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STEADY LAMINAR BOUNDARY LAYER FLOW AND HEAT TRANSFER ALONG AN INFINITE, POROUS, HOT, VERTICAL MOVING PLATE IN PRESENCE OF HEAT SOURCE AND CONSTANT FREE STREAM

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Abstract: An analysis is made to study two- dimensional laminar boundary layer flow of a viscous, incompressible fluid, along an infinite, porous, hot, vertical continuous moving plate. The governing partial differential equations are non-dimensionalized and are solved using Natural Transform Technique. The expressions for velocity field, temperature field, rate of heat transfer and skin-friction have been obtained. The influence of various physical parameters, such as Eckert number Ec, Prandtl number Pr, Grashoff number Gr, plate velocity α and heat source/sink parameters is discussed with the help of graphs to show the physical aspects of the problem. It is found that these parameters significantly affect the flow and heat transfer.

Key Words: Laminar flow, boundary layer, moving porous plate, heat transfer, natural transformation.

Mathematics Classification: 80A20, 76S05, 44A10

1. Introduction

A study of boundary layer (Sparrow [19], Schlichting [17], Bansal [3], Pai [13]) behavior on continuous solid surface has attracted the attention of researchers because such flows may find applications in different areas such as aerodynamic extrusion of plastic sheets, the boundary layer along material handling conveyers, the cooling of an infinite metallic plate in a cool bath etc. The classical problem was introduced by Blasius[5] by studying the boundary layer flow on a fixed flat plate. The flow field due to moving flat surface was developed by Sakiadis [16] where he took the constant velocity of plate. Crane [9] extended the work of Sakiadis[16] for two dimensional problem under some specific conditions. Yao et al. [21] studied the heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions. Cortell [8] investigated the flow and heat transfer of a viscoelastic fluid over a stretching sheet.

Prasad et.al.[15] studied the momentum and heat transfer in viscoelastic fluid flow in porous medium over a non–isothermal stretching sheet. Later Cortell [7] found the similarity solutions for flow and heat transfer of viscoelastic fluid over a porous stretching sheet. Vajravelu [20] obtained the solution of boundary layer flow and heat transfer over a continuous porous surface moving in an oscillating free stream.

The generalization of Laplace-Transform i.e. Natural-Transform, initially was defined by Khan and Khan [10] as N - transform, who studied their properties and applications. Later, Belgacem and Silambarasan [4], Silambarasan and Belgacem [18] defined its inverse and studied some additional fundamental properties of this integral transform and named it the Natural Transformation. Applications of Natural transform in the solution of partial differential equations have been studied by Al-Omari [1], Agarwal et al. [2], Podlubny [14], and Bulu et al. [6]. Loonker and Banerji [11,12] applied the Natural Transform for the distribution and Boehmians spaces.

The aim of the present investigation is to study the steady laminar boundary layer flow and heat transfer through an incompressible viscous fluid along an infinite, porous, hot vertical continuous moving plate in the presence of volumetric rate of heat generation (or absorption) and constant free stream by means of Natural Transformation.

2. Formulation of the Problem

Consider the steady boundary layer flow and heat transfer of a viscous incompressible fluid along an infinite hot vertical continuous moving plate in the presence of constant suction at the surface, constant free stream U_∞ and heat generation (or absorption). The plate is moving in upwards direction i.e.in flow direction with constant velocity and maintained at a constant temperature T_w. The flow is in positive direction of X*-axis in upwards direction and Y*-axis is taken normal to the plate.

