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COSMOLOGICAL MODELS FOR RADIATION DOMINATED PHASE WITH VACUUM ENERGY DENSITY IN FRW SPACE-TIME

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Abstract: Cosmological models for radiation dominated phase with vacuum energy density in the frame work of FRW space-time, are investigated. To get the deterministic result, we have assumed that vacuum energy density $\Lambda \sim R^{-2}$ as considered by Chen and Wu [6] where R is scale factor. The reality condition $\rho > 0$ is satisfied for the models and in special case $\Lambda \sim \frac{1}{t^2}$ leads to the result as obtained by Beesham [5]. The first model represents decelerating phase while in the second model, the particle moves with constant velocity. The other physical aspects are also discussed.

Keywords: Cosmological, radiation phase, vacuum energy.

1. Introduction

The radiation was the dominant influence on the expansion of the universe in early stages of the universe because most of the energy was in the form of radiation. Thus, the examination of the radiation dominated expansion is essential for understanding the early universe. To understand the radiation dominated phase of the universe, it is interesting to know about Microwave Background Radiation (CMBR) and abundance of light nuclei in the early universe (Gamow [9]).

The turning point for cosmology came with the discovery of the microwave background radiation (Penzias and Wilson [13]). It confirmed the early universe scenario and taken together with the extended velocity of Hubble's law.

The cosmic microwave background is considered to be left over radiation from the big-bang or the time when the universe began (Space.Com). The case of radiation dominated universe differs from the matter dominated one where the pressure (p) of radiation makes a significant contribution to gravitational attraction and in this case we have $\rho = 3p$ as mentioned by Narlikar [12] where the speed of light in gravitational unit is considered as $c = 1$.

Einstein added cosmological constant Λ (now vacuum energy density) into his field equations for the study of static universe because at that time universe was assumed as static, homogeneous and isotropic. But Friedmann [8] pointed out that there is no need to add cosmological constant into Einstein field equation for non-static universes because he obtained non-static cosmological model without Λ-term for dust distribution. When Hubble's observations established the expanding picture of the universe, Einstein conceded that there is no special need for Λ-term into his gravitational field equations. Nevertheless, famous cosmologists Eddington [7] and Lemaitre [11] felt that Λ-term introduced into the field equation should have certain attractive feature into cosmology and models based on it should be discussed at length. Zel'dovich [18] in his most innovative way revived the issue of cosmological constant (Λ) by identifying it with the vacuum energy density due to quantum fluctuations. Therefore, people started thinking about cosmological constant (Λ) once again with a new outlook. In 1998 and 1999, two independent groups led by Riess et al. [16] and Perlmutter et al. [14] used Type Ia supernovae showed that universe is accelerating. This discovery provided the first direct evidence that Λ is non-zero with $\Lambda \sim 1.7 \times 10^{-121}$ Planck units. It is now commonly believed by Scientific Community that the cosmological constant – a kind of repulsive pressure dubbed as dark energy, is the most suitable candidate to explain recent observations of the universe that universe is not only expanding but also accelerating. A number of cosmological models with vacuum energy density have been investigated by many authors viz. Beesham [5], Sahni and Starobinsky [17], Bali et al. [1,2,3], Ram and Verma [15], Barrow and Shaw [4].

In this paper, cosmological models for radiation dominated phase with decaying vacuum energy density in the frame work of FRW space-time are investigated. To get the deterministic result, we have assumed that the vacuum energy $\Lambda \sim R^{-2}$ as considered by Chen and Wu [6] where R is scale factor. The physical aspects of the models are also discussed.

2. Metric and Field Equations

We consider FRW metric in the form as

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right] \tag{1}$$

where k = 0, −1,1.

Einstein field equation with time dependent vacuum energy density (Λ) is given by

$$R_i^j - \frac{1}{2}\bar{R}\, g_i^j = -8\pi T_i^j - \Lambda(t)\, g_i^j \tag{2}$$

(in the gravitational unit G = 1, c = 1) where R_{ij} is the Ricci tensor, $\overline{R} = g^{ij} R_{ij}$ (scalar curvature which measures the curvature of space). The energy-momentum tensor T_i^j for perfect fluid is given by

$$T_i^j = (\rho + p) v_i v^i - p g_i^j \tag{3}$$

where p being the isotropic pressure and ρ the matter density. We assume the coordinates to be commoving so that $v^1 = 0 = v^2 = v^3, v^4 = 1.$

The Einstein field equation (2) for the metric (1) leads to

$$\frac{3R_4^2}{R^2} + \frac{3k}{R^2} = 8\pi\rho + \Lambda(t) \tag{4}$$

and

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} + \frac{k}{R^2} = -8\pi p + \Lambda(t) \tag{5}$$

The conservation equation

$$(8\pi T_i^j + \Lambda g_i^j)_{;j} = 0$$

leads to

$$8\pi\left[\rho_4 + 3(\rho + p)\frac{R_4}{R}\right] + \Lambda_4 = 0 \tag{6}$$

where the suffix 4 indicates partial derivative with respect to time t.

