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SPAN-WISE FLUCTUATING HYDROMAGNETIC FREE CONVECTIVE HEAT TRANSFER FLOW PAST A HOT VERTICAL POROUS PLATE WITH THERMAL RADIATION AND VISCOUS DISSIPATION IN SLIP FLOW REGIME

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Abstract: The present study analyses the effect of thermal radiation on hydromagnetic free convective heat transfer flow past a hot vertical porous plate. The plate is subjected to slip flow condition and its temperature is assumed to be varying with space and time. The dimensionless governing equations are solved by using series expansion method. The influence of parameters characterizing the flow on the velocity profile, the temperature profile, the coefficient of skin friction and the Nusselt number are discussed with the help of graphs. It is seen that thermal radiation causes the velocity field and temperature field to decrease and wall slip increases the rate of flow of fluid at the plate.

Keywords: Free convection, thermal radiation, viscous dissipation, slip flow regime.

Mathematical subject classification (2010) 76D05, 76D10

1. Introduction

The phenomenon of MHD free convective flow together with heat transfer has attracted the attention of researchers because of their possible application in many branches of science and technology. Bejan and Khair [4] investigated the vertical free convection boundary layer flow with heat and mass transfer in a porous medium. Chaudhary et al. [5] considered the effect of radiation on MHD heat transfer flow. The effects of heat transfer on MHD free convective flow through porous medium with viscous dissipation have been studied by Poonia and Chaudhary [12]. Jain et al. [7] analyzed an unsteady three dimensional free convection flow with combined heat and mass transfer over a vertical plate embedded in a porous medium with time dependent suction velocity and transverse sinusoidal permeability. Thermal-diffusion and diffusion-thermo effects on unsteady MHD mixed convection flow past an infinite vertical porous plate in the presence of chemical reaction, heat source and Joule effect have been studied numerically by Sharma et al. [17].

The study of fluctuating flow is very important in the paper industry. Singh and Mathew [20] analyzed the heat transfer in free convective fluctuating MHD flow of a viscous, incompressible and electrically conducting fluid past a hot vertical porous plate. Rajesh [14] studied radiation effects on MHD free convection flow near a vertical plate with ramped wall temperature. Heat transfer in viscous, free convective fluctuating MHD flow through porous media past a vertical porous plate with variable temperature has been investigated by Mishra et al. [9]. Magnetohydrodynamic free convection flow past a flat plate with ramped wall temperature and radiative heat transfer embedded in a porous medium in the presence of an inclined magnetic field has been studied by Nandkeolyar and Das [10].

The fluid slippage at solid boundary is another very important phenomenon that is widely encountered in this era of industrialization. Due to practical applications of the slip flow regime, several scholars have carried out their research work in this field. The magnetohydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conducting fluid over a semi-infinite vertical porous plate in a slip-flow regime have been analyzed by Sahin [16]. Pal and Talukdar [11] presented perturbation analysis of unsteady magneto hydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. The unsteady MHD flow of viscous, incompressible and electrically conducting fluid through a porous medium adjacent to an accelerated impermeable plate in a rotating system has been analyzed by Chauhan and Rastogi [6]. In that analysis Hall current was considered and boundary wall slip flow and temperature slip conditions were assumed. The unsteady free convection heat and mass transfer flow through a non-homogeneous porous medium with variable permeability bounded by an infinite porous vertical plate in slip flow regime taking into account the radiation, chemical reaction and temperature gradient dependent heat source has been analyzed by Rao et al.[15]. Ahmed and Das [3] presented the effects of thermal radiation and chemical reaction on MHD unsteady mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium in a slip flow regime with variable suction. The wavering free convective gelatinous incompressible flow past a perpendicular permeable flat plate with cyclic temperature in slip flow system has been discussed by Kalra and Verma [8].

There are many industrial and engineering processes, where the temperature varies in a complex manner. Most of the studies are based on the situations where the wall temperature is either constant or varies with time only. A few studies have been conducted by considering span-wise co-sinusoidal temperature of the surfaces. Singh [18] analyzed an unsteady free convection flow past a hot vertical porous plate with variable temperature. Singh and Chand [19] discussed an unsteady free convective MHD flow past a vertical porous plate with variable temperature. Rafi and Sharma [13] studied the free convective flow of a viscous fluid past a hot vertical porous plate with the assumption that the suction velocity is constant and normal to the plate and the plate temperature is span-wise co-sinusoidal. The study of hydromagnetic unsteady MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been made by Venkateswarlu et al. [21]. Free convective fluctuating MHD flow through porous media past a vertical porous plate with variable temperature and heat source has been studied extensively by Acharya et al. [1].

In the present work an attempt has been made to analyze the effect of thermal radiation on hydromagnetic free convective heat transfer flow past a hot vertical porous plate in slip flow regime. Series expansion method is used to solve the governing non-linear partial differential equations.

2. Formulation of the problem

An unsteady free convective slip flow of a viscous, incompressible, electrically conducting fluid past an insulated, hot porous plate lying vertically on the $x^* - z^*$ plane is considered. The x^*-axis is taken along the plate and y^*-axis is taken perpendicular to the plane of the plate. A uniform magnetic field of strength B_0 is applied along the y^*-axis. Let (u^*, v^*, w^*) be the component of velocity in the (x^*, y^*, z^*) direction respectively. The plate is considered to be infinite in x^* direction, hence all the physical quantities are independent of x^*.

Following Acharya and Padhy [2], w^* is independent of z^* and so we assume $w^* = 0$ throughout.

