_4^j(t)$$

$$A_2^j(t) = q_{21}(t) © A_1^j(t) + q_{24}(t) © A_4^j(t)$$

$$A_3^j(t) = \delta Z_3(t) + q_{30}(t) © A_0^j(t) + q_{34}(t) © A_4^j(t)$$

$$A_4^j(t) = q_{43}(t) © A_3^j(t) + q_{45}(t) © A_5^j(t)$$

$$A_5^j(t) = (1-\delta)W_5(t) + q_{52}(t) © A_2^j(t) + q_{51}^{(0)}(t) © A_1^j(t) \tag{17-22}$$

Here δ is a dichotomous variable taking value ‘0’ in above relations when system is operative due to the unit-1 and takes value ‘1’ in the relations when system is operative due to the unit-2. The values of $Z_0(t)$ and $W_1(t)$ have been already defined in reliability section. The values of $Z_3(t)$ and $W_5(t)$ are as follows :

$$Z_3(t) = e^{-\beta_1 t}\bar{F}_2(t)$$

$$W_5(t) = e^{-\beta_2 t}\bar{F}_1(t) + \int_0^t \beta_2 e^{-\beta_2 u}\,du\,\bar{F}_1(t) = \bar{F}_1(t)$$

Taking Laplace transforms of relations (17-22)and simplifying the resulting equations we get the values of $A_0^{1*}(s)$ and $A_0^{2*}(s)$ as follows;

$$A_0^{1*}(s) = \frac{N_1(s)}{D_1(s)} \quad \text{and} \quad A_0^{2*}(s) = \frac{N_2(s)}{D_1(s)} \tag{23-24}$$

Where,

$$N_1(s) = \Big[1 - q_{34}^* q_{43}^* - q_{24}^* q_{45}^* q_{52}^* - q_{21}^* \big\{ q_{12}^* \big(1 - q_{43}^* q_{34}^*\big) + q_{14}^{(3)*} q_{45}^* q_{52}^* \big\} - q_{45}^* q_{51}^{(0)*} \big(q_{12}^* q_{24}^* + q_{14}^{(3)*} \big) \Big] Z_0^*$$
$$+ q_{01}^* \big(q_{12}^* q_{24}^* + q_{14}^{(3)*} \big) q_{45}^* W_5^*$$

$$N_2(s) = q_{01}^* \big(1 - q_{43}^* q_{34}^* - q_{24}^* q_{45}^* q_{52}^*\big) W_1^* + q_{01}^* \big(q_{12}^* q_{24}^* + q_{14}^{(3)*} \big) q_{43}^* Z_3^*$$

and

$$D_1(s) = 1 - q_{34}^* q_{43}^* - q_{24}^* q_{45}^* q_{52}^* - q_{21}^* \big\{ q_{12}^* \big(1 - q_{34}^* q_{43}^*\big) + q_{14}^{(3)*} q_{45}^* q_{52}^* \big\} - q_{45}^* q_{51}^{(0)*} \big(q_{12}^* q_{24}^* + q_{14}^{(3)*} \big)$$
$$- q_{01}^* q_{10}^{(3)*} \big(1 - q_{34}^* q_{43}^* - q_{24}^* q_{45}^* q_{52}^*\big) - q_{43}^* q_{30}^* q_{01}^* \big(q_{12}^* q_{24}^* + q_{14}^{(3)*} \big)$$

The steady state availability of the system is given by,

$$A_0^1 = \lim_{s \to 0} s A_0^{j*}(s) = \lim_{s \to 0} s \frac{N_1(s)}{D_1(s)}$$

As $D_1(0) = 0$, so by using L. Hospitals rule, we get

$$A_0^1 = \frac{N_1(0)}{D_1'(0)} \text{ and } A_0^2 = \frac{N_2(0)}{D_1'(0)} \tag{25-26}$$

