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A SUFFICIENT CONDITION FOR THE EXCHANGE PRINCIPLE IN MULTICOMPONENT CONVECTION PROBLEM IN COMPLETELY CONFINED FLUIDS

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Abstract: The multicomponent instability problem for fluid completely confined in an arbitrary region with rigid bounding surfaces is considered. A sufficient condition for the validity of the exchange principle is derived, which to the best of our knowledge does not appear to have been reported in the literature on multicomponent convection with the same degree of generality. The result has its importance due to the reason that no numerical computation is possible for the approximations of the solutions in case of arbitrary boundaries.

Keywords: Multicomponent convection, the principle of the exchange of stabilities, completely confined fluids, concentration Rayleigh number.

1. Introduction

Thermosolutal convection or, more generally, the double diffusive convection is a mixing process occurs by the interaction of two components (one of them may be heat) contributing to density that diffuse at different rates. Double diffusive convection has important applications in limnology, oceanography, chemical engineering, geophysics, astrophysics etc. To determine the conditions under which these convective motions will occur, the linear stability of two superposed concentration gradients (or one of them may be temperature gradient) has been studied by Stern [26], Veronis [29], Nield [8], Baines and Gill [1] and Turner [27]. For a broad view of the subject one may referred to Radko [20], Brandt and Fernando [2] and Prakash and Kumar [11].

These researchers have considered only the case of two component system. However, it has been recognized later (Griffiths [3], Turner [28]) that there are many physical systems wherein more than two components are present. Examples of such multiple diffusive convection fluid systems include the solidification of molten alloys, geothermally heated lakes, magmas and their laboratory models and sea water. For the detailed overview of the work done on triply/ multiple diffusive convection one may refer to [3, 7, 9, 11-19, 21-24]. These researchers found that small concentrations of a third component with a smaller diffusivity can have a significant effect upon the nature of diffusive instabilities and direct salt finger (steady convection) and oscillatory modes are simultaneously unstable under a wide range of conditions.

All these researchers have confined themselves to horizontal layer geometry, perhaps, due to the complexity involved in the analysis of the hydrodynamic problems with arbitrary geometries as no numerical calculations/computations is possible for the present problem. However, there are a few researchers (Sherman and Ostrach [25], Gupta et al. [4], Gupta et al. [5], Prakash et al. [16] and Prakash et al. [19]) who have extended the classical work to more general hydrodynamic stability problems with arbitrary boundaries.

The establishment of the nonoccurrence of any slow oscillatory motions which may be neutral or unstable implies the validity of the principle of the exchange of stabilities (PES) or simply known as exchange principle. The validity of this principle in stability problems eliminates the unsteady terms from the linearized perturbation equations which results in notable mathematical simplicity since the transition from stability to instability occurs via a marginal state which is characterized by the vanishing of both real and imaginary parts of the complex time eigenvalue associated with the perturbation. Pellew and Southwell [10] proved the validity of PES (i.e. occurrence of steady convection) for the classical Rayleigh-Benard instability problem for fluids bounded by two infinite, horizontal parallel planes, whereas Sherman and Ostrach [25], Gupta et al. [4] and Prakash et al. [16] respectively, extended their result to Rayleigh-Benard instability problem, thermosolutal convection and triply diffusive convection problems for fluids completely confined in arbitrary regions with rigid boundaries.

In the present communication, which is motivated by the desire to extend the works of Sherman and Ostrach [25], Gupta et al. [4] and Prakash et al. [16] to more complex problems, namely, multicomponent convection problems for fluids completely confined in an arbitrary region bounded by rigid surfaces, a sufficient condition for the validity of the exchange principle is derived. To the authors' knowledge no such results have been reported in the literature for multicomponent convection problem. Further, the results of Sherman and Ostrach [25] for Rayleigh-Benard problem, Gupta et al. [4] for double-diffusive and Prakash et al. [16] for triply diffusive convection problem are obtained as a consequence.

2. Mathematical Formulation and Analysis

Consider a Boussinesq fluid statically confined in an arbitrary completely enclosed region (as shown in Fig. 1) which is maintained at a uniform temperature and concentration gradient parallel to the body force acting on a fluid by applying certain prescribed thermal and concentration boundary conditions on the bounding walls. The problem under investigation is to examine the stability of this physical configuration when the heat and the $n-1$ concentrations make opposing contributions to the vertical density gradient. It is further assumed that the cross diffusion effects can been neglected.