We assume the span wise co-sinusoidal temperature of the form

$$T^* = T_0^* + \varepsilon\,(T_0^* - T_\infty^*)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right) \tag{1}$$

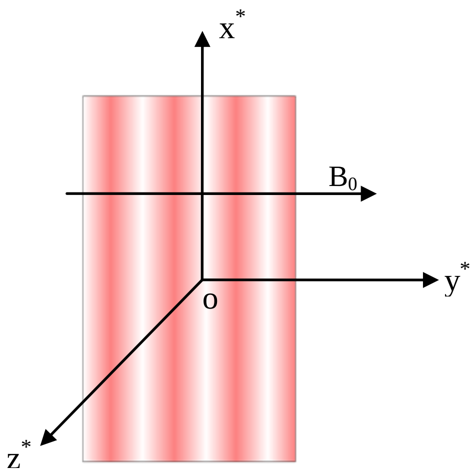

Figure 1: Physical model of the problem

where $\varepsilon \ll 1$ is an amplitude of plate temperature, d is the wavelength, ω^* is the frequency, t^* is the time, T_0^* is the mean temperature of the plate, T_∞^* is the temperature of the free stream.

The mean temperature T_0^* of the plate is supplemented by the secondary temperature $\varepsilon\,(T_0^* - T_\infty^*)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right)$ varying with space and time.

Following Singh and Mathew [20] and under the usual Boussinesq approximation, equations governing the flow are:

$$v_y^* = 0 \qquad \Rightarrow v^* = -V \tag{2}$$

$$u_t^* + v^* u_y^* = \upsilon\left(u_{yy}^* + u_{zz}^*\right) + g\beta(T^* - T_\infty^*) - \frac{\sigma B_0^2 u^*}{\rho} \tag{3}$$

$$T_t^* + v^* T_y^* = \frac{\kappa}{\rho C_p}\left(T_{yy}^* + T_{zz}^*\right) + \frac{\mu}{\rho C_p}\left[\left(u_y^*\right)^2 + (u_z^*)^2\right] - \frac{1}{\rho C_p}\left(q_y^*\right) \tag{4}$$

In above equations u^*,v^*-denote the components of velocity in the boundary layer in x^* and y^* directions respectively; V -the constant suction velocity at the plate; T^*-the temperature in the boundary; t^*-the time; β- the volumetric coefficient of thermal expansion; ρ-the density of fluid; μ-the coefficient of viscosity; g-the acceleration due to gravity; υ-the kinematic viscosity; σ-the electrical conductivity; C_p-the heat capacity of fluid at constant pressure; B_0-the magnetic field strength; κ-the thermal conductivity of the fluid; q^*-the radiative flux vector.

The boundary conditions are given by

$$\left.\begin{aligned} &y^* = 0;\, u^* = L u_y^*,\, T^* = T_0^* + \varepsilon\,(T_0^* - T_\infty^*)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right) \\ &y^* \to \infty;\, u^* = 0,\, T^* = T_\infty^* \end{aligned}\right\} \tag{5}$$

where $L = \left(\frac{2-f_1}{f_1}\right) L_1$, is the mean free path and f_1 is the Maxwell's reflection coefficient.

By using Rosseland approximation, the radiative flux vector q^* for optically thick fluid is given by

$$q^* = -\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^{*4}}{\partial y^*} \tag{6}$$

where κ^* is mean absorption coefficient and σ^* is Stefan-Boltzmann constant. Assuming that the difference between fluid temperature T^* and free stream temperature T_∞^* is small, so expanding T^{*4} by Taylor series about a free stream temperature. After neglecting second and higher order terms in $(T^* - T_\infty^*)$, we get

$$T^{*4} = 4T_\infty^{*3} T^* - 3T_\infty^{*4} \tag{7}$$

On the use of equations (6) and (7), equation (4) becomes

$$T_t^* + v^* T_y^* = \frac{\kappa}{\rho C_p}\left(T_{yy}^* + T_{zz}^*\right) + \frac{\mu}{\rho C_p}\left[\left(u_y^*\right)^2 + (u_z^*)^2\right] + \frac{1}{\rho C_p}\frac{16\sigma^* T_\infty^{*3}}{3\kappa^*}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{8}$$

On Introducing following non-dimensional variables and parameters:

$$\left.\begin{aligned} & y = \frac{y^*}{d}, z = \frac{z^*}{d}, t = \frac{t^* V}{d}, u = \frac{u^*}{V}, \omega = \frac{\omega^* d}{V}, \theta = \frac{T^* - T_\infty^*}{T_0^* - T_\infty^*}, Re = \frac{Vd}{\upsilon}, Pr = \frac{\mu C_p}{\kappa} \\ & M = B_0 d\sqrt{\frac{\sigma}{\mu}}, Gr = \frac{g\beta(T_0^* - T_\infty^*)d^2}{\upsilon V}, Ec = \frac{V^2}{C_p\left(T_0^* - T_\infty^*\right)}, N = \frac{\kappa\kappa^*}{4\sigma^* T_\infty^*}, h = \frac{L}{d} \end{aligned}\right\} \tag{9}$$

where ω is the frequency of temperature fluctuation, θ is the non-dimensional temperature, Re is the Reynolds number, Pr is the Prandtl number, M is the magnetic field parameter, Gr is the Grashof number , Ec is the Eckert number, N is the radiation parameter, h is the slip parameter and d is the wavelength.