Where,

$$N_1(0) = [1 - p_{34}p_{43} - p_{24}p_{45}p_{52} - p_{12}p_{21} + p_{12}p_{21}p_{43}p_{34} - p_{14}^{(3)}p_{45}p_{52}p_{21} - p_{12}p_{24}p_{45}p_{51}^{(0)}$$
$$- p_{51}^{(0)} p_{14}^{(3)} p_{45}]\, n_1 + [p_{12}p_{24} + p_{14}^{(3)}]\, p_{45} n_1$$

$$N_2(0) = [1 - p_{43}p_{34} - p_{24}p_{45}p_{52}] \left[\frac{\theta\psi_3 - \beta_1\psi_1}{\theta - \beta_1} \right] + \left[p_{12}p_{24} + p_{14}^{(3)} \right] p_{43}\psi_3$$

and

$$\begin{aligned}D_1^{'}(0) &= [1 - p_{34}p_{43} - p_{24}p_{45}p_{52} - p_{12}p_{21} + p_{12}p_{21}p_{43}p_{34} - p_{14}^{(3)}p_{45}p_{52}p_{21} - p_{12}p_{24}p_{45}p_{51}^{(0)} \\ &\quad - p_{51}^{(0)}p_{14}^{(3)}p_{45}]n_1 + [1 - p_{34}p_{43} - p_{24}p_{45}p_{52}]\left[\frac{\theta\psi_3 - \beta_1\psi_1}{\theta - \beta_1}\right] + [p_{12} - p_{12}p_{43}p_{34} \\ &\quad + p_{14}^{(3)}p_{45}p_{52}]\psi_2 + \left[p_{12}p_{24} + p_{14}^{(3)}\right]p_{43}\psi_3 + \left[p_{14}^{(3)} + p_{12}p_{24}\right]\psi_4 + \left[p_{12}p_{24} + p_{14}^{(3)}\right]p_{45}n_1\end{aligned} \tag{27}$$

The expected up time of the system due to unit-1 and unit-2 respectively in interval (0, t) aregiven by

$$\mu_{up}^{1}(t) = \int_0^t A_0^1(u)\,du \qquad \text{and} \qquad \mu_{up}^{2}(t) = \int_0^t A_0^2(u)\,du$$

so that

$$\mu_{up}^{1*}(s) = \frac{A_0^{1*}(s)}{s} \text{ and } \mu_{up}^{2*}(s) = \frac{A_0^{2*}(s)}{s} \tag{28-29}$$

c) Busy Period Analysis

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the skilled repairman is busy in repair of unit-1 and regular repairmanis busy in repair of a unit-2 at time t, when the system initially starts from state $S_i \in E$. Using the simple probabilistic arguments in regenerative point technique as in case of availability, we develop the recurrence relations for $B_i^1(t)$ and $B_i^2(t)$;i=0,1,2,3,4,5.Then,taking the Laplace Transforms of these recurrence relations and solving the resulting algebraic equations for $B_0^{1*}(s)$ and $B_0^{2*}(s)$ we get;

$$B_0^{1*}(s) = \frac{N_3(s)}{D_1(s)} \text{ and } B_0^{2*}(s) = \frac{N_4(s)}{D_1(s)} \tag{30-31}$$

Where,

$$N_3(s) = \left(q_{01}^{*}q_{12}^{*}q_{24}^{*}q_{43}^{*} + q_{01}^{*}q_{43}^{*}q_{14}^{(3)*}\right)Z_3^{*} + \left(q_{01}^{*}q_{12}^{*}q_{24}^{*} + q_{01}^{*}q_{14}^{(3)*}\right)Z_4^{*} \tag{32}$$

$$\begin{aligned}N_4(s) &= \left(q_{01}^{*}q_{12}^{*} - q_{01}^{*}q_{12}^{*}q_{43}^{*}q_{34}^{*} + q_{01}^{*}q_{52}^{*}q_{14}^{(3)*}q_{45}^{*}\right)Z_2^{*} + \left(q_{01}^{*}q_{12}^{*}q_{24}^{*} + q_{01}^{*}q_{14}^{(3)*}\right)Z_4^{*} \\ &\quad + \left(q_{01}^{*}q_{12}^{*}q_{24}^{*}q_{45}^{*} + q_{01}^{*}q_{14}^{(3)*}q_{45}^{*}\right)W_5^{*}\end{aligned} \tag{33}$$