The governing boundary layer equations of continuity, motion and energy for the present flow are

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \text{(constant)},\ v_0 > 0 \tag{1}$$

$$\rho\left(-v_0 \frac{\partial u^*}{\partial y^*}\right) = \mu \frac{\partial^2 u^*}{\partial y^{*2}} + \rho g \beta \left(T^* - T_\infty\right) \tag{2}$$

$$\rho C_p \left(-v_0 \frac{\partial T^*}{\partial y^*}\right) = \kappa \frac{\partial^2 T^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*}\right)^2 + Q\left(T^* - T_\infty\right) \tag{3}$$

where u*, v* are the velocity components along X*- axis and Y*- axis, respectively, ρ the density, ν_0 the cross-flow velocity, μ the coefficient of viscosity, C_p the specific heat at constant pressure, κ the thermal conductivity, Q the volumetric rate of heat generation parameter, g the acceleration due to gravity, T_∞ the free stream temperature and β the volume expansion.

The corresponding boundary conditions are

$$\left.\begin{array}{l} y^*=0 \ : u^* = U_w,\ v^*= -v_0,\ T^* = T_w \\ y^*\to\infty : u^*\to U_\infty\ ,\ T^*\to T_\infty \end{array}\right\} \tag{4}$$

3. Method of Solution

Introducing the following non-dimensional quantities

$$\left.\begin{array}{l} y = y^*\dfrac{v_0}{\upsilon}, u = \dfrac{u^*}{U_\infty}, \theta = \dfrac{T^*-T_\infty}{T_w - T_\infty}, \alpha = \dfrac{U_w}{U_\infty} \\ \Pr = \dfrac{\mu C_p}{\kappa}, S = \dfrac{Qv^2}{\kappa v_0^2}, Ec = \dfrac{U_\infty^2}{C_p\left(T_w - T_\infty\right)}, Gr = \dfrac{g\beta\upsilon}{U_\infty v_0^2}\left(T_w - T_\infty\right) \end{array}\right\} \tag{5}$$

to get

$$u''+u' = -Gr\theta \tag{6}$$

$$\theta''+\Pr\theta'+S\theta = -Ec\Pr(u')^2 \tag{7}$$

Where Pr is the Prandtl number, Ec the Eckert number, S the heat source parameter and dashes denote the differentiation w.r.t. y.

The boundary conditions in non- dimensional form are

$$\left.\begin{array}{l} y = 0 : u = \alpha, \theta = 1 \\ y \to \infty : u \to 1, \theta \to 0 \end{array}\right\} \tag{8}$$

Where 'α' is the velocity ratio parameter.

For incompressible fluid flow, the Eckert number is very small therefore u(y) and θ(y) can be expanded in the powers of Ec as given below

$$u(y) = u_0 + Ecu_1 + O(Ec^2) \tag{9}$$

$$\theta(y) = \theta_0 + Ec\theta_1 + O(Ec^2) \tag{10}$$

Zeroth-order equations are

$$u''_0+u'_0 = -Gr\theta_0 \tag{11}$$

$$\theta''_0+\Pr\theta'_0+S\theta_0=0 \tag{12}$$

First-order equations are

$$u''_1+u'_1=-Gr\theta_1 \tag{13}$$

$$\theta''_1+\Pr\theta'_1+S\theta_1=-\Pr\left(u'_0\right)^2 \tag{14}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} y=0 \quad : u_0=\alpha, u_1=0, \theta_0=1, \theta_1=0 \\ y\to\infty : u_0\to 1, u_1\to 0, \theta_0\to 0, \theta_1\to 0 \end{array}\right\} \tag{15}$$

Now we use Natural Transform to solve zeroth- order equation, to get

$$\theta_0(y)=\frac{1}{2}\left\{e^{\frac{-\Pr+\sqrt{\Pr^2-4S}}{2}y}+e^{\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}y}\right\}$$

$$+\frac{1}{\sqrt{\Pr^2-4S}}\left\{\theta'_0(0)+\frac{1}{2}\Pr\right\}\left\{e^{\frac{-\Pr+\sqrt{\Pr^2-4S}}{2}y}-e^{\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}y}\right\}$$

For $m_2=\dfrac{-\Pr-\sqrt{\Pr^2-4S}}{2}$ (16)

$$\theta_0(y)=e^{\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}y}=e^{m_2y},\ u_0(y)=1+A_1e^{-y}-B_1e^{m_2y}, \tag{17}$$