Using above mentioned dimensionless quantities, equations (3) and (8) transform to

$$Re\left(u_t - u_y\right) = \left(u_{yy} + u_{zz}\right) - M^2 u + Gr\theta \tag{10}$$

$$RePr\left(\theta_t - \theta_y\right) = \left(\theta_{yy} + \theta_{zz}\right) + EcPr\left[\left(u_y\right)^2 + (u_z)^2\right] + \frac{4}{3N}\theta_{yy} \tag{11}$$

The corresponding boundary conditions are given by:

$$\left.\begin{aligned} & y = 0\,; u = hu_y, \theta = 1 + \varepsilon\cos(\pi z - \omega t) \\ & y \to \infty; u = 0, \theta = 0 \end{aligned}\right\} \tag{12}$$

3. Method of solution

Since the amplitude ε of the plate temperature is assume to be very small, we represent the velocity and temperature in the neighborhood of the plate as

$$\left.\begin{aligned} & u(y,z,t) = u_0(y) + \varepsilon u_1(y,z,t) + O(\varepsilon^2) \\ & \theta(y,z,t) = \theta_0(y) + \varepsilon\theta_1(y,z,t) + O(\varepsilon^2) \end{aligned}\right\} \tag{13}$$

Comparing the coefficients of like powers of ε after substituting (13) in equations (10) and (11), we get the following set of equations

$$u_{0_{yy}} + Reu_{0_y} - M^2 u_0 = -Gr\theta_0 \tag{14}$$

$$u_{1_{yy}} + u_{1_{zz}} - Reu_{1_t} + Reu_{1_y} - M^2 u_1 = -Gr\theta_1 \tag{15}$$

$$\left(1 + \frac{4}{3N}\right)\theta_{0_{yy}} + RePr\theta_{0_y} + EcPr\left(u_{0_y}\right)^2 = 0 \tag{16}$$

$$RePr\left(\theta_{1_t} - \theta_{1_y}\right) = \left(1 + \frac{4}{3N}\right)\theta_{1_{yy}} + 2EcPr\, u_{0_y} u_{1_y} + \theta_{1_{zz}} \tag{17}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} & y = 0\,; u_0 = hu_{0_y}, u_1 = hu_{1_y}, \theta_0 = 1, \theta_1 = \cos(\pi z - \omega t) \\ & y \to \infty\,; u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0 \end{aligned}\right\} \tag{18}$$

In order to solve coupled equations (14) to (17), we use the following equations with perturbation parameter Ec (the Eckert number):

$$\left.\begin{aligned} u_0(y) &= u_{01}(y) + \mathrm{Ec}\, u_{02}(y) + O(\mathrm{Ec}^2) \\ \theta_0(y) &= \theta_{01}(y) + \mathrm{Ec}\, \theta_{02}(y) + O(\mathrm{Ec}^2) \\ u_1(y,z,t) &= \phi(y) e^{\iota(\pi z - \omega t)} \\ \theta_1(y,z,t) &= \psi(y) e^{\iota(\pi z - \omega t)} \end{aligned}\right\} \tag{19}$$

Substituting (19) in equations (14) to (17), we get the following equations:

$$u_{01_{yy}} + \mathrm{Re}\, u_{01_y} - M^2 u_{01} = -\mathrm{Gr}\, \theta_{01} \tag{20}$$

$$u_{02_{yy}} + \mathrm{Re}\, u_{02_y} - M^2 u_{02} = -\mathrm{Gr}\, \theta_{02} \tag{21}$$

$$\phi_{yy} + \mathrm{Re}\phi_y - (\pi^2 - \iota\omega \mathrm{Re} + M^2)\phi = -\mathrm{Gr}\, \psi \tag{22}$$

$$\left(1 + \frac{4}{3N}\right) \theta_{01_{yy}} + \mathrm{RePr}\, \theta_{01_y} = 0 \tag{23}$$

$$\left(1 + \frac{4}{3N}\right) \theta_{02_{yy}} + \mathrm{RePr}\, \theta_{02_y} + \mathrm{Pr}\left(u_{01_y}\right)^2 = 0 \tag{24}$$

$$\left(1 + \frac{4}{3N}\right) \psi_{yy} + \mathrm{RePr}\, \psi_y + (\iota\omega \mathrm{RePr} - \pi^2)\psi + 2\mathrm{EcPr}\left(u_{01_y}\right)\left(\phi_y\right) = 0 \tag{25}$$

the corresponding boundary conditions are:

$$\left.\begin{aligned} &y = 0;\, u_{01} = h u_{01_y},\, u_{02} = h u_{02_y},\, \phi = h\phi_y,\, \theta_{01} = 1,\, \theta_{02} = 0,\, \psi = 1 \\ &y \to \infty;\, u_{01} = 0,\, u_{02} = 0,\, \theta_{01} = 0,\, \theta_{02} = 0,\, \phi = 0,\, \psi = 0 \end{aligned}\right\} \tag{26}$$

The solutions of equations (20), (21), (23) and (24) under the boundary conditions (26) are as follows:

$$u_{01}(y) = B_2 e^{-A_2 y} + B_1 e^{-A_1 y} \tag{27}$$

$$u_{02}(y) = B_{18} e^{-A_2 y} + B_{10} e^{-A_1 y} + B_{11} e^{-2A_2 y} + B_{12} e^{-2A_1 y} + B_{13} e^{-(A_1 + A_2) y} \tag{28}$$

$$\theta_{01}(y) = e^{-A_1 y} \tag{29}$$

$$\theta_{02}(y) = B_9 e^{-A_1 y} + B_6 e^{-2A_2 y} + B_7 e^{-2A_1 y} + B_8 e^{-(A_1 + A_2) y} \tag{30}$$