3. Solution of Field Equations

To get the deterministic results, we assume that the universe represents radiation dominated phase for which we have

$$\rho = 3p \tag{7}$$

and $\Lambda = \ell / R^2$ as considered by Chen and Wu [6]. Equation (7) leads to

$$\frac{2R_{44}}{R} + \frac{2R_4^2}{R^2} = -\frac{2k}{R^2} + \frac{4\ell}{3R^2} \tag{8}$$

where ℓ is proportionality constant.

Equation (8) leads to

$$2R_{44} + \frac{2}{R} R_4^2 = \frac{2(2\ell - 3k)}{3R} \tag{9}$$

From equation (9), we have

$$f^2 = \left(\frac{dR}{dt} \right)^2 = \frac{\alpha R^2 + \beta}{R^2} \tag{10}$$

where $\alpha = \frac{2\ell - 3k}{3}, R_4 = f(R)$ and β is the constant of integration.

Equation (10) leads to

$$R^2 = (at + b)^2 - L^2 \tag{11}$$

where $\sqrt{\alpha} = a, L^2 = \frac{\beta}{\alpha}$.

Therefore, the metric (1) leads to the form

$$ds^2 = dt^2 - [(at + b)^2 - L^2] \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right] \tag{12}$$

In particular, if we take a = 1, b = 0, L = 0, the metric (12) takes the form

$$ds^2 = dt^2 - t^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \tag{13}$$

3. Physical and Geometrical Aspects

The matter density (ρ) and isotropic pressure (p) for the model (12) are given by

$$8\pi\rho = \frac{3a^2 (at + b)^2}{[(at + b)^2 - L^2]^2} + \frac{3k - \ell}{(at + b)^2 - L^2} \tag{14}$$

$$8\pi p = \frac{8\pi\rho}{3} \tag{15}$$

The Hubble parameter (H), the expansion (θ), the spatial volume (R^3), the deceleration parameter (q), the vacuum energy density (Λ) for the model (12) are given by

$$H = \frac{R_4}{R} = \frac{a(at + b)}{(at + b)^2 - L^2} \tag{16}$$

$$\theta = 3H = \frac{3a(at+b)}{(at+b)^2 - L^2} \tag{17}$$

$$R^3 = [(at+b)^2 - L^2]^{3/2} \tag{18}$$

$$q = \frac{a^2 L^2}{\left[(at+b)^2 - L^2\right]^2} \tag{19}$$

$$\Lambda = \frac{\ell}{(at+b)^2 - L^2} \tag{20}$$

The above mentioned quantities for a = 1, b = 0, L = 0 for the model (13) lead to

$$8\pi\rho = \frac{3}{t^2} + \frac{3k - \ell}{t^2} \tag{21}$$

$$8\pi p = \frac{8\pi\rho}{3} \tag{22}$$

$$H = \frac{1}{t} \tag{23}$$

$$\theta = \frac{3}{t} \tag{24}$$

$$R^3 = t^3 \tag{25}$$

$$q = 0 \tag{26}$$

$$\Lambda = \frac{\ell}{t^2} \tag{27}$$

5. Conclusion

The reality condition $\rho > 0$ for the model (12) leads to

$$3a^2(at+b)^2 > (\ell - 3k)[(at+b)^2 - L^2]$$

which is satisfied for k = 0, 1, -1. The model (12) starts with a bing at $t = \frac{L-b}{a}$ and continues to expand while the model (13) starts expanding at t = 0 and expansion stops at t = ∞. The spatial volume increases with time for both the models. The model (12) represents decelerating phase of universe while in the model (13), the particle moves with constant velocity. The vacuum energy density (Λ) decreases with time in the model (12) while the model (13) for a = 1, b = 0, L = 0, leads to $\Lambda \sim \frac{1}{t^2}$ which matches with the result as obtained by Beesham [5].

The special cases for k = 0, 1, -1 can also be discussed very easily.

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