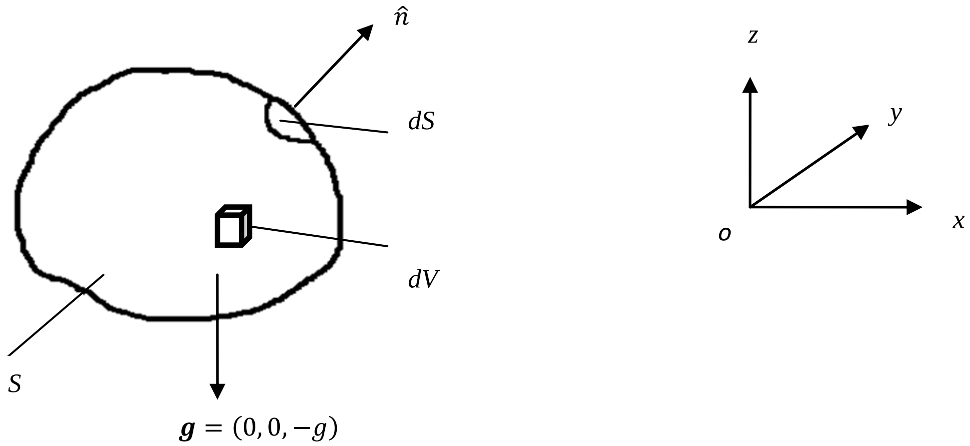

Fig. 1 Geometrical Configuration

The governing linearized perturbation equations in non dimensional form for the problem with time dependence of the form exp (pt) ($p = p_r + ip_i$, being complex in general) are given by (Gupta et al. [4])

$$\frac{p}{\sigma}\vec{U} = \nabla^2\vec{U} - \text{grad}(P) + R\theta\hat{k} - R_1\phi_1\hat{k} - R_2\phi_2\hat{k} - \cdots - R_{n-1}\phi_{n-1}\hat{k}, \tag{1}$$

$$\nabla^2\theta - p\theta = -\vec{U}.\hat{k}, \tag{2}$$

$$\tau_1\nabla^2\phi_1 - p\phi_1 = -\vec{U}.\hat{k}, \tag{3}$$

$$\tau_2\nabla^2\phi_2 - p\phi_2 = -\vec{U}.\hat{k}, \tag{4}$$

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$$\tau_{n-1}\nabla^2\phi_{n-1} - p\phi_{n-1} = -\vec{U}.\hat{k}, \tag{5}$$

$$\text{div}\vec{U} = 0, \tag{6}$$

where $\vec{U}, P, \theta, \phi_1, \phi_2, \dots\dots, \phi_{n-1}$ denote respectively the perturbed velocity, pressure, temperature, concentration for the $n-1$concentration components. R is the thermal Rayleigh number, R_1, R_2 ,…, R_{n-1} are the concentration Rayleigh numbers for $n-1$ concentration components respectively. σ is the Prandtl number, τ_1, τ_2,…, τ_{n-1} are the Lewis numbers for $n-1$concentration components respectively and $\hat{k}$ is a unit vector in the vertical direction. The equations have been written in dimensionless forms by using the scale factors $\frac{d^2}{\kappa}, \frac{\kappa}{d}, \frac{\rho\nu\kappa}{d^2}, \beta d, \beta_1 d, \beta_2 d, \dots, \beta_{n-1} d$ for time, velocity, pressure, temperature, and the $n-1$ concentrations respectively, where d is a characteristic length, κ the thermal diffusivity, ρ the density, ν the kinematic viscosity, β the constant temperature gradient, $\beta_1, \beta_2, \ldots, \beta_{n-1}$ are the constant concentration gradients for $n-1$ concentration components.

Equations $(1)-(6)$ are to be solved in a simply connected subset V of R^3 with boundary S subject to the following homogeneous time independent boundary conditions:

$\vec{U} = 0 = \theta = \phi_1 = \phi_2 = \cdots = \phi_{n-1}$ on S (rigid bounding surface with fixed temperature and mass concentrations). (7)

Equations (1) – (6) together with boundary conditions (7) describe an eigenvalue problem for p for prescribed values of the other parameters and the system is stable, neutral or unstable according as p_r is negative, zero or positive. Further if $p_r = 0$ implies $p_i = 0$, then the principle of the exchange of stabilities (PES) is valid otherwise we will have overstability.