In the long run, the probabilities that the skilled and regular repairmen will be busy, are respectively given by :

$$B_0^1 = \frac{N_3(0)}{D_1^{'}(0)} \text{ and } B_0^2 = \frac{N_4(0)}{D_1^{'}(0)} \tag{34-35}$$

Where,

$$N_3(0)=\left(p_{12}p_{24}+p_{14}^{(3)}\right)\left(p_{43}\psi_3+\psi_4\right)$$

$N_4(0)=\left\{p_{12}\left(1-p_{43}p_{34}\right)+p_{52}p_{14}^{(3)}p_{45}\right\}\psi_2+\left(p_{12}p_{24}+p_{14}^{(3)}\right)\left(\psi_4+p_{45}n_1\right)$ The value of $D_1^{'}(0)$ is same as given by expression (27).

The expected busy period of skilledrepairman and regular repairman during (0,t) are given by,

$$\mu_b^1(t)=\int_0^t B_0^1(u)du \text{ and } \mu_b^2(t)=\int_0^t B_0^2(u)du$$

So that

$$\mu_b^{1*}(s)=\frac{B_0^{1*}(s)}{s} \text{ and } \mu_b^2(s)=\frac{B_0^{2*}(s)}{s} \tag{36-37}$$

7. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred during time (0, t) by considering the characteristics obtained in earlier sections.

Let us consider,

K_0 = revenue per unit time by the system when it is operative.

K_1 = cost per unit time whenskilled repairman is busy in repair of unit-1.

K_2 = cost per unit timeinrepair of unit-2 by regular repairman.

K_3 = salary paid to regular repairman per-unit time.

Then, the net expected profit incurred during time (0, t),

$$P(t)=\text{Expected total revenue in }(0,t)-\text{Expected amount spent on repair in }(0,t)$$

$$=K_0\mu_{up}(t)-K_1\mu_b^1(t)-K_2\mu_b^2(t)-K_3(t)$$

The expected profit per-unit time in steady state is given by,

$$P=K_0A_0-K_1B_0^1-K_2B_0^2-K_3$$

8. Particular Case

Case 1: When failure times of unit-1 and unit-2 also follow exponential distributions then,

$$f_1(t)=\alpha_1 e^{-\alpha_1 t}, \qquad f_2(t)=\alpha_2 e^{-\alpha_2 t}$$

The Laplace Transform of above density function is as given below;

$$f_1^*(s) = \tilde{F}_1(s) = \frac{\alpha_1}{s+\alpha_1}, \qquad f_2^*(s) = \tilde{F}_2(s) = \frac{\alpha_2}{s+\alpha_2}$$

In view of above changed values of transition probabilities and mean sojourn times given in sec.4 and sec.5 are as follows,

$$p_{12} = \frac{\alpha_2}{\theta+\alpha_2}, \qquad p_{30} = \frac{\beta_1}{\beta_1+\alpha_2}$$

$$p_{34} = \frac{\alpha_2}{\beta_1+\alpha_2}, \qquad p_{52} = \frac{\alpha_1}{\beta_2+\alpha_1}$$

$$p_{10}^{(3)} = \frac{\theta\beta_1}{(\beta_1+\alpha_2)(\theta+\alpha_2)}, \qquad p_{14}^{(3)} = \frac{\theta\alpha_2}{(\beta_1+\alpha_2)(\theta+\alpha_2)}$$

$$p_{51}^{(0)} = \frac{\beta_2}{(\beta_2+\alpha_1)},$$

$$\psi_0 = \frac{1}{\alpha_1} = n_1(\text{say}), \qquad \psi_1 = \frac{1}{\alpha_2+\theta}$$

$$\psi_3 = \frac{1}{\alpha_2+\beta_1} \qquad \psi_5 = \frac{1}{\alpha_1+\beta_2}$$