Where $A_1=\alpha-1+\dfrac{Gr}{m_2(1+m_2)}$ and $B_1=\dfrac{Gr}{m_2(1+m_2)}$ (18)

First order equation for temperature become

$$\theta''_1+\Pr\theta'_1+S\theta_1+\Pr\left(-A_1e^{-y}-B_1m_2e^{m_2y}\right)^2=0$$

Using Natural Transform, we get

$$\theta_1(y)=\frac{2\theta'_1(0)e^{-\frac{\Pr}{2}y}}{\sqrt{\Pr^2-4S}}\left\{\frac{e^{\frac{y}{2}\sqrt{\Pr^2-4S}}-e^{-\frac{y}{2}\sqrt{\Pr^2-4S}}}{2}\right\}$$

$$-\left[\frac{A_1^2\Pr}{(S-2\Pr+4)}e^{-2y}+\frac{2A_1^2\Pr}{\left(\Pr+\sqrt{\Pr^2-4S}-4\right)\sqrt{\Pr^2-4S}}e^{m_2y}+\frac{2A_1^2\Pr}{\left(-\Pr+\sqrt{\Pr^2-4S}+4\right)\sqrt{\Pr^2-4S}}e^{-(\Pr+m_2)y}\right]$$

$$-\left[-\frac{2B_1^2\Pr m_2}{\left(\Pr+3\sqrt{\Pr^2-4S}\right)}e^{2m_2y}+\frac{B_1^2\Pr m_2}{\sqrt{\Pr^2-4S}}e^{m_2y}+\frac{2B_1^2\Pr m_2^2}{\left(\Pr+3\sqrt{\Pr^2-4S}\right)\sqrt{\Pr^2-4S}}e^{-(\Pr+m_2)y}\right]$$

$$-\left[\frac{2A_1B_1\Pr m_2}{\left(1+\sqrt{\Pr^2-4S}\right)}e^{(m_2-1)y}-\frac{2A_1B_1\Pr m_2}{\sqrt{\Pr^2-4S}}e^{m_2y}+\frac{2A_1B_1\Pr m_2}{\left(1+\sqrt{\Pr^2-4S}\right)\sqrt{\Pr^2-4S}}e^{-(\Pr+m_2)y}\right]$$

After applying boundary conditions, we get

$$\theta_1(y)=A_2e^{-2y}+B_2e^{m_2y}+C_2e^{(m_2-1)y}+G_2e^{2m_2y} \tag{19}$$

Where

$$A_2=-\frac{A_1^2\Pr}{(S-2\Pr+4)},\quad C_2=\frac{-2A_1B_1\Pr m_2}{1-(\Pr+2m_2)},\quad G_2=\frac{-B_1^2\Pr m_2}{\Pr+3m_2}\ \text{and}$$

$$B_2=-(A_2+G_2+C_2) \tag{20}$$

Velocity equation becomes

$$u''_1+u'_1=-Gr\left[A_2e^{-2y}+B_2e^{m_2y}+C_2e^{(m_2-1)y}+G_2e^{2m_2y}\right]$$

Solving it using Natural Transform, we get

$$u_1(y) = u'_1(0)\left[1-e^{-y}\right] - \frac{Gr}{2}\left[\begin{array}{c} A_2\left(1-2e^{-y}+e^{-2y}\right) \\ +2B_2\left(-\frac{1}{m_2}+\frac{1}{(m_2+1)}e^{-y}+\frac{1}{m_2(m_2+1)}e^{m_2 y}\right) \\ +2C_2\left(-\frac{1}{(m_2-1)}+\frac{1}{m_2}e^{-y}+\frac{1}{m_2(m_2-1)}e^{(m_2-1)y}\right) \\ +G_2\left(-\frac{1}{m_2}+\frac{2}{(2m_2+1)}e^{-y}+\frac{1}{m_2(2m_2+1)}e^{2m_2 y}\right) \end{array}\right]$$