Now we prove the following theorem:

Theorem: If $(p, \vec{U}, \theta, \phi_1, \phi_2, \ldots, \phi_{n-1})$, $p = p_r + ip_i, p_r \geq 0$, is a solution of equations (1) - (7) with $R > 0, R_1 > 0, R_2 > 0, \ldots, R_{n-1} > 0$ and $\frac{R_1 \sigma l^4}{\tau_1^2 \Lambda^2} + \frac{R_2 \sigma l^4}{\tau_2^2 \Lambda^2} + \cdots + \frac{R_{n-1} \sigma l^4}{\tau_{n-1}^2 \Lambda^2} \leq 1$, then $p_i = 0$, where l is the smallest distance between two parallel planes that just contain V and $\Lambda (> 2)$ is a constant. In particular PES is valid if

$\frac{R_1 \sigma l^4}{\tau_1^2 \Lambda^2} + \frac{R_2 \sigma l^4}{\tau_2^2 \Lambda^2} + \cdots + \frac{R_{n-1} \sigma l^4}{\tau_{n-1}^2 \Lambda^2} \leq 1.$

Proof: We rewrite system of equations (1) – (5) in the following alternate forms:

$$\frac{p}{\sigma}\vec{U} + \text{grad}(P) - \nabla^2 \vec{U} - R\theta\hat{k} + R_1\phi_1\hat{k} + R_2\phi_2\hat{k} + \cdots + R_{n-1}\phi_{n-1}\hat{k} = 0, \tag{8}$$

$$-R\left[\nabla^2\theta - p\theta + \vec{U}.\hat{k}\right] = 0, \tag{9}$$

$$R_1\left[\tau_1\nabla^2\phi_1 - p\phi_1 + \vec{U}.\hat{k}\right] = 0, \tag{10}$$

$$R_2\left[\tau_2\nabla^2\phi_2 - p\phi_2 + \vec{U}.\hat{k}\right] = 0, \tag{11}$$

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$$R_{n-1}\left[\tau_{n-1}\nabla^2\phi_{n-1} - p\phi_{n-1} + \vec{U}.\hat{k}\right] = 0. \tag{12}$$

Forming the dot product of equation (8) with $\vec{U}^*$ ($*$ denotes complex conjugation) and integrating over the domain V, we obtain

$$\frac{p}{\sigma}\int_V (\vec{U}.\vec{U}^*)dV + \int_V [(\text{grad}P).\vec{U}^*]dV - \int_V (\vec{U}^*.\nabla^2\vec{U})dV - R\int_V [(\theta\hat{k}).\vec{U}^*]dV + R_1\int_V [(\phi_1\hat{k}).\vec{U}^*]dV + R_2\int_V [(\phi_2\hat{k}).\vec{U}^*]dV + \cdots + R_{n-1}\int_V [(\phi_{n-1}\hat{k}).\vec{U}^*]dV = 0. \tag{13}$$

Subsequently, for convenience in writing, we omit V and the infinitesimal volume dV from the integral sign and the integrand, respectively.

Multiplying equations (9) - (12) by $\theta^*, \phi_1^*, \phi_2^*,, \ldots, \phi_{n-1}^*$ respectively, integrating over the domain V, we get

$$-R\int \theta^*\left[\nabla^2\theta - p\theta + \vec{U}.\hat{k}\right] = 0, \tag{14}$$

$$R_1\int \phi_1^*\left[\tau_1\nabla^2\phi_1 - p\phi_1 + \vec{U}.\hat{k}\right] = 0, \tag{15}$$

$$R_2\int \phi_2^*\left[\tau_2\nabla^2\phi_2 - p\phi_2 + \vec{U}.\hat{k}\right] = 0. \tag{16}$$

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$$R_{n-1}\int \phi_{n-1}^*\left[\tau_{n-1}\nabla^2\phi_{n-1} - p\phi_{n-1} + \vec{U}.\hat{k}\right] = 0. \tag{17}$$