Case 2: When failure times of unit-1 and unit-2 follows Lindley distributions then,

$$f_1(t) = \frac{\alpha_1^2}{(1+\alpha_1)}(1+t)e^{-\alpha_1 t}, \qquad f_2(t) = \frac{\alpha_2^2}{(1+\alpha_2)}(1+t)e^{-\alpha_2 t}$$

The Laplace Transform of above density function is as given below;

$$f_1^*(s) = \tilde{F}_1(s) = \frac{(s+\alpha_1+1)\alpha_1^2}{(1+\alpha_1)(s+\alpha_1)^2}, \qquad f_2^*(s) = \tilde{F}_2(s) = \frac{(s+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(s+\alpha_2)^2}$$

In view of above changed values of transition probabilities and mean sojourn times given in sec.4 and sec.5 are as follows,

$$p_{12} = \frac{(\theta+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\theta+\alpha_2)^2}, \qquad p_{30} = 1 - \frac{(\beta_1+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\beta_1+\alpha_2)^2}$$

$$p_{34} = \frac{(\beta_1+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\beta_1+\alpha_2)^2}, \qquad p_{52} = \frac{(\beta_2+\alpha_1+1)\alpha_1^2}{(1+\alpha_1)(\beta_2+\alpha_1)^2}$$

$$p_{10}^{(3)} = \frac{\theta\beta_1}{\theta-\beta_1}\left[\frac{1}{\beta_1}\left(1-\frac{(\beta_1+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\beta_1+\alpha_2)^2}\right)-\frac{1}{\theta}\left(1-\frac{(\theta+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\theta+\alpha_2)^2}\right)\right]$$

$$p_{14}^{(3)} = \frac{\theta}{\theta-\beta_1}\left[\frac{(\beta_1+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\beta_1+\alpha_2)^2}-\frac{(\theta+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\theta+\alpha_2)^2}\right]$$

$$p_{51}^{(0)} = 1-\frac{(\beta_2+\alpha_1+1)\alpha_1^2}{(1+\alpha_1)(\beta_2+\alpha_1)^2}$$

$$\psi_0 = \frac{\alpha_1+2}{\alpha_1(\alpha_1+1)}, \qquad \psi_1 = \frac{1}{\theta}\left[1-\frac{(\theta+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\theta+\alpha_2)^2}\right]$$

$$\psi_3 = \frac{1}{\beta_1}\left[1-\frac{(\beta_1+\alpha_2+1)\alpha_2^2}{(1+\alpha_2)(\beta_1+\alpha_2)^2}\right],$$

$$\psi_5 = \frac{1}{\beta_2}\left[1-\frac{(\beta_2+\alpha_1+1)\alpha_1^2}{(1+\alpha_1)(\beta_2+\alpha_1)^2}\right]$$

9. Graphical Study of System Behaviour

The curves for MTSF and profit function have been drawn for the two particular cases 1 and 2 in respect of different parameters. **In Case 1, when failure times of both the units follow exponential distribution**. Fig. 2 depicts the variations in MTSF with respect to the failure rate α_1 of unit-1 for three different values of repair rate of unit-1 (β_1) and two different values of arrival rate of skilled repairman (θ) when the other parameters are kept fixed as $\alpha_2 = 0.01,\ \beta_2 = 0.02.$ From these curves we observed that MTSF decreases uniformly as the failure rate α_1 increases. Further as the value of β_1 increases the expected life time of the system increasesand θ increases the expected life time of the system also increases.

Similarly Fig.3 reveals the variations in profit (P) with respect to α_1 for varying values of β_1 and θ when the other parameter are kept fixed as $\alpha_2 = 0.037,\ \beta_2 = 0.026,.$ Here also the same trend in respect of α_1, β_1 and θ are obtained as in case of MTSF.