Equations (22) and (25) are still coupled, so we assume the following perturbed forms:

$$\left.\begin{aligned} \phi(y) &= \phi_0(y) + \mathrm{Ec}\, \phi_1(y) + O(\mathrm{Ec}^2) \\ \psi(y) &= \psi_0(y) + \mathrm{Ec}\, \psi_1(y) + O(\mathrm{Ec}^2) \end{aligned}\right\} \tag{31}$$

Substituting (31) into equations (22) and (25) and equating like powers of Ec, we get the following set of equations:

$$\phi_{0_{yy}} + \mathrm{Re}\phi_{0_y} - (\pi^2 - \iota\omega \mathrm{Re} + M^2)\phi_0 = -\mathrm{Gr}\, \psi_0 \tag{32}$$

$$\phi_{1_{yy}} + \mathrm{Re}\phi_{1_y} - (\pi^2 - \iota\omega\mathrm{Re} + M^2)\phi_1 = -\mathrm{Gr}\,\psi_1 \quad (33)$$

$$\left(1 + \frac{4}{3N}\right)\psi_{0_{yy}} + \mathrm{RePr}\,\psi_{0_y} + (\iota\omega\mathrm{RePr} - \pi^2)\psi_0 = 0 \quad (34)$$

$$\left(1 + \frac{4}{3N}\right)\psi_{1_{yy}} + \mathrm{RePr}\,\psi_{1_y} + (\iota\omega\mathrm{RePr} - \pi^2)\psi_1 + 2\mathrm{Pr}\,u_{01_y}\phi_{0_y} = 0 \quad (35)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} y = 0; \phi_0 = h\phi_{0_y}, \phi_1 = h\phi_{1_y}, \psi_0 = 1, \psi_1 = 0 \\ y \to \infty; \phi_0 = 0, \phi_1 = 0, \psi_0 = 0, \psi_1 = 0 \end{array}\right\} \quad (36)$$

The solutions of equations (32) to (35) under the boundary conditions (36) are as follows:

$$\phi_0 = B_{20}e^{-A_4y} + B_{19}e^{-A_3y} \quad (37)$$

$$\phi_1 = B_{40}e^{-A_4y} + B_{30}e^{-A_3y} + B_{31}e^{-(A_2+A_4)y} + B_{32}e^{-(A_1+A_4)y} + B_{33}e^{-(A_2+A_3)y} + B_{34}e^{-(A_1+A_3)y} \quad (38)$$

$$\psi_0 = e^{-A_3y} \quad (39)$$

$$\psi_1 = B_{29}e^{-A_3y} + B_{25}e^{-(A_2+A_4)y} + B_{26}e^{-(A_1+A_4)y} + B_{27}e^{-(A_2+A_3)y} + B_{28}e^{-(A_1+A_3)y} \quad (40)$$

Using equations (27), (28), (37) and (38), the following expression for the velocity profile is obtained:

$$\begin{aligned} u(y,z,t) = {} & B_2e^{-A_2y} + B_1e^{-A_1y} + \varepsilon\left(B_{20}e^{-A_4y} + B_{19}e^{-A_3y}\right)e^{\iota(\pi z-\omega t)} + \mathrm{Ec}\,[B_{18}e^{-A_2y} + B_{10}e^{-A_1y} \\ & + B_{11}e^{-2A_2y} + B_{12}e^{-2A_1y} + B_{13}e^{-(A_1+A_2)y} + \varepsilon e^{\iota(\pi z-\omega t)}(B_{40}e^{-A_4y} + B_{30}e^{-A_3y} \\ & + B_{31}e^{-(A_2+A_4)y} + B_{32}e^{-(A_1+A_4)y} + B_{33}e^{-(A_2+A_3)y} + B_{34}e^{-(A_1+A_3)y})] \end{aligned} \quad (41)$$

Using equations (29), (30), (39) and (40), we get the following expression for the temperature profile:

$$\begin{aligned} \theta(y,z,t) = {} & e^{-A_1y} + \varepsilon(e^{-A_3y})e^{\iota(\pi z-\omega t)} + \mathrm{Ec}\,[B_9e^{-A_1y} + B_6e^{-2A_2y} + B_7e^{-2A_1y} + B_8e^{-(A_1+A_2)y} \\ & + \varepsilon e^{\iota(\pi z-\omega t)}(B_{29}e^{-A_3y} + B_{25}e^{-(A_2+A_4)y} + B_{26}e^{-(A_1+A_4)y} + B_{27}e^{-(A_2+A_3)y} \\ & + B_{28}e^{-(A_1+A_3)y})] \end{aligned} \quad (42)$$

4. Some important characteristics of flow

From the velocity field, the expression for the non-dimensional shear stress (τ) at the plate is given by

$$\tau = \left(u_y\right)_{y=0}$$

$$= -(A_2B_2 + A_1B_1) - Ec[A_2B_{18} + A_1B_{10} + 2A_2B_{11} + 2A_1B_{12} + (A_1 + A_2)B_{13}]$$
$$-\varepsilon|F|\cos(\pi z - \omega t + \alpha_1) \quad (43)$$

where

$$F = F_i + \iota F_r = A_4B_{20} + A_3B_{19} + Ec[A_4B_{40} + A_3B_{30} + (A_2 + A_4)B_{31} + (A_1 + A_4)B_{32}$$
$$+(A_2 + A_3)B_{33} + (A_1 + A_3)B_{34}] \quad (44)$$

$$|F| = \sqrt{F_i^2 + F_r^2} \text{ and } \alpha_1 = \tan^{-1}\left(\frac{F_r}{F_i}\right) \quad (45)$$