After boundary conditions, we get

$$u_1(y) = A_3 e^{-2y} + B_3 e^{m_2 y} + C_3 e^{(m_2-1)y} + G_3 e^{2m_2 y} + H_3 e^{-y} \tag{21}$$

Where

$$A_3 = -\left(\frac{GrA_2}{2}\right),\quad B_3 = -\left(\frac{GrB_2}{m_2(m_2+1)}\right),\quad C_3 = -\left(\frac{GrC_2}{m_2(m_2-1)}\right),$$

$$G_3 = -\left(\frac{GrG_2}{2m_2(2m_2+1)}\right) \quad \text{and} \quad H_3 = -(A_3+B_3+C_3+G_3) \tag{22}$$

Hence the velocity and temperature fields become

$$u(y) = 1 + A_1 e^{-y} - B_1 e^{m_2 y} + Ec\left\{A_3 e^{-2y} + B_3 e^{m_2 y} + C_3 e^{(m_2-1)y} + G_3 e^{2m_2 y} + H_3 e^{-y}\right\} \tag{23}$$

$$\theta(y) = e^{m_2 y} + Ec\left\{A_2 e^{-2y} + B_2 e^{m_2 y} + C_2 e^{(m_2-1)y} + G_2 e^{2m_2 y}\right\} \tag{24}$$

4. Skin-Friction Coefficient

The skin-friction coefficient C_f at the plate is given by

$$C_f = \frac{\tau_w}{\rho U_\infty v_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$C_f = -A_1 - m_2 B_1 + Ec\left\{-2A_3 + m_2 B_3 + (m_2-1)C_3 + 2m_2 G_3 - H_3\right\} \tag{25}$$

5. Nusselt Number

The rate of heat transfer in terms of Nusselt number at the plate is given by

$$N_u = \frac{v}{v_0}\frac{q}{\kappa(T_w - T_\infty)} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$N_u = -m_2 - Ec\left\{-2A_2 + m_2 B_2 + (m_2 - 1)C_2 + 2m_2 G_2\right\} \tag{26}$$

6. Results and Discussion

Figure 1.01-Velocity Profile for Different Values of Grashoff Number

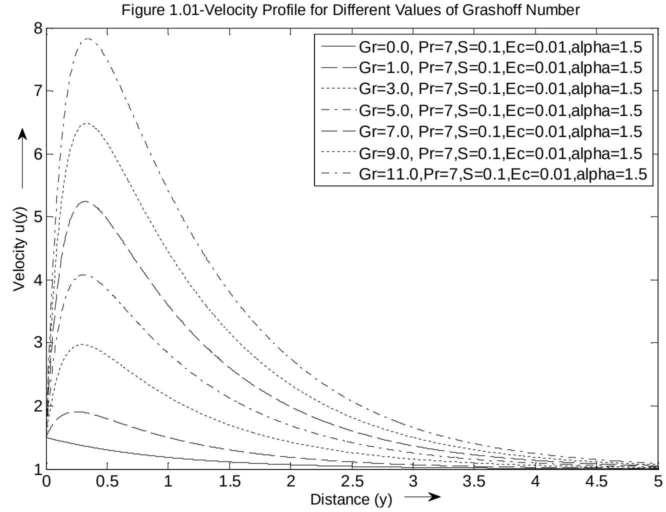
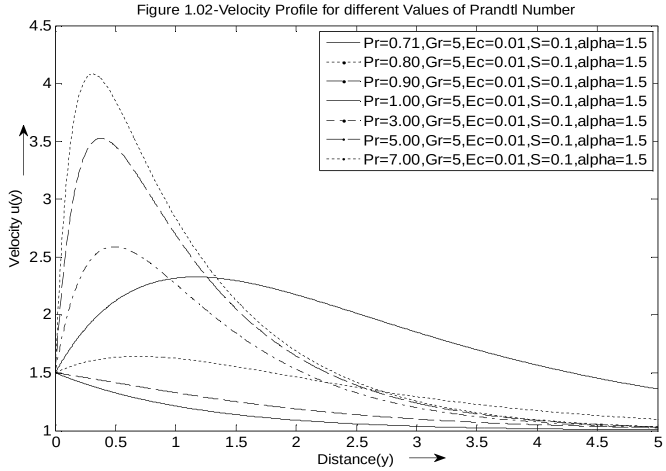