In Case 2, when failure times of both the units follow Lindley distribution. Fig. 4 depicts the variations in MTSF with respect to the failure parameter α_1 of unit-1 for three different values of repair rate of unit-1 (β_1) and two different values of arrival rate of skilled repairman (θ) when the other parameters are kept fixed as $\alpha_2 = 0.01$, $\beta_2 = 0.02$. from these curves we observed that MTSF decreases uniformly as the failure parameter α_1 increases.Further as the value of β_1 increases the expected life time of the system increases and θ increases the expected life time of the system also increases.

Similarly Fig.5 reveals the variations in profit (P) with respect to α_1 for varying values of β_1 and θ when the other parameters are kept fixed as $\alpha_2 = 0.037$, $\beta_2 = 0.026$. Here also the same trends in respect of α_1, β_1 and θ are obtained as in case of MTSF.

10. Conclusion

From Fig.2, we conclude from dotted curves that to achieve MTSF at least 300 units the failure rate (α_1) of unit-1 must be less than 0.018, 0.022, 0.027 respectively for $\beta_1 = 0.04$, 0.06 and 0.10 when θ is kept fixed as 0.09. Similarly, we can find the upper bounds for α_1 corresponding to $\beta_1 = 0.04$, 0.06 and 0.10 when $\theta =$ 0.03 to achieve at least a particular value of MTSF.

From Fig.3the dotted curves reveals that the system is profitable only if α_1 is less than 0.09, 0.14, 0.21 respectively for $\beta_1 =$0.019, 0.027, 0.033 for fixed value of θ=0.029. From smooth curves, we conclude that the system is profitable only if α_1 is less than 0.015, 0.02, 0.025 respectively for $\beta_1 =$0.019, 0.027, 0.033 for fixed value of θ=0.011.

From Fig.4, we conclude from dotted curves that to achieve MTSF at least 400 units the failure rate (α_1) of unit-1 must be less than 0.016, 0.021, 0.030 respectively for $\beta_1 = 0.04$, 0.06 and 0.10 when θ is kept fixed as 0.09. Similarly, we can find the upper bounds for α_1 corresponding to $\beta_1 =$0.04, 0.06 and 0.10 when $\theta =$ 0.03 to achieve at least a particular value of MTSF.

From Fig.5 the dotted curves reveals that the system is profitable only if α_1 is less than 0.06, 0.11, 0.15 respectively for $\beta_1 =$0.019, 0.027, 0.033 for fixed value of θ=0.029. From smooth curves, we conclude that the system is profitable only if α_1 is less than 0.012, 0.016, 0.02 respectively for $\beta_1 =$0.019, 0.027, 0.033 for fixed value of θ_1=0.011.