Now, adding equations (14) – (17) to equation (13), we have

$$\frac{p}{\sigma}\int\left(\vec{U}.\vec{U}^*\right) + \int(\mathrm{grad}P).\vec{U}^* - \int\left(\vec{U}^*.\nabla^2\vec{U}\right) - R\int \theta^*(\nabla^2 - p)\theta + R_1\int \phi_1^*\,(\tau_1\nabla^2 - p)\phi_1 + R_2\int \phi_2^*(\tau_2\nabla^2 - p)\phi_2 + \cdots + R_{n-1}\int \phi_{n-1}^*(\tau_{n-1}\nabla^2 - p)\phi_{n-1} - RI + R_1I_1 + R_2I_2 + \cdots + R_{n-1}I_{n-1} = 0, \tag{18}$$

where $I = 2Re\left[\int\left(\theta\hat{k}.\vec{U}^*\right)\right], I_1 = 2Re\left[\int\left(\phi_1\hat{k}.\vec{U}^*\right)\right], I_2 = 2Re\left[\int\left(\phi_2\hat{k}.\vec{U}^*\right)\right],$

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$I_{n-1} = 2Re\left[\int\left(\phi_{n-1}\hat{k}.\vec{U}^*\right)\right],$

and Re denotes 'the real part of'. Using Gauss' theorem and boundary conditions (7), we have

$$\int(\mathrm{grad}\,P).\vec{U}^* = \int_S P\vec{U}^*.\hat{n}dS - \int P\mathrm{div}\vec{U}^* = 0, \tag{19}$$

$$\int\left(\vec{U}^*.\nabla^2\vec{U}\right) = -\int\left(\mathrm{curlcurl}\vec{U}.\vec{U}^*\right) = -\int \mathrm{curl}\vec{U}.\mathrm{curl}\vec{U}^* - \int_S\left(\mathrm{curl}\vec{U}\right)\times\vec{U}^*.\hat{n}dS$$

$$= -\int \mathrm{curl}\vec{U}.\mathrm{curl}\vec{U}^*, \tag{20}$$

$$\int(\theta^*\nabla^2\theta) = \int_S(\theta^*\nabla\theta).\hat{n}dS - \int\nabla\theta.\nabla\theta^* = -\int\nabla\theta.\nabla\theta^*, \tag{21}$$

$$\int(\phi_1^*\nabla^2\phi_1) = \int_S(\phi_1^*\nabla\phi_1).\hat{n}dS - \int\nabla\phi_1.\nabla\phi_1^* = -\int\nabla\phi_1.\nabla\phi_1^*, \tag{22}$$

$$\int(\phi_2^*\nabla^2\phi_2) = \int_S(\phi_2^*\nabla\phi_2).\hat{n}dS - \int\nabla\phi_2.\nabla\phi_2^* = -\int\nabla\phi_2.\nabla\phi_2^*, \tag{23}$$

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$$\int(\phi_{n-1}^*\nabla^2\phi_{n-1}) = \int_S(\phi_{n-1}^*\nabla\phi_{n-1}).\hat{n} - \int\nabla\phi_{n-1}.\nabla\phi_{n-1}^* = -\int\nabla\phi_{n-1}.\nabla\phi_{n-1}^*, \tag{24}$$

where $\hat{n}$ is a unit outward drawn normal at any point on S. Using integral relations (19) – (24) in equation (18), we have

$$\frac{p}{\sigma}\int\left(\vec{U}.\vec{U}^*\right) + \int\mathrm{curl}\vec{U}.\mathrm{curl}\vec{U}^* + R\int(\nabla\theta.\nabla\theta^* + p|\theta|^2) - R_1\int(\tau_1\nabla\phi_1.\nabla\phi_1^* + p|\phi_1|^2) - R_2\int(\tau_2\nabla\phi_2.\nabla\phi_2^* + p|\phi_2|^2) - \cdots - R_{n-1}\int(\tau_{n-1}\nabla\phi_{n-1}.\nabla\phi_{n-1}^* + p|\phi_{n-1}|^2) - RI + R_1I_1 + R_2I_2 + \cdots + R_{n-1}I_{n-1} = 0. \tag{25}$$

Equating the imaginary part of equation (25) to zero and since, $p_i \neq 0$, we have

$$\frac{1}{\sigma}\int(\vec{U}.\vec{U}^*) + R\int|\theta|^2 = R_1\int|\phi_1|^2 + R_2\int|\phi_2|^2 + \cdots + R_{n-1}\int|\phi_{n-1}|^2. \qquad (26)$$

Multiplying equation (3) by ϕ_1^*, integrating over V, utilizing the relation (22), and then equating the real parts of the resulting equation, we obtain

$$\tau_1\int\nabla\phi_1.\nabla\phi_1^* + p_r\int|\phi_1|^2 = Re\left[\int(\vec{U}.\hat{k})\phi_1^*\right] \leq \int|\vec{U}.\hat{k}|\,|\phi_1|$$

$$\leq \left\{\int|\vec{U}.\hat{k}|^2\right\}^{1/2}\{\int|\phi_1|^2\}^{1/2}. \qquad (27)$$