From the temperature field, the expression for the non-dimensional rate of heat transfer coefficient at the plate is given by

$$Nu = -(\theta_y)_{y=0}$$
$$= A_1 + Ec[A_1B_9 + 2A_2B_6 + 2A_1B_7 + (A_1 + A_2)B_8] + \varepsilon|G|\cos(\pi z - \omega t + \alpha_2) \quad (46)$$

where $G = G_i + \iota G_r$

$$= A_3 + Ec[A_3B_{29} + (A_2 + A_4)B_{25} + (A_1 + A_4)B_{26} + (A_2 + A_3)B_{27} + (A_1 + A_3)B_{28}] \quad (47)$$

$$|G| = \sqrt{G_i^2 + G_r^2} \text{ and } \alpha_2 = \tan^{-1}\left(\frac{G_r}{G_i}\right) \quad (48)$$

All constants used above have been listed in the appendix.

5. Results and Discussion

In order to analyze the results, numerical computations has been carried out for variations in the governing parameters such as the Reynolds number (Re), the Prandtl number (Pr), the Grashof number (Gr), the Eckert number (Ec), the magnetic field parameter (M), the radiation parameter (N) and the slip parameter (h).For illustration of these results, numerical values are plotted in figures (2-7).

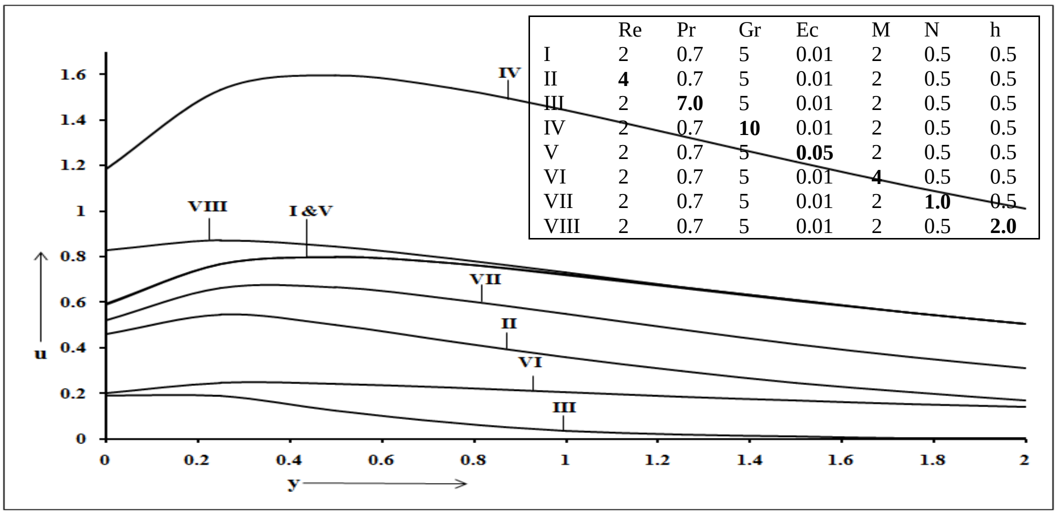

Figure 2: Velocity profile for different values of Re, Pr, Gr, Ec, M, N and h for ω = 0.5, ε = 0.01.

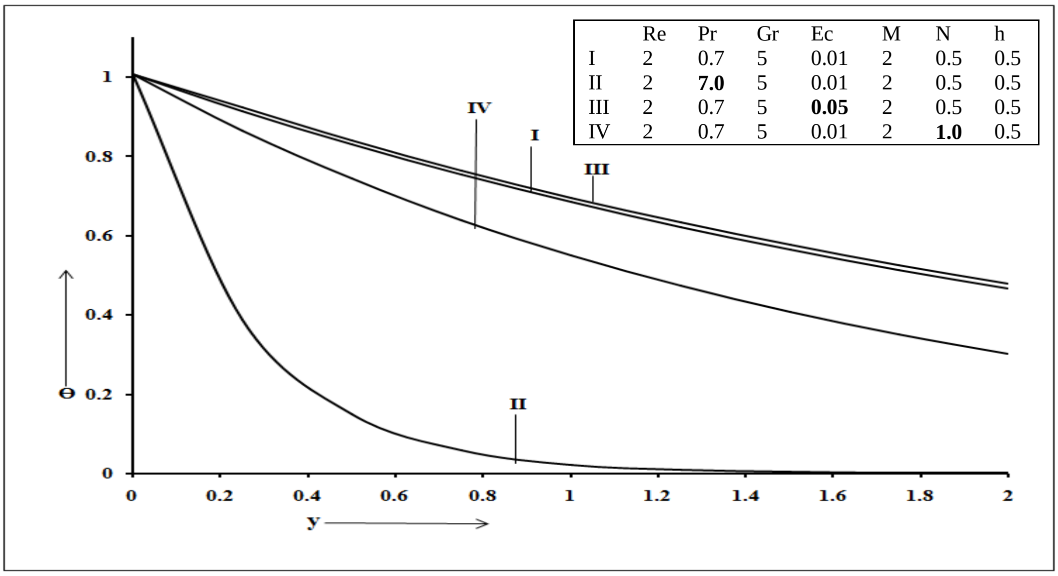

Figure 3: Temperature profile for different values of Pr, Ec and N for ω = 0.5, ε = 0.01.