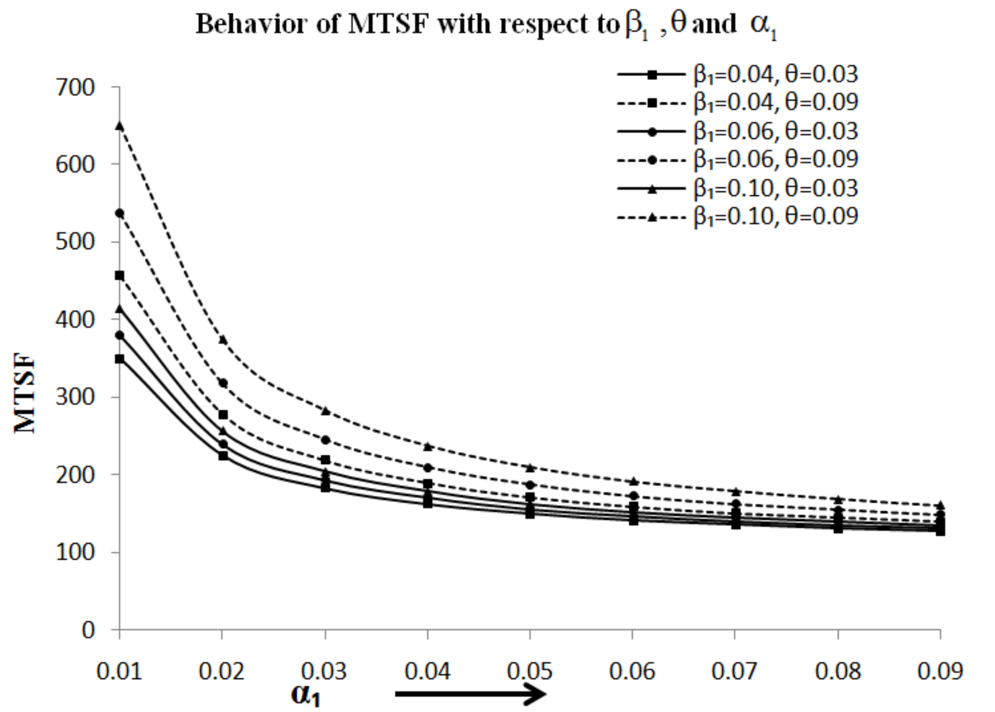
Behavior of MTSF with respect to β1 , θ and α1
β1=0.04, θ=0.03
β1=0.04, θ=0.09
β1=0.06, θ=0.03
β1=0.06, θ=0.09
β1=0.10, θ=0.03
β1=0.10, θ=0.09
MTSF
700
600
500
400
300
200
100
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
α1

Fig. 2

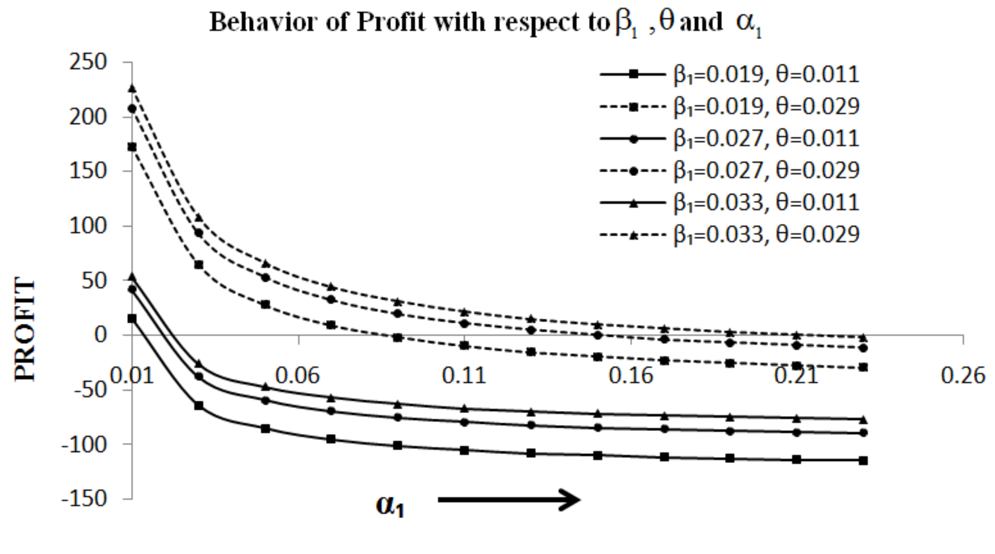
Behavior of Profit with respect to β1 , θ and α1
β1=0.019, θ=0.011
β1=0.019, θ=0.029
β1=0.027, θ=0.011
β1=0.027, θ=0.029
β1=0.033, θ=0.011
β1=0.033, θ=0.029
PROFIT
250
200
150
100
50
0
-50
-100
-150
0.01
0.06
0.11
0.16
0.21
0.26
α1