Since $p_r \geq 0$, we have from inequality (27) that

$$\tau_1\int\nabla\phi_1.\nabla\phi_1^* \leq \left\{\int|\vec{U}.\hat{k}|^2\right\}^{1/2}\{\int|\phi_1|^2\}^{1/2},$$

which upon utilizing Poincare inequality (Joseph [6]) viz.

$$\int\nabla\phi_1.\nabla\phi_1^* \geq \frac{\Lambda}{l^2}\int|\phi_1|^2, \qquad (28)$$

yields

$$\int|\phi_1|^2 \leq \frac{l^4}{\tau_1^2\Lambda^2}\int|\vec{U}.\hat{k}|^2 \leq \frac{l^4}{\tau_1^2\Lambda^2}\int\vec{U}.\vec{U}^*. \qquad (29)$$

Using the same procedure, it follows from equations (4) and (5) that

$$\int|\phi_2|^2 \leq \frac{l^4}{\tau_2^2\Lambda^2}\int\vec{U}.\vec{U}^*, \qquad (30)$$

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$$\int|\phi_{n-1}|^2 \leq \frac{l^4}{\tau_{n-1}^2\Lambda^2}\int\vec{U}.\vec{U}^*. \qquad (31)$$

Utilizing inequalities (29)-(31) in equation (26), we have

$$\left(\frac{1}{\sigma} - \frac{R_1 l^4}{\tau_1^2\Lambda^2} - \frac{R_2 l^4}{\tau_2^2\Lambda^2} - \cdots - \frac{R_{n-1}l^4}{\tau_{n-1}^2\Lambda^2}\right)\int\vec{U}.\vec{U}^* + R\int|\theta|^2 < 0, \qquad (32)$$

which clearly implies that

$$\frac{R_1\sigma l^4}{\tau_1^2\Lambda^2} + \frac{R_2\sigma l^4}{\tau_2^2\Lambda^2} + \cdots + \frac{R_{n-1}\sigma l^4}{\tau_{n-1}^2\Lambda^2} > 1.$$

Hence, if $\frac{R_1\sigma l^4}{\tau_1^2\Lambda^2} + \frac{R_2\sigma l^4}{\tau_2^2\Lambda^2} + \cdots + \frac{R_{n-1}\sigma l^4}{\tau_{n-1}^2\Lambda^2} \leq 1$, then we must have $p_i = 0$.

This proves the theorem.

The above theorem states from the physical point of view that for the problem of multicomponent convection for the completely confined fluids, an arbitrary neutral or unstable mode of the system is definitely nonoscillatory in character and in particular the principle of the exchange of stabilities is valid if $\frac{R_1 \sigma l^4}{\tau_1^2 \Lambda^2} + \frac{R_2 \sigma l^4}{\tau_2^2 \Lambda^2} + \cdots + \frac{R_{n-1} \sigma l^4}{\tau_{n-1}^2 \Lambda^2} \leq 1.$

3. Special Cases

It follows from above theorem that an arbitrary neutral or unstable mode is non oscillatory in character and in particular PES is valid for:

1. Rayleigh - Benard convection problem in completely confined fluids ($R > 0, R_1 = R_2 = R_3 = \cdots = R_{n-1} = 0$).

   (Sherman and Ostrach [25])

2. Thermohaline convection ($R > 0, R_1 > 0, R_2 = R_3 = \cdots = R_{n-1} = 0$) if

   $\frac{R_1 \sigma l^4}{\tau_1^2 \Lambda^2} \leq 1.$

   (Gupta et al. [4])

3. Triply diffusive convection ($R > 0, R_1 > 0, R_2 > 0, R_3 = \cdots = R_{n-1} = 0$) if

   $\frac{R_1 \sigma l^4}{\tau_1^2 \Lambda^2} + \frac{R_2 \sigma l^4}{\tau_2^2 \Lambda^2} \leq 1.$

   (Prakash et al. [16])

4. Conclusion

The multicomponent instability problem for fluid completely confined in an arbitrary region with rigid bounding surfaces has been mathematically analysed. A sufficient condition for the validity of the exchange principle is derived in terms of the parameters of the system alone.

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