Figure 3 exhibits continuously decreasing behavior of the temperature distribution. For higher value of the Prandtl number (Pr =7.0), the temperature of fluid near the plate reduces quickly (curve II).Curves III and IV reveal that increase in the Eckert number, increases the temperature profile whereas increase in the radiation parameter decreases it. This result qualitatively agrees with expectation, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

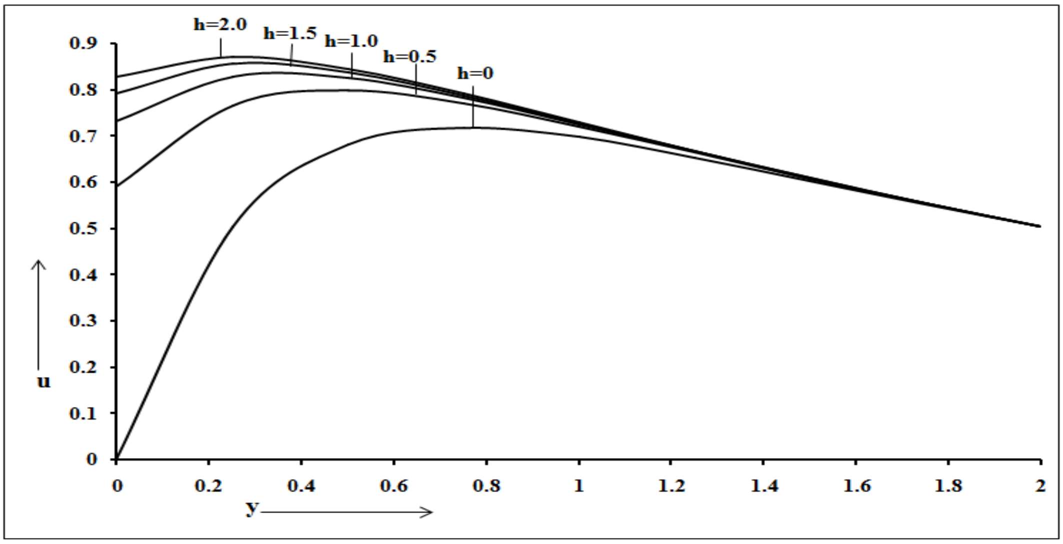

Figure 4: Effect of wall slip on velocity of fluid in the boundary layer for Re = 2, Pr = 0.7, Gr =5, Ec = 0.01, M =2, N = 0.5, ω = 0.5, ε = 0.01.

Figure 4 depicts the velocity profile variations with the slip parameter h, it is observed that the velocity is high near the plate and there after it decreases and reaches the stationary value far away from the plate. As expected, velocity increases with an increase in the slip parameter h.

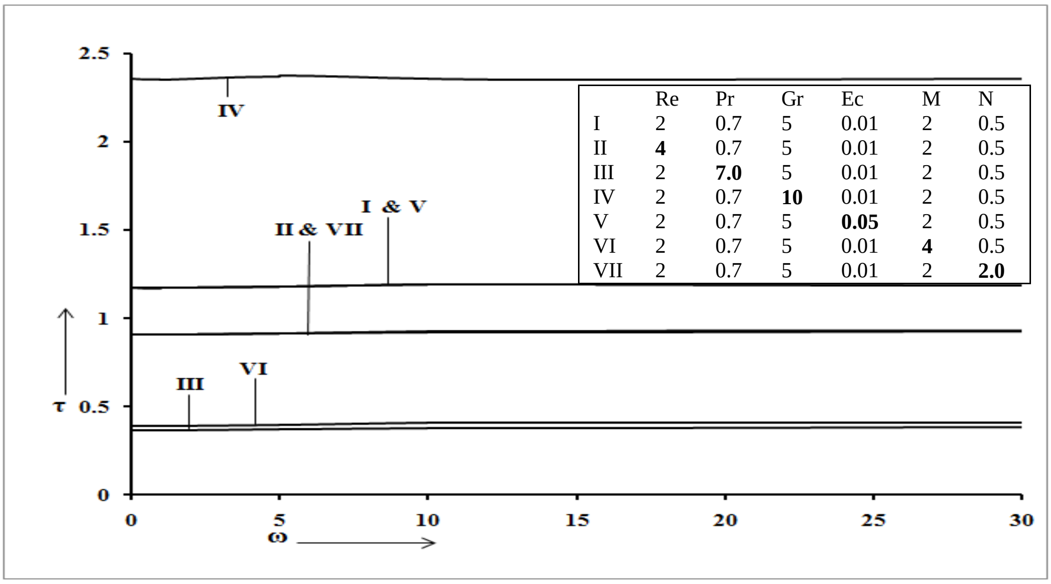

Figure 5: The shear stress (τ) for different values of parameters viz. Re, Pr, Gr, Ec, M and N for h = 0.5, ε = 0.01.