Fig.3

Behavior of MTSF with respect to β_1 , θ and α_1

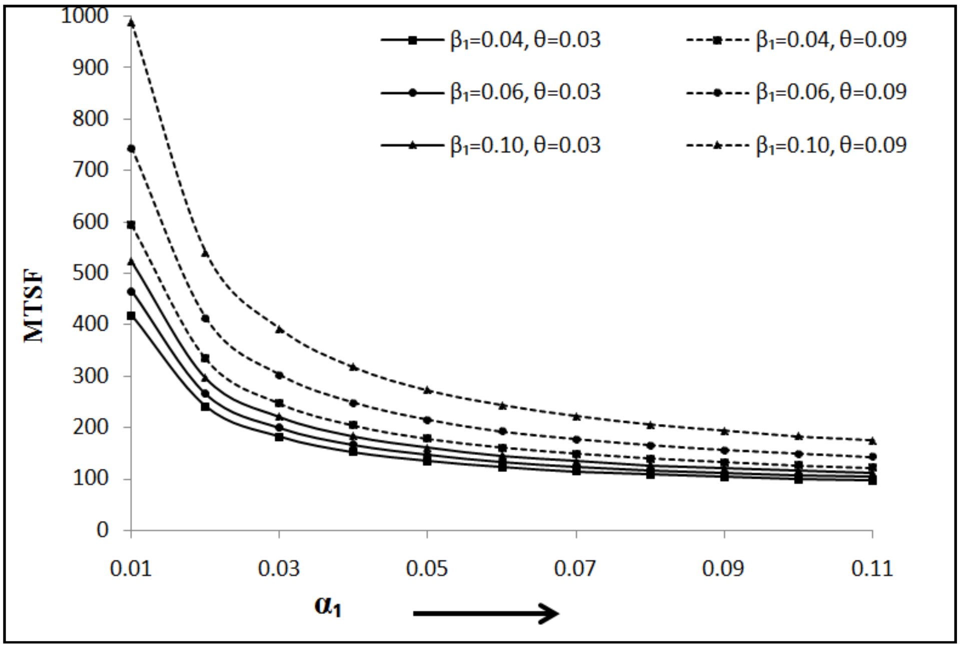

Fig. 4

Behavior of Profit with respect to β_1 , θ and α_1

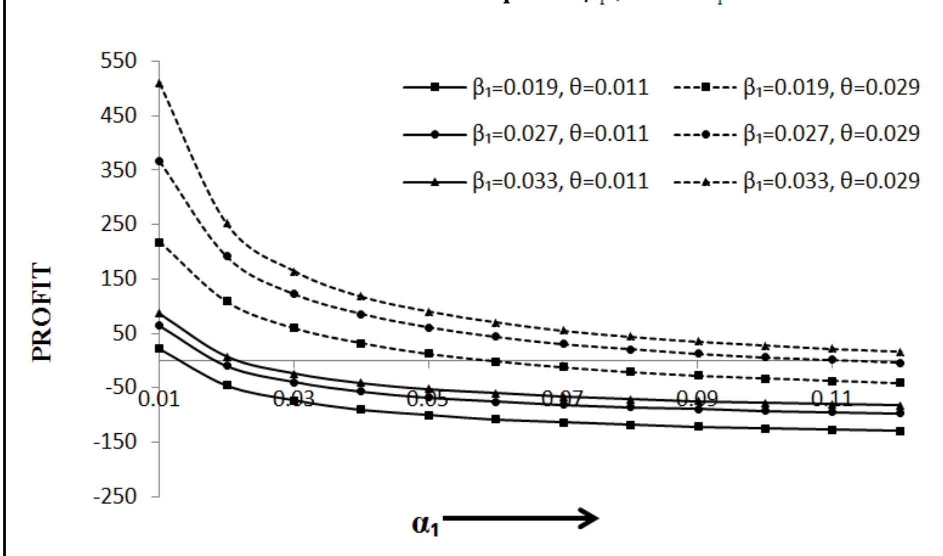

Fig.5

Acknowledgement: The authors are thankful to the Refree for valuable suggestions.

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