Figure 1.02-Velocity Profile for different Values of Prandtl Number



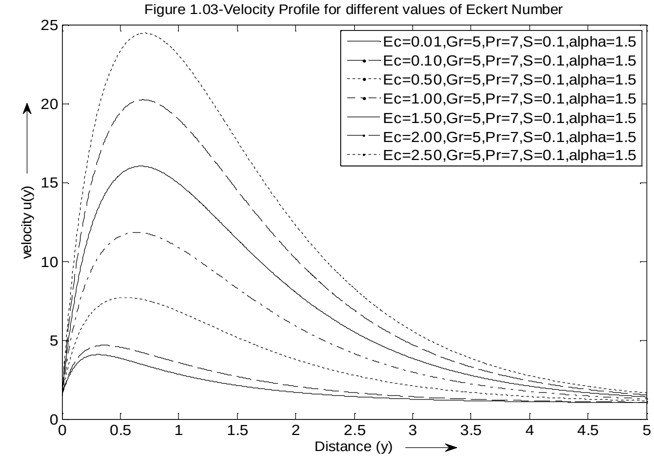


Figure 1.03-Velocity Profile for different values of Eckert Number

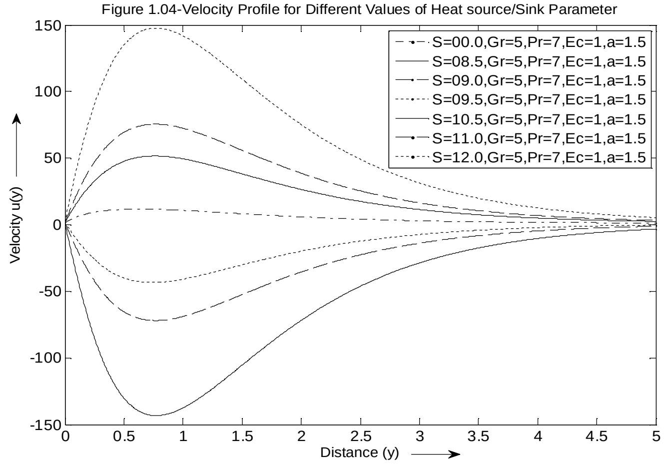


Figure 1.04-Velocity Profile for Different Values of Heat source/Sink Parameter

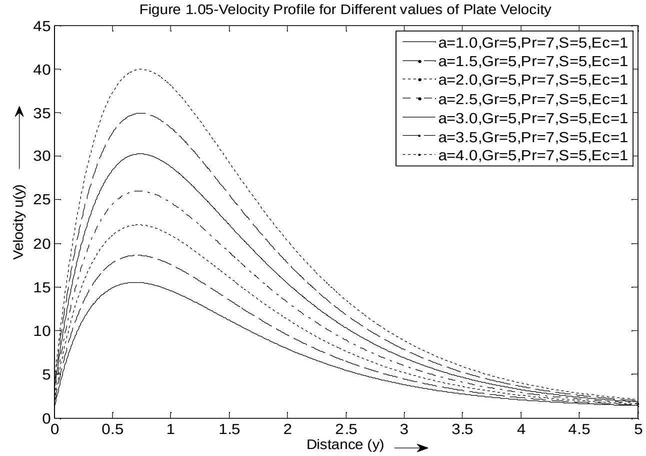


Figure 1.05-Velocity Profile for Different values of Plate Velocity