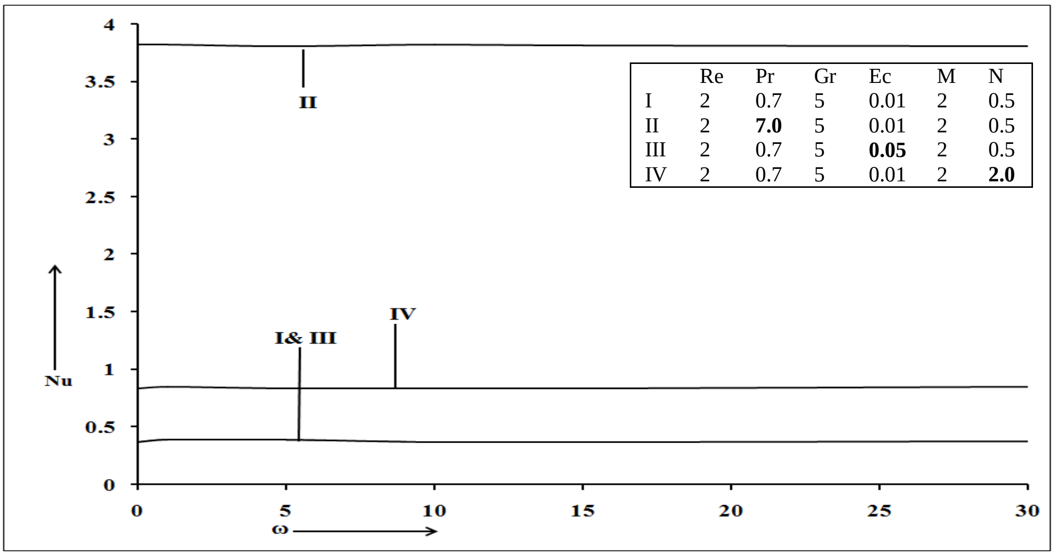

Figure 6: The Nusselt number (Nu) for different values of Pr, Ec and N for h = 0.5, ε = 0.01.

From figure 5 and figure 6, it is observed that the shear stress and the rate of heat transfer coefficients at the plate are almost linear for all values of frequency of fluctuation. The shear stress increases when the Grashof number increases while it decreases with increase in the Reynolds number, the Prandtl number, the magnetic parameter and the radiation parameter and remains unaffected with increase in the Eckert number. From figure 6 it is concluded that the rate of heat transfer coefficient at the plate increases with the Prandtl number and the radiation parameter.

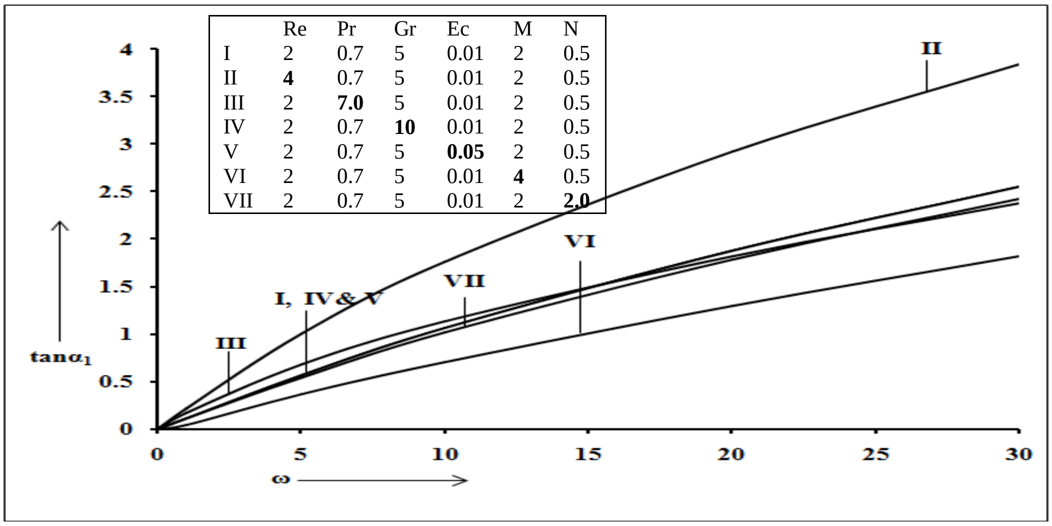

Figure 7: The phase angle of shear stress ($\tan\alpha_1$)for different values of Re, Pr, Gr, Ec, M and N for h = 0.5, ε = 0.01.

Figure 7 shows a steady increasing behavior of the phase angle with an increasing frequency parameter. A slight decrease in it is observed when ω exceed 15.0(approx.). A decrease in the phase angle is noticed due to increase in the radiation parameter and the magnetic parameter but the reverse effect is observed in case of the Prandtl number and the Reynolds number. The phase angle remains unaffected due to buoyancy effect and dissipative loss that is the Grashof number and the Eckert number are not affecting it.

6. Conclusions

From this study we concluded that

- The fluid velocity in the boundary layer increases with the Grashof number but decreases with increase in the Reynolds number, the Prandtl number, the magnetic field parameter and the radiation parameter and it remains unaffected by the variation in the Eckert number.
- The temperature in the boundary layer increases with the Eckert number, whereas it decreases with increase of the Prandtl number and the radiation parameter.
- The fluid velocity near the plate increases with increase in the slip parameter.
- The shear stress at the plate increases when the Grashof number increases while decreases with increase in the Reynolds number, the Prandtl number, the magnetic parameter and the radiation parameter.
- The rate of heat transfer coefficient at the plate increases with the Prandtl number and the radiation parameter.

