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BIANCHI TYPE IX COSMOLOGICAL MODELS FOR BAROTROPIC FLUID DISTRIBUTION WITH VACUUM ENERGY DENSITY

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Abstract. Bianchi Type IX cosmological models for barotropic fluid distribution with vacuum energy density are investigated. To get the deterministic model of the universe, we assume that $a = b^2$ where a and b are metric potentials. The cosmological constant (vacuum energy density Λ) is assumed to be proportional to R^{-3} where R is scale factor. The physical and geometrical aspects of the models with singularities are also discussed. The vacuum energy density (Λ) decreases with time in both the models.
Key words and Phrases: Bianchi IX, Cosmological, barotropic, vacuum energy density.

1. Introduction

Bianchi Type IX cosmological models are interesting in the study because familiar models like FRW model with positive curvature, the de-Sitter model for positive curvature and Taub-NUT solutions are of Bianchi Type IX space-time. Vaidya and Patel [14] investigated a general scheme for the derivation of exact solutions of Einstein’s field equations corresponding to perfect fluid plus pure radiation fields. Several authors viz. Chakraborty [6], Bali et al. [1,2], King [9] have studied Bianchi Type IX models in different contexts.

The current accelerating expansion of the universe suggests that our universe is dominated by unknown dark energy. The cosmological constant is the most favoured candidate of dark energy representing energy density of vacuum in the context of quantum field theory. The observations for distant type Ia Supernovae (Perlmutter et al. [10], Riess et al. [11,12] strongly favour a positive value of cosmological constant (Λ) in order to measure the expansion rate of universe and now it is believed that the universe is not only expanding but also accelerating. Several authors viz. Sahni and Starobinsky [13], Verma and Ram [15], Bali and Singh [3,4,5] investigated cosmological models with decaying vacuum energy density (Λ).

Seeing the importance of vacuum energy density (Λ) and Bianchi Type IX space-time of physical interest, it is interesting to investigate cosmological models using Bianchi Type

IX space-time with vacuum energy density (Λ). Some physical and geometrical aspects of the models with singularity in the models are also discussed.

2. Metric and Field Equations

We consider Bianchi Type IX metric in the form

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + (b^2\sin^2 y + a^2\cos^2 y)dz^2 - 2a^2\cos y\, dx\, dz \quad (1)$$

where a and b are functions of t-alone.

The energy momentum tensor for perfect fluid is given by

$$T_i^j = (\rho + p)v_i v^j + p\, g_i^j \quad (2)$$

where p is the isotropic pressure, ρ the matter density and v^i the flow vector satisfying

$$g_{ij}v^i v^j = -1 \quad (3)$$

We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1$$

The Einstein's field are equation given by

$$R_i^j - \frac{1}{2}Rg_i^j - \Lambda g_i^j = -8\pi T_i^j (c = 1, G = 1 \text{ in gravitational unit}). \quad (4)$$

For the line element (1), the field equation (4) leads to

$$\frac{2\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{3a^2}{4b^4} = -8\pi p + \Lambda \quad (5)$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} + \frac{a^2}{4b^4} = -8\pi p + \Lambda \quad (6)$$

$$\frac{2\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} - \frac{a^2}{4b^4} + \frac{1}{b^2} = 8\pi\gamma\rho + \Lambda \quad (7)$$

where over head dot indicates partial derivative with respect to t.

3. Solution of the Field Equations

To get a deterministic solution, we assume $a = b^2, p = \rho\gamma \text{ and } \Lambda \sim 1/R^3$, where R is the scale factor. Equations (5) and (7) lead to

$$2\ddot{b}+\frac{(1+5\gamma)}{b}\dot{b}^2=(1+6\gamma)\frac{1}{b^3}-(1+\gamma)\frac{1}{b}+(3+\gamma)\frac{b}{4} \tag{8}$$

The equation (8) leads to

$$\frac{d}{db}(f^2)+\frac{1+5\gamma}{b}(f^2)=(1+6\gamma)\frac{1}{b^3}-(1+\gamma)\frac{1}{b}+(3+\gamma)\frac{b}{4} \tag{9}$$

where

$$\dot{b}=f(b) \tag{10}$$

Equation (9) leads to

$$\left(\frac{db}{dt}\right)^2=\frac{1+6\gamma}{5\gamma-1}b^{-2}-\frac{1+\gamma}{5\gamma+1}+\frac{3+\gamma}{4(5\gamma+3)}b^2+\delta b^{-(5\gamma+1)} \tag{11}$$

where δ is the constant of integration.