Figure 1.06-Temparature Distribution Profile for Varying Grashoff Number

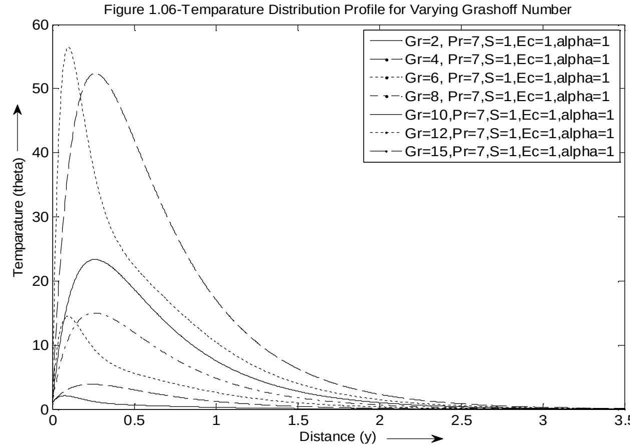


Figure 1.07-Temperature Distribution for Different Values of Prandtl Number

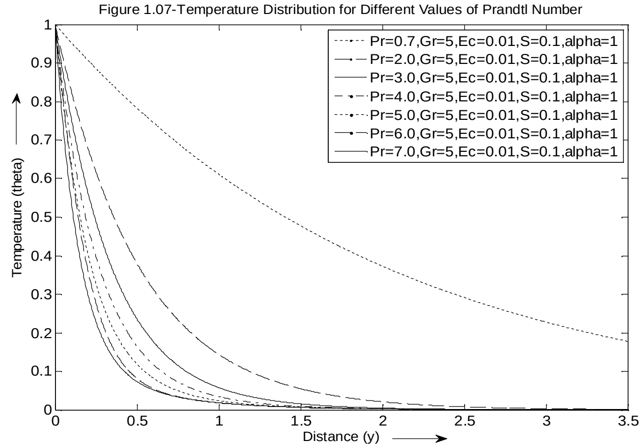


Figure 1.08-Temperature Distribution for Different Values of Eckert Number

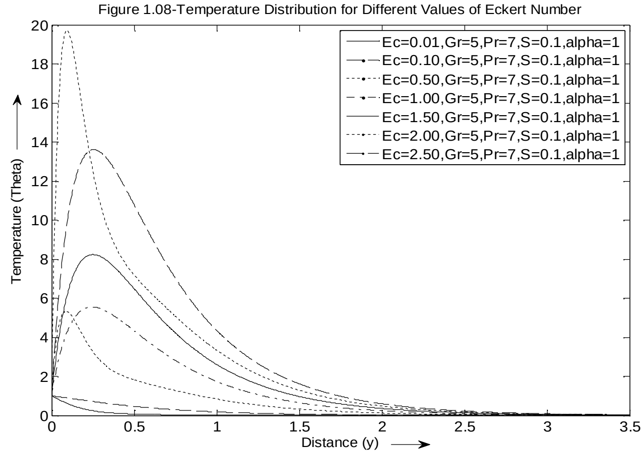


Figure 1.09-Temperature Distribution for Different Values of Heat Source/Sink Parameter