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Appendix

$A_1 = \frac{Re\,Pr}{\left(1+\frac{4}{3N}\right)}$, $A_2 = \frac{Re+\sqrt{Re^2+4M^2}}{2}$, $A_3 = \frac{RePr+\sqrt{RePr^2-4\left(1+\frac{4}{3N}\right)(\iota\omega RePr-\pi^2)}}{2\left(1+\frac{4}{3N}\right)}$

$A_4 = \frac{Re+\sqrt{Re^2+4(\pi^2-\iota\omega Re+M^2)}}{2}$, $B_1 = \frac{-Gr}{A_1^2-ReA_1-M^2}$, $B_2 = \frac{-B_1(1+hA_1)}{(1+hA_2)}$,

$B_3 = -PrB_2{}^2A_2{}^2$, $B_4 = -PrB_1{}^2A_1{}^2$, $B_5 = -2PrA_1B_1A_2B_2$,

$B_6 = \frac{B_3}{4A_2{}^2\left(1+\frac{4}{3N}\right)-2A_2RePr}$, $B_7 = \frac{B_4}{4A_1{}^2\left(1+\frac{4}{3N}\right)-2A_1RePr}$, $B_8 = \frac{B_5}{(A_1+A_2)^2\left(1+\frac{4}{3N}\right)-RePr(A_1+A_2)}$

$B_9 = -(B_6+B_7+B_8)$, $B_{10} = \frac{-GrB_9}{A_1^2-ReA_1-M^2}$, $B_{11} = \frac{-GrB_6}{4A_2^2-2ReA_2-M^2}$, $B_{12} = \frac{-GrB_7}{4A_1^2-2ReA_1-M^2}$,

$B_{13} = \frac{-GrB_8}{\left[(A_1+A_2)^2-Re(A_1+A_2)-M^2\right]}$, $B_{14} = \frac{-B_{10}(1+hA_1)}{(1+hA_2)}$, $B_{15} = \frac{-B_{11}(1+2hA_2)}{(1+hA_2)}$, $B_{16} = \frac{-B_{12}(1+2hA_1)}{(1+hA_2)}$,

$B_{17} = \frac{-B_{13}\left(1+h(A_1+A_2)\right)}{(1+hA_2)}$, $B_{18} = B_{14}+B_{15}+B_{16}+B_{17}$, $B_{19} = \frac{-Gr}{A_3^2-Re\,A_3-\left(\pi^2-\iota\omega Re+M^2\right)}$

$B_{20} = -\frac{(1+hA_3)}{(1+hA_4)}$, $B_{21} = -2\,\Pr A_2A_4B_2B_{20}$, $B_{22} = -2\,\Pr A_1A_4B_1B_{20}$, $B_{23} = -2\,\Pr A_2A_3B_2B_{19}$,

$B_{24} = -2\Pr A_1A_3B_1B_{19}$, $B_{25} = \frac{B_{21}}{\left(1+\frac{4}{3N}\right)(A_2+A_4)^2-\mathrm{RePr}(A_2+A_4)+(\iota\omega\mathrm{RePr}-\pi^2)}$,

$B_{26} = \frac{B_{22}}{\left(1+\frac{4}{3N}\right)(A_1+A_4)^2-\mathrm{RePr}(A_1+A_4)+(\iota\omega\mathrm{RePr}-\pi^2)}$, $B_{27} = \frac{B_{23}}{\left(1+\frac{4}{3N}\right)(A_2+A_3)^2-\mathrm{RePr}(A_2+A_3)+(\iota\omega\mathrm{RePr}-\pi^2)}$,

$B_{28} = \frac{B_{21}}{\left(1+\frac{4}{3N}\right)(A_1+A_3)^2-\mathrm{RePr}(A_1+A_3)+(\iota\omega\mathrm{RePr}-\pi^2)}$, $B_{29} = -(B_{25} + B_{26} + B_{27} + B_{28})$,

$B_{30} = \frac{-\mathrm{Gr}B_{29}}{A_3^2-\mathrm{Re}A_3-\left(\pi^2-\iota\omega\mathrm{Re}+M^2\right)}$, $B_{31} = \frac{-\mathrm{Gr}B_{25}}{(A_2+A_4)^2-\mathrm{Re}(A_2+A_4)-\left(\pi^2-\iota\omega\mathrm{Re}+M^2\right)}$,

$B_{32} = \frac{-\mathrm{Gr}B_{26}}{(A_1+A_4)^2-\mathrm{Re}(A_1+A_4)-\left(\pi^2-\iota\omega\mathrm{Re}+M^2\right)}$, $B_{33} = \frac{-\mathrm{Gr}B_{27}}{(A_2+A_3)^2-\mathrm{Re}(A_2+A_3)-\left(\pi^2-\iota\omega\mathrm{Re}+M^2\right)}$,

$B_{34} = \frac{-\mathrm{Gr}B_{28}}{(A_1+A_3)^2-\mathrm{Re}(A_1+A_3)-\left(\pi^2-\iota\omega\mathrm{Re}+M^2\right)}$, $B_{35} = -\frac{B_{30}(1+hA_3)}{(1+hA_4)}$, $B_{36} = -\frac{B_{31}\left(1+h(A_2+A_4)\right)}{(1+hA_4)}$,

$B_{37} = -\frac{B_{32}\left(1+h(A_1+A_4)\right)}{(1+hA_4)}$, $B_{38} = -\frac{B_{33}\left(1+h(A_2+A_3)\right)}{(1+hA_4)}$, $B_{39} = -\frac{B_{34}\left(1+h(A_1+A_3)\right)}{(1+hA_4)}$,

$B_{40} = B_{35} + B_{36} + B_{37} + B_{38} + B_{39}$