Hence, the metric (1) reduces to the form

$$ds^2=-\left(\frac{dt}{db}\right)^2db^2+b^4dx^2+b^2dy^2+(b^2\sin^2y+b^4\cos^2y)dz^2-2b^4\cos y\,dx\,dz, \tag{12}$$

where $a = b^2$. After using (11) in (12), we get

$$ds^2=-\left(\frac{1}{\frac{1+6\gamma}{5\gamma-1}T^{-2}-\frac{1+\gamma}{5\gamma+1}+\frac{3+\gamma}{4(5\gamma+3)}T^2+\frac{\delta}{T^{5\gamma+1}}}\right)dT^2+T^4dX^2+T^2dY^2$$

$$+T^2\sin^2Y+T^4\cos^2Y)dZ^2-2T^4\cos Y\,dX\,dZ \tag{13}$$

where $b = T$, $x = X$, $y = Y$, $z = Z$.

4. Special Model

If $\delta = 0$, then the equation (11) leads to

$$\left(\frac{db}{dt}\right)^2=\frac{1+6\gamma}{5\gamma-1}\frac{1}{b^2}-\frac{1+\gamma}{5\gamma+1}+\frac{(3+\gamma)b^2}{4(5\gamma+3)} \tag{14}$$

which leads to

$$\frac{bdb}{\sqrt{Bb^4 - Db^2 + A}} = dt \tag{15}$$

where

$$A = \frac{1+\gamma}{5\gamma - 1}, B = \frac{3+\gamma}{4(5\gamma+3)}$$

$$D = \frac{1+\gamma}{5\gamma + 1} \tag{16}$$

Thus, we have

$$\frac{2bdb}{\sqrt{(b^2)^2 - \frac{D}{B}b^2 + \frac{A}{B}}} = 2\sqrt{B}\, dt \tag{17}$$

which leads to

$$\frac{2bdb}{\sqrt{\left(b^2 - \frac{D}{2B}\right)^2 + \left(\frac{A}{B} - \frac{D^2}{4B^2}\right)}} = 2\sqrt{B}dt \tag{18}$$

Thus, we have

$$b^2 = \frac{D}{2B} + \alpha \sinh(2\sqrt{B}\tau) \tag{19}$$

where

$$\frac{A}{B} - \frac{D^2}{4B^2} = \alpha^2, 4AB - D^2 > 0$$

and $t + t_0 = \tau$

$$a = b^2 = \frac{D}{2B} + \alpha \sinh(2\sqrt{B}\tau) \tag{20}$$

The metric (1) leads to the form

$$ds^2 = -d\tau^2 + \left[\frac{D}{2B} + \alpha \sinh(2\sqrt{B}\tau)\right]^2 dx^2 + \left[\frac{D}{2B} + \alpha \sinh(2\sqrt{B}\tau)\right] dy^2$$

$$+ \left[\frac{D}{2B} + \alpha \sinh(2\sqrt{B}\tau)\right] \sin^2 y + \left[\frac{D}{2B} + \alpha \sinh(2\sqrt{B}\tau)\right]^2 dz^2$$

$$- 2\left[\frac{D}{2B} + \alpha \sinh(2\sqrt{B}\tau)\right]^2 dx\, dz \qquad (21)$$