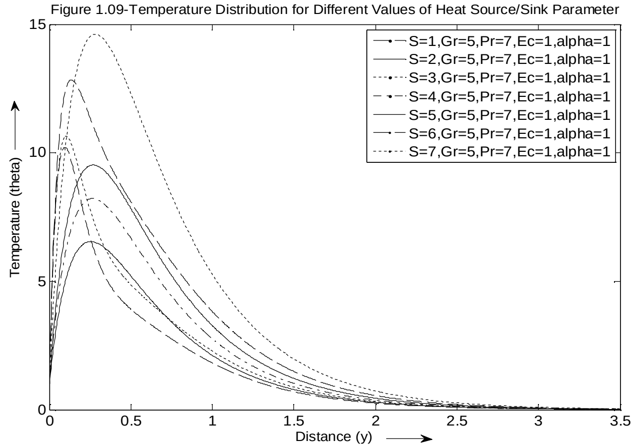
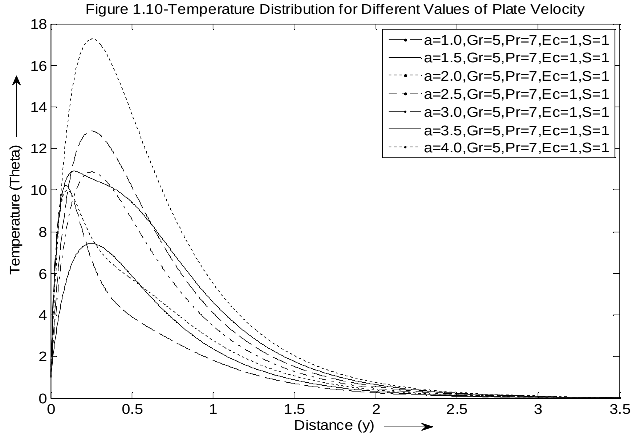


Figure 1.10-Temperature Distribution for Different Values of Plate Velocity



From the figure 1.01 it is clear that for zero Grashoff Number (i.e. when the fluid temperature at wall is same as the free stream temperature or $T_w = T_\infty$) the fluid velocity decreases continuously with distance (y), but for positive Grashoff Numbers the velocity attains its maximum value at y=0.5 (approx.) and then it decreases exponentially.

From the figure 1.02 we observe that the mean stream velocity is minimum for Prandtl Number one (Pr=1).For Pr >1 higher the Prandtl Number greater will the velocity. The phenomena get reversed for Pr <1.

It is evident from figure 1.03 that the mean fluid velocity increases with increasing the Eckert Number. i.e. under the same circumstances the fluid with large Eckert Number flows fast as compare to the fluid with smaller Eckert Number.

For the given set of values, figure 1.04 shows that the mean velocity increases with increasing the value of heat source parameter for S <10, and the direction of flow becomes opposite for S >10.

It is evident from the figure 1.05 that the mean fluid velocity increases with increasing the plate velocity. Also for y=0.7 (approx.) the fluid gets its maximum velocity. The graph between distance and velocity is skewed in right and changes its nature from platykurtic to leptokurtic as we increase the plate velocity.

The figure 1.06 shows that as we increase the Grashoff Number of the fluid, the mean temperature distribution increases in the thin boundary layer near the plate where the viscous forces are confined.

From the figure 1.07 we observe that the temperature distribution decreases as we increase the value of Prandtl number. For the given set of data it is maximum for Pr=0.7 i.e. for air.

It is evident from the figure 1.08 that the temperature distribution increases with increasing the value of Eckert Number. For very small Eckert Number the temperature decreases continuously with distance from plate.

The figure 1.09 shows that as we increase the value of heat source parameter, the mean temperature distribution increases and sharp changes take place for large value of heat source parameter.

It is evident from the figure 1.10 that the mean temperature distribution increases with increasing the plate velocity. From this phenomenon we conclude that more the plate velocity greater will the heat transfer.

7. Conclusion

It is evident from these figures that the mean fluid velocity increases with increasing the Eckert Number, Grashoff number, Prandtl number (Pr>1), for small values of heat source or sink parameter and with the velocity of the moving plate. The phenomena gets reverse for small Prandtl number (Pr<1) and for large values of heat source or sink parameter.

On the other hand we also observe that the temperature distribution decreases as we increase the value of Prandtl number or Eckert number but increases with increasing the values of Grashoff number, heat source or sink parameter and the plate velocity.