5. Physical and geometrical aspects

The matter density (ρ) and isotropic pressure (p) for the model (13) are given by

$$8\pi\rho = \left(\frac{25\gamma + 6}{5\gamma - 1}\right)\frac{1}{T^4} - \frac{4}{5\gamma + 1}\frac{1}{T^2} + \frac{5(3+\gamma)}{4(5\gamma + 3)} + \frac{5\delta}{T^{5\gamma+3}} - \frac{1}{4} \qquad (22)$$

and

$$8\pi p = 8\pi\gamma\rho,\ 0 \le \gamma \le 1 \qquad (23)$$

The expansion (θ) and shear (σ) are given by

$$\theta = \frac{\dot{a}}{a} + \frac{2\dot{b}}{b}$$

$$= \frac{4\dot{b}}{b}$$

$$= 4\sqrt{\frac{6\gamma + 1}{5\gamma - 1}\frac{1}{T^4} - \frac{(1+\gamma)}{5\gamma + 1}\frac{1}{T^2} + \frac{(3+\gamma)}{4(5\gamma + 3)} + \delta\frac{1}{T^{5\gamma+3}}} \qquad (24)$$

and

$$\sigma = \frac{1}{\sqrt{3}}\left|\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right|$$

$$= \frac{1}{\sqrt{3}}\sqrt{\frac{6\gamma + 1}{5\gamma - 1}\frac{1}{T^4} - \frac{(1+\gamma)}{(5\gamma + 1)}\frac{1}{T^2} + \frac{(3+\gamma)}{4(5\gamma + 3)} + \frac{\delta}{T^{5\gamma+3}}} \qquad (25)$$

Thus

$$\frac{\sigma}{\theta}=\frac{1}{4\sqrt{3}}\neq 0 \tag{26}$$

The reality conditions (i) ρ+p > 0, (ii) ρ+3 > 0 given by Ellis [7] lead to

$$\left(\frac{5\gamma+6}{5\gamma-1}\right)\frac{1}{T^4}+\frac{5(3+\gamma)}{4(5\gamma+3)}+\frac{5\delta}{T^{5\gamma+3}}>\frac{4}{5\gamma+1}\frac{1}{T^2}+\frac{1}{4} \tag{27}$$

The spatial volume $=R^3=ab^2=b^4=T^4$ (28)

The vacuum energy density $(\Lambda)\simeq\frac{1}{T^4}$ (29)

The matter density (ρ), the isotropic pressure (p), the expansion (θ), the shear (σ) and spatial volume (R^3) for the model (21) are given by

$$8\pi\rho=\frac{10\beta^2 B\cosh^2(2\sqrt{B}\tau)+4\left\{\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau)\right\}-\left\{\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau)\right\}^2}{4\left\{\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau)\right\}^2} \tag{30}$$

$$8\pi p=8\pi\gamma\rho \tag{31}$$

$$\theta=\frac{\dot{a}}{a}+\frac{2\dot{b}}{b}=4\frac{\dot{b}}{b}$$

$$=\frac{4\beta\sqrt{B}\cosh(2\sqrt{B}\tau)}{\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau)} \tag{32}$$

$$\sigma=\frac{1}{\sqrt{3}}\left|\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right|$$

$$=\frac{1}{\sqrt{3}}\frac{\beta\sqrt{B}\cosh(2\sqrt{B}\tau)}{\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau)} \tag{33}$$

$$R^3=ab^2=b^4$$

$$=\left[\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau)\right]^2 \quad (34)$$

The vacuum energy density $(\Lambda)\simeq\frac{1}{R^3}\simeq\frac{1}{ab^2}\simeq\frac{1}{b^4}$

$$\frac{\sigma}{\theta}=\frac{1}{4\sqrt{3}}\neq 0.$$

The reality conditions ρ + p > 0, ρ+3 p > 0 given by Ellis [7] lead to

$$10\beta^2 B\cosh^2(2\sqrt{B}\tau)+4\left\{\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau\right\}>\left\{\frac{D}{2B}+\beta\sinh(2\sqrt{B}\tau)\right\}^2 \quad (35)$$

6. Conclusion

The model (13) starts with a big-bang at T = 0 and continues to expand indefinitely. Since $\frac{\sigma}{\theta}\neq 0$, hence, anisotropy is maintained throughout. The spatial volume increases with time. The model (13) has Point Type singularity at T = 0 (MacCallum [9]). The reality condition ρ > 0 is satisfied. The model (21) starts with finite expansion at τ = 0 and continues to expand. The reality condition ρ > 0 is satisfied in the model (21). The spatial volume increases with time. The anisotropy is maintained throughout in the model (21). The metric potentials a and b are finite at τ = 0. The vacuum energy density (Λ) decreases with time for both the models.

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