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References

[1] Al-Omari, S K Q (2013). "On the applications of Natural transform, International Journal of Pure and Applied Mathematics", 85 (4), 729-744.

[2] Agarwal, S K., Jhankal, A K and Mathur R, (2016). "Boundary Layer Flow and Heat Transfer along an Infinite Porous Hot Horizontal Continuous Moving Plate by Means of the Natural Transformation Method", Int. J. Eng. Res. and App., 6, 83-87.

[3] Bansal, J L (1977). "Viscous Fluid Dynamics", Oxford & IBH Pub. Co., New Delhi, India.

[4] Belgacem, F B M and Silambarasan, R (2012). “Theory of Natural Transform”, Math. Engg, Sci. Aerospace (MESA), 3 (1), 99-124.

[5] Blasius, H (1908). “Grenzschichten in Flussigkeiten mit kleiner Reibung Zeitschrift for Mathematik Physik”, 56, 1-37.

[6] Bulu, H, Baskonus H M and Belgacem F B M (2013). “The analytical solution of some fractional ordinary differential equations by the Sumudu transform method”, Abstract and Applied Analysis 2013, 1-6.

[7] Cortell, R (1994). “Similarity Solutions For Flow And Heat Transfer Of A Viscoelastic Fluid Over A Stretching Sheet”, Int. J. Non-Linear Mech.,29, 155-161.

[8] Cortell, R (2006). “A Note on Flow and Heat Transfer of A Viscoelastic Fluid Over A Stretching Sheet”, Int. J. Non-Linear Mech.,41, 78-85.

[9] Crane, L (1970). “Flow past a stretching plate“”, Z. Angew Math Phys (ZAMP), Vol.21, 645-647.

[10] Khan, Z.H. and Khan, W.A (2008). “N-Transform-Properties and Applications”, NUST J. Engg. Sci. 1 (1), 127-133.

[11] Loonker, D. and Banerji P.K. (2013). “Natural transform for distribution and Boehmian spaces”, Math. Engg. Sci. Aerospace 4 (1), 69-76.

[12] Loonker, D. and Banerji P.K. (2013). “Applications Of Natural Transform To Differential Equations”, J. Indian Acad. Math. 35 (1), 151-158.

[13] Pai, S. I. (1956). “Viscous Flow Theory: I – Laminar Flow”, D.VanNostrand Co., New York, USA.

[14] Podlubny I (1999). “Fractional Differential Equations”, Vol. 198, Mathematics in Science and Engineering, Academic Press, San Diego, USA.

[15] Prasad, K V., Abel, M S and Khan, S K., (2000). “Momentum And Heat Transfer In Viscoelastic Fluid Flow In Porous Medium Over A Non-Dimensional Stretching Sheet”, Int. J. Num. Meth.Heat Fluid Flow, 10, 786-801.

[16] Sakiadis, B.C. (1961). “Boundary-Layer Behavior On Continues Solid Surfaces: II. The Boundary-Layer On Continuous Flat Surface”, AIChE J.vol. 7, 221-225.

[17] Schlichting, H (1964). “Boundary Layer Theory”, sixth ed., McGraw-Hill, New York.

[18] Silambarasan R and Belgacem F B M (2011). “Applications of the Natural transform to Maxwell's equations”, Prog. Electromagnetic Research Symposium Proc. Suzhou, China, 899- 902.

[19] Sparrow,E M and Cess R D, (1961). “Temperature-dependent heat sources or sinks in a stagnation point flow”. Appl. Sci. Res. A10, 185.

[20] Vajravelu, K, (1987). “Boundary layer flow and heat transfer over a continuous, porous, surface moving in an oscillating free stream-I”. ZAMM. Z.angew. Math. Mech.67(7), 342-344.

[21] Yao, S., Fang,T. and Zhong, Y (2011). Heat transfer of a generalized stretching/shriking wall problem with convective boundary conditions. Commun. Nonlinear Sci. Number. Simulat. 16, 752-760.